Asia Mathematika
Volume: 3 Issue: 3 , (2019) Pages: 16 – 22
Available online at **www.asiamath.org**

$\widetilde{G}\alpha$-CLOSED SETS IN TOPOLOGICAL SPACES

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Received: 22 Sep 2019 • **Accepted:** 05 Dec 2019 • **Published Online:** 31 Dec 2019

Abstract: In this paper, we introduce the notion of $\widetilde{g}\alpha$-*closed* sets in topological spaces and investigate some of their basic properties.

Key words: $\widetilde{g}\alpha$-*closed* set and $^\sharp gs$-*closed* set.

1. Introduction and Preliminaries

Levine [6,7] introduced the concept of *generalized closed* sets and *semi-closed* sets in topological spaces. Maki et al. introduced *generalized* α-*closed* sets (briefly $g\alpha$-*closed* sets) [9] and α-*generalized closed* sets (briefly αg-closed sets) [8]. The concept of $\widehat{g}$-*closed* sets [16,17], $^*g$-*closed* sets [14] and $^\sharp gs$-*closed* sets [15] are introduced by M.K.R.S. Veera Kumar. In this paper, we introduce a new class of sets, namely, $\widetilde{g}\alpha$-*closed* sets and present some of its properties.

Throughout this paper (X,τ), (Y,σ) and (Z,η) represent topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space (X,τ), $cl(A)$ and $int(A)$ denote the closure of A and the interior of A, respectively. $P(X)$ denotes the power set of X.

We recall the following definitions which are useful in the sequel.

Definition 1.1. A subset A of a space (X,τ) is called

1. a *pre-open* set [10] if $A \subseteq int(cl(A))$ and a *pre-closed* set if $cl(int(A)) \subseteq A$,
2. a *semi-open* set [7] if $A \subseteq cl(int(A))$ and a *semi-closed* set [7] if $int(cl(A)) \subseteq A$,
3. an α-*open* set [11] if $A \subseteq int(cl(int(A)))$ and an α-*closed* set [11] if $cl(int(cl(A))) \subseteq A$,
4. a *semi-preopen* set [1] if $A \subseteq cl(int(cl(A)))$ and a *semi-preclosed* set [1] if $int(cl(int(A))) \subseteq A$ and
5. a *regular open* set if $A = int(cl(A))$ and a *regular closed* set if $cl(int(A)) = A$.

The pre-closure (resp. semi-closure, α-closure, semi-preclosure) of a subset A of a space (X,τ) is the intersection of all *pre-closed* (resp. *semi-closed*, α-*closed*, *semi-preclosed*) sets that contain A and is denoted by $pcl(A)$ (resp. $scl(A)$, $\alpha cl(A)$, $spcl(A)$).

Definition 1.2. A subset A of a space (X,τ) is called a

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1. a *generalized closed* (briefly *g-closed*) set [6] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is *open* in (X, τ); the complement of a *g-closed* set is called a *g-open* set,
2. a *semi-generalized closed* (briefly *sg-closed*) set [3] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is *semi-open* in (X, τ),
3. a *generalized semi-closed* (briefly *gs-closed*) set [2] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is *open* in (X, τ),
4. an *α-generalized closed* (briefly *αg-closed*) set [8] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is *open* in (X, τ),
5. a *generalized α-closed* (briefly *$g\alpha$-closed*) set [9] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is *α-open* in (X, τ),
6. a *$g\alpha^*$-closed* set [9] if $\alpha cl(A) \subseteq int(U)$ whenever $A \subseteq U$ and U is *α-open* in (X, τ),
7. a *generalized semi-preclosed* (briefly *gsp-closed*) set [4] if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is *open* in (X, τ),
8. a *generalized preregular-closed* (briefly *gpr-closed*) set [5] if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is *regular open* in (X, τ),
9. a *g^*-closed* set [13] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is *g-open* in (X, τ),
10. a *$\widehat{g}$-closed* set [16,17] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is *semi-open* in (X, τ); the complement of a *$\widehat{g}$-closed* set is called a *$\widehat{g}$-open* set,
11. a *${}^*g$-closed* set [14] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is *$\widehat{g}$-open* in (X, τ); the complement of a *${}^*g$-closed* set is called a *${}^*g$-open* set,
12. a *${}^\sharp gs$-closed* set [15] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is *${}^*g$-open* in (X, τ); the complement of a *${}^\sharp gs$-closed* set is called a *${}^\sharp gs$-open* set and
13. a *$\widetilde{g}s$-closed* set [12] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is *${}^\sharp gs$-open* in (X, τ).

Notation 1.3. For a topological space (X, τ), $C(X, \tau)$ (resp. $\alpha C(X, \tau)$, $GC(X, \tau)$, $SGC(X, \tau)$, $GSC(X, \tau)$, $\alpha GC(X, \tau)$, $G\alpha C(X, \tau)$, $G\alpha^* C(X, \tau)$, $GSPC(X, \tau)$, $GPRC(X, \tau)$, $G^*C(X, \tau)$, ${}^*GC(X, \tau)$, ${}^\sharp GSC(X, \tau)$, $\widetilde{G}SC(X, \tau)$) denotes the class of all *closed* (resp. *α-closed*, *g-closed*, *sg-closed*, *gs-closed*, *αg-closed*, *$g\alpha$-closed*, *$g\alpha^*$-closed*, *gsp-closed*, *gpr-closed*, *g^*-closed*, *${}^*g$-closed*, *${}^\sharp gs$-closed*, *$\widetilde{g}s$-closed*) subsets of (X, τ).

2. $\widetilde{g}\alpha$-*closed* sets

We introduce the following definition.

Definition 2.1. A subset A of (X, τ) is called a *$\widetilde{g}\alpha$-closed* set if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is *${}^\sharp gs$-open* in (X, τ).

Theorem 2.2. *Every α-closed set is a $\widetilde{g}\alpha$-closed set and thus every closed set is $\widetilde{g}\alpha$-closed.*

Proof. Let A be an α-*closed* set in (X,τ), then $A = \alpha cl(A)$. Let $A \subseteq U$ such that U is $^{\sharp}gs$-*open* in (X,τ). Since A is α-*closed*, $A = \alpha cl(A) \subseteq U$. This shows that A is $\widetilde{g}\alpha$-*closed* set. The second part of the theorem follows from the fact that every *closed* set is α-*closed*.

The converse of Theorem 2.2 is not true as it can be seen by the following example.

Example 2.3. Let $X = \{a,b,c\}$ and $\tau = \{X,\phi,\{a,b\}\}$. Here $\alpha C(X,\tau) = \{X,\phi,\{c\}\}$ and $\widetilde{G}\alpha C(X,\tau) = \{X,\phi,\{c\},\{b,c\},\{a,c\}\}$ and let $A = \{b,c\}$. Then A is not an α-*closed* and thus it is not *closed*. However A is a $\widetilde{g}\alpha$-*closed* set.

Thus the class of $\widetilde{g}\alpha$-*closed* sets properly contains the classes of α-*closed* sets and *closed* sets.

Theorem 2.4.

(a) *Every $\widetilde{g}\alpha$-closed set is a gs-closed set and thus gsp-closed and gpr-closed.*

(b) *Every $\widetilde{g}\alpha$-closed set is a $g\alpha$-closed set and thus αg-closed.*

(c) *Every $\widetilde{g}\alpha$-closed set is a sg-closed set and thus semi-preclosed.*

Proof. It follows from the definitions.

The following examples show that these implications are not reversible.

Example 2.5. Let $X = \{a,b,c\}$ and $\tau = \{X,\phi,\{a\},\{b,c\}\}$. Here $GSC(X,\tau) = P(X)$, $GSPC(X,\tau) = P(X)$, $GPRC(X,\tau) = P(X)$ and $\widetilde{G}\alpha C(X,\tau) = \{X,\phi,\{a\},\{b,c\}\}$ and let $A = \{b\}$. Then A is *gs-closed*, *gsp-closed* and *gpr-closed*. However A is not a $\widetilde{g}\alpha$-*closed* set.

Example 2.6. Let $X = \{a,b,c\}$ and $\tau = \{X,\phi,\{b\},\{b,c\}\}$. Here $G\alpha C(X,\tau) = \{X,\phi,\{a\},\{c\},\{a,b\},\{a,c\}\}$, $\alpha GC(X,\tau) = \{X,\phi,\{a\},\{c\},\{a,b\},\{a,c\}\}$ and $\widetilde{G}\alpha C(X,\tau) = \{X,\phi,\{a\},\{c\},\{a,c\}\}$ and let $A = \{a,b\}$. Then A is $g\alpha$-*closed* and αg-*closed*. However A is not a $\widetilde{g}\alpha$-*closed* set.

Example 2.7. Let $X = \{a,b,c\}$ and $\tau = \{X,\phi,\{a\},\{b\},\{a,b\}\}$. Here $SGC(X,\tau) = \{X,\phi,\{a\},\{b\},\{c\},\{b,c\},\{a,c\}\}$, $SPC(X,\tau) = \{X,\phi,\{a\},\{b\},\{c\},\{b,c\},\{a,c\}\}$ and $\widetilde{G}\alpha C(X,\tau) = \{X,\phi,\{c\},\{b,c\},\{a,c\}\}$ and let $A = \{a\}$. Then A is *sg-closed* and *semi-preclosed*. However A is not a $\widetilde{g}\alpha$-*closed* set.

Theorem 2.8. *Every $\widetilde{g}\alpha$-closed set is $\widetilde{g}s$-closed set.*

Proof. It follows from the definitions.

The converse of Theorem 2.8 need not be true by the following example.

Example 2.9. Let $X = \{a,b,c\}$ and $\tau = \{X,\phi,\{a\}\}$. Here $\widetilde{G}SC(X,\tau) = \{X,\phi,\{a\},\{b\},\{c\},\{a,b\},\{b,c\}\}$, $\widetilde{G}\alpha C(X,\tau) = \{X,\phi,\{b\},\{c\},\{b,c\}\}$. Let $A = \{a\}$. Then A is $\widetilde{g}s$-*closed* but not a $\widetilde{g}\alpha$-*closed* set.

Theorem 2.10.

(a) $\widetilde{g}\alpha$*-closedness is independent of g-closedness, g^*-closedness and $^*g$-closedness.*

(b) $\widetilde{g}\alpha$*-closedness is independent of $\widehat{g}$-closedness.*

(c) $\widetilde{g}\alpha$*-closedness is independent of $g\alpha^*$-closedness.*

Proof. It follows from the following examples.

Example 2.11. Let $X = \{a, b, c\}$ and $\tau = \{X, \phi, \{a\}, \{a, c\}\}$. Here $GC(X, \tau) = \{X, \phi, \{b\}, \{a, b\}, \{b, c\}\}$, $G^*C(X, \tau) = \{X, \phi, \{b\}, \{a, b\}, \{b, c\}\}$, $^*GC(X, \tau) = \{X, \phi, \{b\}, \{a, b\}, \{b, c\}\}$ and $\widetilde{G}\alpha C(X, \tau) = \{X, \phi, \{b\}.\{c\}, \{b, c\}\}$. Then $\{a, b\}$ is *g-closed*, g^**-closed* and $^*g$*-closed*, but not $\widetilde{g}\alpha$*-closed* set and also $\{c\}$ is $\widetilde{g}\alpha$*-closed*, but not even a *g-closed*, g^**-closed* and $^*g$*-closed*.

Example 2.12. Let $X = \{a, b, c\}$ and $\tau = \{X, \phi, \{a\}, \{a, c\}\}$. Here $\widehat{G}C(X, \tau) = \{X, \phi, \{b\}, \{b, c\}\}$ and $\widetilde{G}\alpha C(X, \tau) = \{X, \phi, \{b\}, \{c\}, \{b, c\}\}$. Then $\{c\}$ is $\widetilde{g}\alpha$*-closed*, but not a $\widehat{g}$*-closed* set.

Let $X = \{a, b, c\}$ and $\tau = \{X, \phi, \{a\}, \{b, c\}\}$. Here $\widehat{G}C(X, \tau) = P(X)$ and $\widetilde{G}\alpha C(X, \tau) = \{X, \phi, \{a\}, \{b, c\}\}$. Then $\{b\}$ is $\widehat{g}$*-closed*, but not a $\widetilde{g}\alpha$*-closed* set.

Example 2.13. Let $X = \{a, b, c\}$ and $\tau = \{X, \phi, \{a\}, \{b, c\}\}$. Here $G\alpha^*C(X, \tau) = P(X)$ and $\widetilde{G}\alpha C(X, \tau) = \{X, \phi, \{a\}, \{b, c\}\}$. Then $\{b\}$ is $g\alpha^*$*-closed*, but not a $\widetilde{g}\alpha$*-closed* set.

Let $X = \{a, b, c\}$ and $\tau = \{X, \phi, \{a\}\}$. Here $G\alpha^*C(X, \tau) = \{X, \phi, \{b, c\}\}$ and $\widetilde{G}\alpha C(X, \tau) = \{X, \phi, \{b\}, \{c\}, \{b, c\}\}$. Then $\{b\}$ is $\widetilde{g}\alpha$*-closed*, but not a $g\alpha^*$*-closed* set.

Theorem 2.14. *Let A be a subset of (X, τ).*

(a) *If A is $\widetilde{g}\alpha$-closed, then $\alpha cl(A) - A$ does not contain any non-empty $^\sharp gs$-closed set.*

(b) *If A is $\widetilde{g}\alpha$-closed and $A \subseteq B \subseteq \alpha cl(A)$, then B is $\widetilde{g}\alpha$-closed.*

Proof.

(a) Suppose that A is $\widetilde{g}\alpha$*-closed* and let F be a non-empty $^\sharp gs$*-closed* set with $F \subseteq \alpha cl(A) - A$. Then $A \subseteq X - F$ and so $\alpha cl(A) \subseteq X - F$. Hence $F \subseteq X - \alpha cl(A)$, a contradiction.

(b) Let U be a $^\sharp gs$*-open* set of (X, τ) such that $B \subseteq U$. Then $A \subseteq U$. Since A is $\widetilde{g}\alpha$*-closed*, $\alpha cl(A) \subseteq U$. Now $\alpha cl(B) \subseteq \alpha cl(\alpha cl(A)) \subseteq U$. Therefore B is also a $\widetilde{g}\alpha$*-closed* set of (X, τ).

Theorem 2.15. *Let A and B be subsets of a topological space (X, τ). Then the union of two $\widetilde{g}\alpha$-closed set is $\widetilde{g}\alpha$-closed set in (X, τ).*

Proof. Let A and B be $\widetilde{g}\alpha$*-closed* sets. Let $A \cup B \subseteq U$ such that U is $^\sharp gs$*-open*. Since A and B are $\widetilde{g}\alpha$*-closed* sets, $\alpha cl(A) \subseteq U$ and $\alpha cl(B) \subseteq U$. This implies that $\alpha cl(A \cup B) = \alpha cl(A) \cup \alpha cl(B) \subseteq U$, (since $\tau^\alpha = \alpha$*-open* set forms a topology [9]) and so $\alpha cl(A \cup B) \subseteq U$. Therefore $A \cup B$ is $\widetilde{g}\alpha$*-closed*.

We need the following notations:

For a subset E of a space (X,τ), we define the following subsets of E.
$E_\tau = \{x \in E/\{x\} \in \tau\}$;
$E_{\mathcal{F}} = \{x \in E/\{x\}$ is *closed* in $(X,\tau)\}$;
$E_{\widetilde{g}\alpha o} = \{x \in E/\{x\}$ is $\widetilde{g}\alpha$-*open* in $(X,\tau)\}$;
$E_{\sharp gsc} = \{x \in E/\{x\}$ is $^\sharp gs$-*closed* in $(X,\tau)\}$.

Lemma 2.16. *For any space* (X,τ), $X = X_{\sharp gsc} \cup X_{\widetilde{g}\alpha o}$ *holds.*

Proof. Let $x \in X$. Suppose that $\{x\}$ is not $^\sharp gs$-*closed* set in (X,τ). Then X is a unique $^\sharp gs$-*open* set containing $X-\{x\}$. Thus $X-\{x\}$ is $\widetilde{g}\alpha$-*closed* in (X,τ) and so $\{x\}$ is $\widetilde{g}\alpha$-*open*. Therefore $x \in X_{\sharp gsc} \cup X_{\widetilde{g}\alpha o}$ holds.

We need more notations:
For a subset A of (X,τ), $ker(A) = \cap\{U/U \in \tau$ and $A \subseteq U\}$;
$^\sharp GSO\text{-}ker(A) = \cap\{U/U \in {}^\sharp GSO(X,\tau)$ and $A \subseteq U\}$.

Theorem 2.17. *For a subset* A *of* (X,τ), *the following conditions are equivalent.*

(1) A *is* $\widetilde{g}\alpha$-*closed in* (X,τ).

(2) $\alpha cl(A) \subseteq {}^\sharp GSO\text{-}ker(A)$ *holds.*

(3) (i) $\alpha cl(A) \cap X_{\sharp gsc} \subseteq A$ *and (ii)* $\alpha cl(A) \cap X_{\sharp gso} \subseteq {}^\sharp GSO\text{-}ker(A)$ *holds.*

Proof.
(1) ⇒ (2) Let $x \notin {}^\sharp GSO\text{-}ker(A)$. Then there exists a set $U \in {}^\sharp GSO(X,\tau)$ such that $x \notin U$ and $A \subseteq U$. Since A is $\widetilde{g}\alpha$-*closed*, $\alpha cl(A) \subseteq U$ and so $x \notin \alpha cl(A)$. This shows that $\alpha cl(A) \subseteq {}^\sharp GSO\text{-}ker(A)$.
(2) ⇒ (1) Let $U \in {}^\sharp GSO(X,\tau)$ such that $A \subseteq U$. Then we have that $^\sharp GSO\text{-}ker(A) \subseteq U$ and so by (2) $\alpha cl(A) \subseteq U$. Therefore A is $\widetilde{g}\alpha$-*closed*.
(2) ⇒ (3) (i) First we claim that $^\sharp GSO\text{-}ker(A) \cap X_{\sharp gsc} \subseteq A$. Indeed, let $x \in {}^\sharp GSO\text{-}ker(A) \cap X_{\sharp gsc}$ and assume that $x \notin A$. Since the set $X-\{x\} \in {}^\sharp GSO(X,\tau)$ and $A \subseteq X-\{x\}$, $^\sharp GSO\text{-}ker(A) \subseteq X-\{x\}$. Then we have that $x \in X-\{x\}$ and so this is a contradiction. Thus we show that $^\sharp GSO\text{-}ker(A) \cap X_{\sharp gsc} \subseteq A$. By using (2), $\alpha cl(A) \cap X_{\sharp gsc} \subseteq {}^\sharp GSO\text{-}ker(A) \cap X_{\sharp gsc} \subseteq A$.
(ii) It is obtained by (2).
(3) ⇒ (2) By lemma 2.16 and (3),

$$\begin{aligned} \alpha cl(A) &= \alpha cl(A) \cap X = \alpha cl(A) \cap (X_{\sharp gsc} \cup X_{\widetilde{g}\alpha o}) \\ &= (\alpha cl(A) \cap X_{\sharp gsc}) \cup (\alpha cl(A) \cap X_{\widetilde{g}\alpha o}) \\ &= A \cup {}^\sharp GSO\text{-}ker(A) \\ &= {}^\sharp GSO\text{-}ker(A) \text{ holds.} \end{aligned}$$

Theorem 2.18. Let (X,τ) be a space and A and B are subsets.

(i) If A is $^\sharp gs$-*open* and $\widetilde{g}\alpha$-*closed*, then A is α-*closed* in (X,τ).

(ii) Suppose that (X,τ) is an α-space. A $\widetilde{g}\alpha$-*closed* set A is α-*closed* in (X,τ) if and only if $\alpha cl(A)-A$ is α-*closed* in (X,τ).

(iii) For each $x\in X$, $\{x\}$ is $^\sharp gs$-*closed* or $X-\{x\}$ is $\widetilde{g}\alpha$-*closed* in (X,τ).

(iv) Every subset is $\widetilde{g}\alpha$-*closed* in (X,τ) if and only if $^\sharp gs$-*open* set is α-*closed*.

Proof.

(ii) (Necessity) If A is α-*closed*, then $\alpha cl(A)-A=\phi$.
(Sufficiency) Suppose that A is $\widetilde{g}\alpha$-*closed* and $\alpha cl(A)-A$ is α-*closed*. It follows from assumptions that $\tau=\tau^{\alpha}$. Then, $\alpha cl(A)-A$ is $^\sharp gs$-*closed* in (X,τ) and by Theorem 2.14., $\alpha cl(A)-A=\phi$. Therefore A is α-*closed* in (X,τ).

(iii) If $\{x\}$ is not $^\sharp gs$-*closed*, then $X-\{x\}$ is not $^\sharp gs$-*open*. Therefore $X-\{x\}$ is $\widetilde{g}\alpha$-*closed* in (X,τ).

(iv) (Necessity) Let U be a $^\sharp gs$-*open* set. Then we have that $\alpha cl(U)\subseteq U$ and hence U is α-*closed*.
(Sufficiency) Let A be a subset and U is a $^\sharp gs$-*open* set such that $A\subseteq U$. Then $\alpha cl(A)\subseteq \alpha cl(U)=U$ and hence A is $\widetilde{g}\alpha$-*closed*.

Remark 2.19. The following diagram shows the relationships established between $\widetilde{g}\alpha$-*closed* sets and some other sets. $A\to B$ represents A implies B but not conversely.

$$\begin{array}{ccccc}
\alpha-closed & & gs-closed & & g\alpha-closed \\
 & \searrow & \uparrow & \nearrow & \\
closed & \longrightarrow & \widetilde{g}\alpha-closed & \longrightarrow & gsp-closed \\
 & \swarrow & \downarrow & \searrow & \\
semi-preclosed & & sg-closed & & gpr-closed
\end{array}$$

References

[1] Andrijevic, D., *Semi-preopen* sets, *Mat. vesnik*, 38(1)(1986), 24-32.

[2] Arya, S. P. and Nour, T., Characterizations of s-normal spaces, *Indian J.Pure.Appl.math.*, 21(8)(1990), 717-719.

[3] Bhattacharya, P. and Lahiri, B. K., *Semi-generalized closed* sets in topology, *Indian J. Math.*, 29(3)(1987) 375-382.

[4] Dontchev, J., On generalizing *semi-preopen* sets, *Mem. Fac. Sci. Kochi Univ. Ser. A. Math.*, 16(1995), 35-48.

[5] Gnanambal, Y., On *generalized preregular closed* sets in topological spaces, *Indian J. Pure Appl. Math.*, 28(3)(1997), 351-360.

[6] Levine, N., *Generalized closed* sets in topology, *Rend. Circ. Math. Palermo*, 19(2)(1970),89-96.

[7] Levine, N., *Semi-open* sets and *semi-continuity* in topological spaces, *Amer. Math. Monthly*, 70(1963), 36-41.

[8] Maki, H., Devi, R. and Balachandran, K., Associated topologies of *generalized α-closed* sets and *α-generalized closed* sets, *Mem. Fac. Sci. Kochi. Univ. Ser.A Math.*, 15(1994), 51-63.

[9] Maki, H., Devi, R. and Balachandran, K., *Generalized α-closed* sets in topology, *Bull. Fukuoka Univ. Ed. Part III*, 42(1993), 13-21.

[10] Mashhour, A. S., Abd. El-Monsef, M. E. and El-Deeb, S. N., On *pre-continuous* and weak *pre-continuous* mappings, *Proc. Math. and Phys. Soc. Egypt*, 53(1982), 47-53.

[11] Njastad, O., On some classes of nearly *open* sets, *Pacific J. Math.*, 15(1965), 961- 970.

[12] Sundaram, P., Rajesh, N. and Dusynski, Z., $\widetilde{g}$-*semi-closed* sets in topological spaces (submitted)

[13] Veera kumar, M. K. R. S., Between *closed* sets and *g-closed* sets, *Mem. Fac. Sci. Kochi Univ. Ser. A. Math.*, 21(2000), 1-19.

[14] Veera kumar, M. K. R. S., Between g^*-*closed* sets and *g-closed* sets, *Antartica J. Math.*, 3(1)(2006), 43-65.

[15] Veera Kumar, M. K. R. S., $^\sharp g$-*semi-closed* sets in topological spaces, *Antartica J. Math.*, 2:2(2005), 201-222.

[16] Veera Kumar, M. K. R. S., $\widehat{g}$-locally *closed* sets and $\widehat{GLC}$-functions, *Indian J. Math.*, 43(2)(2001), 231-247.

[17] Veera Kumar, M. K. R. S.,On $\widehat{g}$-*closed* sets in topological spaces, *Allahabad Math. Soc.*, 18(2003), 99-112.

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