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An updated survey on distance-based domination parameters in graphs

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Abstract: A subset D of a graph $G = (V, E)$ is said to be a dominating set if every vertex not in D is adjacent to at least one vertex in D. The minimum cardinality of a dominating set of G is the domination number $\gamma(G)$ of G. A set $D \subseteq V(G)$ is a distance k-dominating set of G, if for every $v \in V \setminus D$ there exists $u \in D$ such that $d(u, v) \leq k$. The distance k-domination number $\gamma_{\leq k}(G)$ is the minimum cardinality of a distance k-dominating set of G. In this survey, we present recent results on distance based dominating sets of graphs.

Key words: Dominating sets, distance, distance dominating sets.

1. Introduction

Domination is one of the interesting and emerging areas in Graph Theory which is widely applied to solve several problems arising in various branches of science and engineering such as computer communication networks, inter connection networks, Mobile Ad-Hoc Network(MANET) etc., Since its evolution, many categories of domination have been introduced. Let $G = (V(G), E(G))$ be a simple, undirected and non-trivial graph with a Vertex set $V(G)$ whose members are called as vertices(or a point, or a node), and a Edge set $E(G)$ whose members are unordered pair of vertices called edges(or a line, or a link) of G.

The distance $d(u, v)$ between two vertices u and v is defined to be length of the shortest $(u - v)$ path(a finite alternating sequence of vertices and edges whose origin is u and terminus is v with no internal vertices being repeated) in G. The open k-neighborhood of v is $N_k(v) = \{u \in V(G)/d(u, v) \leq k\}$ and the closed k-neighborhood of v is $N_k[v] = \{v\} \cup N_k(v)$. The eccentricity of a vertex v is defined to be $e(v) = max\{d(u, v) : u \in V\}$, whereas the radius of G is $rad(G) = min\{e(v) : v \in V\}$ and the diameter of G is $diam(G) = max\{e(v) : v \in V\}$. For notation and graph theoretic terminology not defined here, we generally follow [5].

A subset $D \subseteq V(G)$ is a dominating set if every vertex not in D is adjacent to at least one vertex in D. The domination number of G is the minimum cardinality of a minimal dominating set which is denoted by $\gamma(G)$. Several domination parameters are explored with reference to graph theoretical features like cycle, independence, clique etc.

Distance based domination is one such concept in theory of domination in graphs whose applications to

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problems involving the placement of a minimum number of objects (hospitals, fire stations, post offices, police stations, warehouses, service centers etc.,) within acceptable distances of a given population, or conversely, the placement of undesirable objects (e.g., toxic wastes, nuclear , airports etc.,) at maximum distances from a given population.

In 1975, Meir and Moon[25] introduced the notion of (distance) k-dominating set. The problem of finding a minimum distance k-dominating set was discussed by P.J. Slater[45] in 1976 while the term distance k-dominating set was introduced formally by Henning et al. [14] in 1991 as: A subset $D \subseteq V$ of vertices is said to be a distance k-dominating set of G if for each vertex v not in D there is at least one vertex u in D whose distance is at most k from v in G. The minimum cardinality of a distance k-dominating set in G is the distance k-domination number of G is denoted by $\gamma_{\leq k}(G)$
.

In [14], the authors used the notation $\gamma_k(G)$ to denote the distance k-domination number of a graph G. Since there is another domination parameter called k-domination number of a graph bearing the same notation in the literature, let us denote the distance-k domination number of G by $\gamma_{\leq k}$ for the sake of clarity.

For a detailed survey on various domination parameters, refer [11, 12]. A recent book on "Topics in Domination in Graphs" by Haynes et al. [13] is yet to be launched in 2020. In this survey, we have carefully excluded those results to be appeared in [13].

2. Distance domination in graphs- A survey of selected recent results

In this survey, we focus on distance based domination and present here several recent results as much as possible. For further details about these domination parameters, one may refer the respective articles listed in reference section. Please note that the present survey is done based on recent publications and based on some new results which have not been discussed in the existing surveys([11, 12]) on domination in graphs.

2.1. Distance-k dominating sets

First, we present some selected results on distance k-domination in graphs.

Vaidya et al. [47] had investigated distance-k domination number of total graph, shadow graph and middle graph of path P_n.

Theorem 2.1. *[47] For $n > 2k$, then*

$$\gamma_{\leq k}(T(P_n)) = \begin{cases} \lfloor \frac{n}{2k} \rfloor, & n \equiv 0, 1 \ (mod\ 2k); \\ \lfloor \frac{n}{2k} \rfloor + 1, & n \equiv 2, 3, \dots 2k-1 \ (mod\ 2k). \end{cases}$$

For $n \leq 2k$, then $\gamma_{\leq k}(T(P_n)) = 1$.

Theorem 2.2. *[47] For $n > 2k+1$, then*

$$\gamma_{\leq k}(D_2(P_n)) = \begin{cases} \lfloor \frac{n}{2k+1} \rfloor + 1, & n \equiv 1, 2, \dots 2k \ (mod(2k+1)); \\ \frac{n}{2k+1}, & n \equiv 0 \ (mod(2k+1)). \end{cases}$$

For $n \leq 2k+1$, then $\gamma_{\leq k}(D_2(P_n)) = 1$.

Theorem 2.3. *[47] For $n > 2k$, then*

$$\gamma_{\leq k}(M(P_n)) = \begin{cases} \lfloor \frac{n}{2k} \rfloor + 1, & n \equiv 1, 2, \dots 2k-1 \ (mod\ 2k); \\ \frac{n}{2k}, & n \equiv 0 \ (mod\ 2k). \end{cases}$$

For $n \leq 2k$ *, then* $\gamma_{\leq k}(M(P_n)) = 1$*.*

For a graph G, the splitting graph $S'(G)$ of G is obtained by adding a new vertex v' corresponding to each vertex v of G such that $N(v) = N(v')$.

A Duplication of a vertex v_i by a new edge $e = v_i'v_i''$ in graph G produces a new graph G' such that $N(v_i') \cap N(v_i'') = \{v_i\}$. A Duplication of an edge $e = uv$ by a new vertex w in a graph G produces a new graph G' such that $N(w) = \{u, v\}$.

Also Vaidhya et al. [48] determined the distance k-domination number for splitting graph of path and cycle as well as the graphs obtained by duplication of a vertex by an edge and duplication of an edge by a vertex as given in the following.

Theorem 2.4. *[48] If* $n \leq 2k+1$ *,* $k \neq 1$*, then* $\gamma_{\leq k}(S'(P_n)) = 1$*.*

Theorem 2.5. *[48] If* $n > 2k+1$*, then*

$$\gamma_{\leq k}(S'(P_n)) = \begin{cases} \lfloor \frac{n}{2k+1} \rfloor, & n \equiv 1, 2, \ldots, 2k \, mod(2k+1); \\ \frac{n}{2k+1}, & m \geq 2k+1, n \equiv 0 \, mod(2k+1). \end{cases}$$

Theorem 2.6. *[48] Let* G *be a graph obtained by duplication of each vertex of path* P_n*,* $n > 2k-1$ *by an edge*

$$\gamma_{\leq k}(P_n) = \begin{cases} \lfloor \frac{n}{2k-1} \rfloor, & n \equiv 1, 2, \ldots, 2k-2 \, mod(2k-1); \\ \frac{n}{2k-1}, & n \equiv 0 \, mod(2k-1). \end{cases}$$

Theorem 2.7. *[49] If* $n > 2k+1$ *, then*

$$\gamma_{\leq k}(S'(C_n)) = \begin{cases} \lfloor \frac{n}{2k+1} \rfloor, & n \equiv 1, 2, \ldots, 2k \quad mod(2k+1); \\ \frac{n}{2k+1}, & m \geq 2k+1, n \equiv 0 \quad mod(2k+1); \end{cases}$$

Theorem 2.8. *[49] Let* G *be a graph obtained by duplication of each vertex of cycle* C_n*,* $n > 2k-1$ *by an edge*

$$\gamma_{\leq k}(G) = \begin{cases} \lfloor \frac{n}{2k-1} \rfloor, & n \equiv 1, 2, \ldots, 2k-2 \quad mod(2k-1); \\ \frac{n}{2k-1}, & n \equiv 0 \quad mod(2k-1); \end{cases}$$

A vertex v in G is a central vertex if $e(v) = rad(G)$. Let S be a subset of $V(G)$ with k vertices. Let $v \in V$. Then the distance of S from v is defined as $d(S, v) = min\{d(x, v) : x \in S\}$. If $v \in S$, then $d(S, v) = 0$. The eccentricity of S is the maximum of $d(S, v)$ over all $v \in V$. That is, $e(S) = max\{d(S, v) : v \in V\}$. Consider the family F_k of the subset S of k vertices $(1 \leq k \leq n-1)$ of G. The k-center of the graph G is the set S^* of k vertices of G such that, $e(S^*) = min\{e(S), S \in F_k\}$.

Anto Kinsley et al.[2] determined the distance-k domination number for a given graph using the k-center.

Theorem 2.9. *[2] For any connected graph* G*,* $\gamma_{\leq k}(G) = 1$ *if and only if there exists a vertex in* G *with eccentricity* $\leq k$*.*

Theorem 2.10. *[2] Every central vertex with radius* k *forms a distance k-dominating set.*

Theorem 2.11. *[2] Every k-center of* G *with radius* i *is a distance i-dominating set.*

The edge comb product of two graphs G_1 and G_2, denoted by $G_1 \trianglerighteq G_2$ is a graph obtained by taking one copy of G_1 and $|E(G_1)|$ copies of G_2 and grafting the i-th copy of G_2 at the i-th edges of G_1. The friendship graph F_n can be constructed by joining n copies of the cycle graph C_3 with a common vertex.

Slamin [44] determined the exact value of distance k-dominating number of $G_1 \trianglerighteq C_m$, where G is a Friendship graph F_n, Wheel graph W_n and for triangular book and a cycle.

Theorem 2.12. *[44] If F_n is a friendship graph on n+1 vertices for $n \geq 3$ and C_m is a cycle on $m \geq 3$ vertices, then the distance k-dominating number of $G = F_n \trianglerighteq C_m$ with grafting edge $c_1c_2 \in V(C_m)$ is*

$$\gamma_{\leq k}(G) = \begin{cases} 1, & m \leq 2k; \\ 2n(\gamma_{\leq k}(C_m) - 1) + n\gamma_{\leq k}(C_m) + 1, & m \geq 2k+1, m \equiv 0 \, mod(2k+1); \\ 3n(\gamma_{\leq k}(C_m) - 2) + 2n + 1, & m \geq 2k+1, m \equiv 1 \, mod(2k+1); \\ 3n(\gamma_{\leq k}(C_m) - 1) + 1, & otherwise. \end{cases}$$

Theorem 2.13. *[44] If W_n is a Wheel graph on n+1 vertices for $n \geq 3$ and C_m is a cycle on $m \geq 3$ vertices, then the distance k-dominating number of $G = W_n \trianglerighteq C_m$ with grafting edge $c_1c_2 \in V(C_m)$ is*

$$\gamma_{\leq k}(G) = \begin{cases} 1, & m \leq 2k; \\ 2n(\gamma_{\leq k}(C_m) - 1) + \lceil \frac{n}{2} \rceil + 1, & m \geq 2k+1, m \equiv 0 \, mod(2k+1); \\ 2n(\gamma_{\leq k}(C_m) - 2) + n + 1, & m \geq 2k+1, m \equiv 1 \, mod(2k+1); \\ 2n(\gamma_{\leq k}(C_m) - 1) + 1, & otherwise. \end{cases}$$

2.2. Distance-two domination and its variants

Sridharan et al.[46] obtained various upper bounds for distance-two domination number $\gamma_{\leq 2}(G)$ and also characterized the classes of graphs attaining these bounds.

Theorem 2.14. *[46] If each component of G contains at least three vertices then $\gamma_{\leq 2}(G) \leq \lfloor \frac{p}{3} \rfloor$, where p is the order of G.*

The equality is acheived for class of graphs H^{+2}, which is a graph obtained from the given graph H by attaching a path of length 2 at every vertex of H.

Theorem 2.15. *[46] Let G be a connected graph of order p, $\gamma_{\leq 2}(G) = p/3$ if and only if either G is a cycle C_3, C_6 or $G = H^{+2}$ for some connected graph H.*

Theorem 2.16. *[46] If G has p vertices and no isolated vertex, then $\gamma_{\leq 2}(G) = (p - \Delta + 1)/2$ if and only if $\Delta = p - 1$ and $\gamma_2(G) < (p - \Delta +)/2$ if $\Delta \neq (p-1)$.*

The corona $G \Box H$ of two graphs G and H where G has p vertices and q edge is defined as the graph G obtained by taking one copy of G and p-copies of H, and then joining by an edge the i-th point of G to every point in the i-th copy of H.

Reni Umilasari [36] determined the dominating number of distance two of corona product of path with any graph.

Theorem 2.17. *[36] Let $P_m \Box G$ be a corona product of path P_m and any graphs G of order n. Then $\gamma_{\leq 2}(P_m \Box G) = \lceil \frac{m}{3} \rceil$ for $m \geq 2$.*

For any permutation π of the vertex set of G, the prism of G with respect to π is the graph πG obtained from G and a copy G' of G by joining $u \in V(G)$ with $v' \in V(G') = \{w' | w \in V(G)\}$ by joining $u \in V(G)$ to $v' \in V(G')$ if and only if $v = \pi(u)$. A graph G is called a universal $\gamma_{\leq 2}$-fixer if $\gamma_{\leq 2}(\pi G) = \gamma_{\leq 2}(G)$ for every permutation π of $V(G)$.

Ferran Hurtado et al.[10] studied distance 2-domination in prisms and obtained charaterization of all paths and cycles that are universal $\gamma_{\leq 2}$-fixers.

Theorem 2.18. *[10] For each integer* $s \geq 2$ *there is a permutation* π *such that* $\gamma_{\leq 2}(\pi(3s-1)K_s) < \gamma((3s-1)K_s)$.

Theorem 2.19. *[10] The path* P_n *is a universal* $\gamma_{\leq 2}$*-fixer if and only if* $n \in \{1, 2, 3, 6\}$. *The cycle* C_n *is a universal* $\gamma_{\leq 2}$*-fixer if and only if* $n \in \{3, 6, 7\}$.

Ameenal Bibi [1] proposed algorithm for minimum distance -2 dominating set of a graph.

Step 1: All the vertices in V are initialized to White color.

Step 2: We select a vertex in V which has maximum degree, (Here we have two vertices, we can choose arbitrarily any one) change its color to Red color and sends a notification to all its neighbors within a distance two. On receiving this notification, the white color neighbor vertices within a distance two turn into Green color.

Step 3: Now we select any one White color vertex in V.

Case 1: If the white color vertex has maximum degree, (In case if we have any vertices are equal maximum degree, we select any one) and not adjacent to any Green color vertex in the remaining vertices of V.

Case 2: If the Green color vertex which is exactly at the distance two and has more than one pendent vertex then change the Green color vertex into Red color vertex.

Step 4: Repeat the above process (Steps 2-3) until there are no more White color vertex in the graph.

Step 5: Now, all the Red color vertices in the graph form a minimum distance-2 dominating set.

A distance-2 dominating set $D \subseteq V$ of a graph G is a split distance-2 dominating set if the induced subgraph $G[V-D]$ is disconnected. The split distance - 2 domination number $\gamma_{S\leq 2}(G)$ is the minimum cardinality of a split distance -2 dominating set. A. Lakshmi [21] found some bounds on distance - 2 split domination number.

Theorem 2.20. *[21] For any graph G,* $\gamma_{\leq 2}(G) + \gamma_{S\leq 2}(G) \leq n$.

The sharpness of this equality can be seen with the path P_2 and cycle C_3. The next theorem relates domination number and split domination number of G.

Theorem 2.21. *[21] For any graph* G, $\gamma_{S\leq 2}(G) = \gamma_S(G) = \gamma(G)$ *if and only if* G *is a friendship graph* F_n, *where* $\gamma_S(G)$ *denotes the split domination number of* G.

2.3. Step domination in Graphs

Two vertices u and v in a graph G for which the distance $d(u, v) = 2$ are said to 2-step dominate each other. The set of vertices of G that are 2-step dominated by v is denoted by $N_2(v) = \{u \in V(G) | d(v, u) = 2\}$. A set S of vertices of G is called a 2-step domination set if $\cup_{v \in S} N_2(v) = V(G)$. A 2-step step domination set S such that the sets $N_2(v)$, $v \in S$ are pairwise disjoint is called an (exact) 2-step domination set. A graph G is an exact n-step domination graph if there is some set of vertices in G such that each vertex in the graph is distance n from exactly one vertex in the set. G.Chatrand et al. [6] had constructed 2-step domination graph and determined all those paths and cycles that are 2-step domination graphs.

Theorem 2.22. *[6] For every non-negative integer n, none of the paths P_{8n+1}, P_{8n+2}, and P_{8n+3} are 2-step domination graphs.*

Theorem 2.23. *[6] A cycle C_n is a 2-step domination graph if and only if $n = 4$ or $n \equiv 0(mod\, 8)$.*

Patricia Hersh[33] generalized to n steps the notion of exact 2-step domination introduced by Chartrand[6] and obtained some interesting results.

Proposition 2.1. *[33] All n-step dominating sets of a graph have equal order.*

Theorem 2.24. *[33] The order of an n-step dominating set of an exact n-step domination graph is at least $\lfloor log_2 n \rfloor + 2$.*

A set $S = \{v_1, v_2, \ldots, v_n\}$ of vertices in a graph G is defined as a step domination set for G if there exist non-negative integers $k_1, k_2, \ldots, k_n$ such that the k-th neighborhood set $\{N_{k_i}(v_i)\}$ forms a partition of $V(G)$. This partition is called the step domination partition associated with S. Let G be a graph with $V(G) = \{v_1, v_2, \ldots, v_n\}$. The minimum cardinality of a step domination set for G is called step domination number $\gamma_S(G)$.

Dror et al.[7] had discussed the step domination number of any tree T.

Theorem 2.25. *[7] Let T be a tree with diameter $D \geq 5$ and $\alpha = (D+1)/n \leq 1$. Then, $\gamma_S(T) \leq$*
$$\begin{cases} (1-\frac{\alpha}{2})n + \frac{1}{2}, & D \equiv 0, 2(mod\, 4) \\ (1-\frac{\alpha}{2})n + 1, & D \equiv 1(mod\, 4) \\ (1-\frac{\alpha}{2})n, & D \equiv 3(mod\, 4). \end{cases}$$

Farhadi et al.[9] obtained new upper bounds on the size of exact 1-step domination graphs and also presented an upper bound on the total domination number of an exact 1-step domination tree and characterized trees achieving equality for this bound.

Theorem 2.26. *[9] Let G be an exact 1-step domination graph of order n and size m. Then, $m \leq {}_{n-\gamma_t(G)}C_2 + n - \gamma_t(G)/2$.*

Theorem 2.27. *[9] Let G be an exact 1-step domination graph of order n and size m. If $\delta \geq 2$, then $m \leq (n(\Delta+1) - \gamma_t(G)/2$.*

Let Γ be family of trees obtained from a sequence $T_1, T_2, \ldots, T_k$ for some integer k, where $T_1 = P_3$ and if $k \geq 2$, then each T_i is an exact 1-step domination tree with exact 1-step dominating set S_i and T_{i+1} is obtained from T_i using one of the following operations, for $i = 1, 2, \ldots, k-1$.

(σ_1) Let y be a support vertex of degree 2 of an exact 1-step dominating set S_i of T_i. Then T_{i+1} is obtained from T_i by adding a new vertex x and joining x to y and let $S_{i+1} = S_i$.

(σ_2) Let $y \in V(T_i) - S_i$ be a leaf of $T_i = P_3$ or a vertex of $T_i \neq P_3$ such that it is not a leaf adjacent to a support vertex of degree 2. Then T_{i+1} is obtained from T_i by adding a path $P_3 : vwx$ and joining v to y, and let $S_{i+1} = S_i \cup \{w, x\}$.

(σ_3) Let $y \in S_i$ be a leaf adjacent to a support vertex of degree 2 in $T_i \neq P_3$. Then T_{i+1} is obtained from T_i by adding a path $P_4 : uvwx$ and joining u to y, let $S_{i+1} = S_i \cup \{w, x\}$.

2.4. Distance Closed Domination in Graphs

Janakiraman et al.[17] introduced a new distance based domination parameter named as distance closed dominating set motivated by concept of ideal sets in graph theory, which is due to the related concept of ideals in ring theory in algebra. Let I be a vertex subset of G. Then I is said to be an ideal set of G if

(i) For each vertex $u \in I$ and for each $w \in V - I$, there exists at least one vertex $v \in I$ such that $d_{<I>}(u,v) = d_G(u,w)$.

(ii) I is minimal.

A ideal set without the minimality condition was considered as a distance closed set which can be defined as: A vertex subset S of G is said to be a *distance closed set* of G if for each vertex $u \in S$ and for each $w \in V-S$, there exists at least one vertex $v \in S$ such that $d_{<S>}(u,v) = d_G(u,w)$. A dominating set S is called a *distance closed dominating (D.C.D) set*, if the induced subgraph $< S >$ is distance closed. The cardinality of a minimum D.C.D set of G is called the distance closed domination number of G and is denoted by γ_{dcl}. D.C.D is distance preserving in most of the cases, it is useful to find a sub-network of a given communication network, which is fault tolerant and has lots of application in communication networks, social and economic networks and signal processing.

Theorem 2.28. *[17] Let G be a graph of order p. Then*

(i) $\gamma_{dcl}(G) = 2$ if and only if G has at least two vertices of degree $p-1$.

(ii) $\gamma_{dcl}(G) = 3$ if and only if G has exactly one vertex of degree $p-1$.

Theorem 2.29. *[17] If a graph G is connected and diameter $d(G) \geq 3$, then $\gamma_{dcl}(\overline{G}) = 4$.*

A graph G is said to be a D.C.D critical if for every edge $e \notin E(G)$, $\gamma_{dcl}(G+e) < \gamma(G)$. If G is a D.C.D critical graph with $\gamma_{dcl}(G) = k$, then G is said to be k-D.C.D critical. There is no 2-D.C.D critical graph. The structural properties of k-distance closed domination critical graphs for $k = 3$ and 4 were studied.

Theorem 2.30. *[18] G is 3-D.C.D critical if and only if*

(i) G is connected.

(ii) G has $\gamma_{dcl}(G) = 3$.

(iii) G has exactly one vertex with eccentricity equal to 1.

(iv) For every pair of non-adjacent vertices at least one of them is of degree p-2.

Theorem 2.31. *[18] A graph G is 4-D.C.D critical if and only if*

(i) G is connected.

(ii) G has $\gamma_{dcl}(G) = 4$.

(iii) For any two non adjacent vertices at least one of them is of degree p–2.

A set $S \subseteq V(G)$ is said to be a distance closed restrained dominating set if (i) $< S >$ is distance closed and (ii) S is a restrained dominating set. The cardinality of the minimum distance closed restrained dominating set is called the distance closed restrained domination number and denoted by $\gamma_{rdcl}(G)$. Janakiraman et al.[19] gave the bounds for the distance closed restrained domination number of some special class of graphs.

Theorem 2.32. *[19] For any graph G with n vertices, we have the following*

(i) $\gamma_{dcl}(G) \leq \gamma_{rdcl}(G)$

(ii) If v is a vertex of degree 1 in G, then v belongs to all D.C.R.D sets of G.

Theorem 2.33. *[19] If G is a $(p-2)$ regular graph with $p \geq 7$, then $\gamma_{rdcl}(G) = 4$.*

V.Sangeetha et al. [39] introduced the *Perfect Distance Closed Dominating (P.D.C.D)* set, if S is a distance closed set. The cardinality of the minimum perfect distance closed dominating set is called the perfect distance closed domination number and it is denoted by γ_{pdcl}. For any graph G, $\gamma_{dcl} \leq \gamma_{pdcl}$. It was observed that complete graphs, cycles, paths and ciliates(A ciliate $C_{p,q}$ is a graph obtained from p disjoint copies of the path P_{q+1} by linking together one end point of each in a cycle C_p) have $\gamma_{pdcl} = p$.

Proposition 2.2. *[39] For any tree T, $\gamma_{pdcl}(T) = p - k + 2$, where k is the number of pendant vertices in T.*

Theorem 2.34. *[39] Let G be a graph with p vertices. Then $\gamma_{pdcl} = 3$ iff*

(i) G has a vertex of degree p-1 and

(ii) G has at least two pendant vertices.

A dominating set D is called a *restrained dominating set* if every vertex in $V \setminus D$ is adjacent to a vertex in D as well as a vertex in $V \setminus D$. A graph G is said to be a *distance closed restrained domination critical(D.C.R.D)* if for every edge $e \notin E(G)$, $\gamma_{rdcl}(G+e) < \gamma_{rdcl}(G)$. If G is a D.C.R.D critical graph with $\gamma(G) = k$, then G is said to be *k-D.C.R.D critical*. Sangeetha et al.[40] characterised some interesting relationship between D.C.D and D.C.R.D.

Theorem 2.35. *[40] If G is a 3-D.C.D critical graph with $p \geq 5$ and having a cut vertex, then G cannot be 3-D.C.R.D critical.*

Theorem 2.36. *[40] If G is a 4-D.C.D critical graph with $p \geq 6$, then G is 4-D.C.R.D critical.*

2.5. Hop Domination in Graphs

In 2015, Ayyasamy et al.[3] introduced another distance related domination parameter named Hop domination number. A subset $S \subseteq V(G)$ of a graph G is a hop dominating set(hd-set) if every $v \in V - S$ there is at least one vertex $u \in S$ such that $d(u, v) = 2$. The minimum cardinality of a hop dominating set of G is called a Hop domination number of G and is denoted by $\gamma_h(G)$.

The hop-degree of a vertex v in a graph G, denoted $d_h(v)$, to be the number of vertices at distance 2 from v in G and the minimum hop-degree and maximum hop-degree is denoted by $\delta_h(G)$ and $\Delta_h(G)$ respectively. Ayyasamy et al.[3] related it with total domination for triangle free graphs and obtained a characterisation for minimal hop dominating set.

Theorem 2.37. *[3] For every traingle-free graph G without isolated vertices,* $\gamma_h(G) \leq \gamma_t(G)$.

Theorem 2.38. *[3] Let* S *be a hop dominating set of a ntc graph* G. *Then* S *is a minimal hop dominating set of* G *if and only if each* $u \in S$ *satisfies at least one of the following:*

(i) there exists a vertex $v \in V \setminus S$ *for which* $N_2(v) \cap S = u$.

(ii) $d_G(u, w) \neq 2$ *for every* $w \in S \setminus \{u\}$.

Natarajan et al. [29] also gave characterisation of graphs with equal hop domination and total domination number $\gamma_t(G)$ and for graphs with equal hop domination and connected domination number $\gamma_c(G)$.

A family of $\mathcal{F}$ of trees that can be obtained from the disjoint union of $k \geq 1$ double stars. Let C be the set of central vertices of the double stars. Add $(k-1)$ edges between the vertices of $(V \setminus C)$ so that the resulting graph is a tree.

Theorem 2.39. *[29] For any tree T,* $\gamma_h(T) = \gamma_t(T)$ *iff* $T \in \mathcal{F}$.

Let $\mathcal{F}^* = \{T \cup \{e\}$: $T \in \mathcal{F}$, e is an edge joining any two leaves or a leaf and an internal vertex which is not support vertex. $\}$

Theorem 2.40. *[29] Let G be a unicyclic graph with the unique cycle C. Then* $\gamma_h(G) = \gamma_t(G)$ *iff* G *is* C_4 *or* $G \in \mathcal{F}^*$.

Theorem 2.41. *[29] Let* T *be a tree of order* $n > 3$. *Then* $\gamma_h(T) = \gamma_c(T)$ *if and only if* T *is a double star.*

Later Ayyaswamy et al.[4] in their subsequent paper derived the bounds on the hop domination number of a tree by recursive operations as $(n-l-s+4)/3 \leq \gamma_h(G) \leq n/2$, where n is the number of vertices, l is the number of leaves and s is the number of support vertices.(A vertex of degree one is called a leaf, its neighbor a support vertex).

A family $\mathcal{T}$ of trees $T = T_k$ that can be obtained as follows. Let T_1 be a path P_6. If k is a positive integer, then T_{k+1} can be obtained recursively from T_k by one of the following operations:

- **Operation O1**: Attach a vertex by joining it to any support vertex of T_k.
- **Operation O2**: Attach a P_2 by joining one of its vertices to a vertex of T_k adjacent to path P_2 or P_3.
- **Operation O3**: Attach a path P_6 by joining one of the leaf to a leaf of T_k adjacent to a weak support vertex.

Theorem 2.42. *[4] If* T *is a tree of order* n *with* l *leaves and* s *support vertices,* $\gamma_h(T) \geq (n-l-s+4)/3$ *with equality iff* $T \in \mathcal{T}$.

A Family $\mathcal{F}^*$ of trees $T = T_K$ can be obtained as follows. Let $T_1 \in \{P_3, P_4\}$. If k is a positive integer, then T_{k+1} can be obtained recursively from T_k by attaching a path P_4 by joining one of its leaf to a leaf of T_k or to a vertex adjacent to P_3 or P_4.

Mojdeh et al.[26] defined hop domination polynomial of G is the polynomial

$D_h(G, x) = \sum_{i=\gamma_h(G)}^{|V(G)|} d_h(G, i)x^i$ and obtained some results.

Theorem 2.43. *[4] If T is a tree of order n with l leaves and s support vertices, $\gamma_h(T) \leq (n/2)$ with equality iff $T \in \mathcal{F}^{**}$.*

Theorem 2.44. *[26] Let G be a graph $K_{1,n-1}, n \geq 3$. Then*

(i) $d_h(G, n-i) = \begin{pmatrix} n-1 \\ n-i-1 \end{pmatrix}$, $0 \leq i \leq n-2$.

(ii) $D_h(G,x) = \sum_{i=0}^{n-2} \begin{pmatrix} n-1 \\ n-i-1 \end{pmatrix} x^{n-i}$

Theorem 2.45. *[26] For a complete bipartite graph $K_{m,n}, (2 \leq m \leq n)$, $D_h(K_{m,n}, x) = ((1+x)^n - 1)((1+x)^m - 1)$.*

Theorem 2.46. *[26] The hop domination polynomial of the Petersen graph is $D_h(P,X) = X^{10} + 10X^9 + 45X^8 + 120X^7 + 210X^6 + 252X^5 + 210X^4 + 110X^3 + 15X^2$.*

Henning et al.[15] characterised the family of graphs in which for each graph G hop domination number is half its order. Let $\mathcal{F}_n$ be the family of all graphs F of order n such that $F \ncong C4$ and every component of F is a 4-cycle or the corona $H \Box K_1$ for some connected graph H.

Theorem 2.47. *[15] If G is a graph of order n with $\delta_h(G) \geq 1$, then the following holds, $\gamma_h(G) = n/2$ if and only if $Dist(G;2) \in \mathcal{F}_n$, where $Dist(G;2) = G^2 - E(G)$, where where G^2 is the square of G with the same set of vertices of G, but in which two vertices are adjacent in G^2 when their distance in G is at most 2.*

Also Henning et al. [15] obtained probabilistic bounds for hop domination number to show that the difference between $\gamma_h(G)$ and $\gamma_t(G)$ is arbitrarily large.

Theorem 2.48. *[15] If G is a graph of order n with $\delta_h = \delta_h(G) \geq 1$, then $\gamma_h(G) \leq (\frac{ln(\delta_h+1)+1}{\delta_h+1})n$.*

Corollary 2.1. *If G is a triangle-free graph of order n with minimum degree $\delta = \delta(G) \geq 2$, then $\gamma_h(G) \leq (\frac{1+\ln\delta}{\delta})n$.*

Theorem 2.49. *[15]*

(i) For almost all random graphs G = G(n, p(n)),$\gamma_h(G) \leq \gamma_t(G)$, if $p(n) << 1/n$, where $x << y$ to mean that x is much less than y.

(ii) For almost all random graphs G = G(n, p), $\gamma_h(G) \leq 1 + np(1+o(1))$, if p is a constant.

A subset S of $V(G)$ is a total hop dominating set if for every vertex $v \in V(G)$, there exists $u \in S$ such that $d(u,v) = 2$. The smallest cardinality of a total hop dominating set of G is denoted by $\gamma_{th}(G)$. If S is a total hop dominating set of G and $N_G(S) = V(G)$, then we call S a total hop total dominating set of G. The smallest cardinality of a total hop total dominating set of G, denoted by $\gamma_{tht}(G)$ is called the total hop total domination number of G. A subset S of $V(G)$ is a total point-wise non-dominating set of G if for every $v \in V(G)$, there is $u \in S$ such that $v \notin N_G(u)$. Pabilona et al.[31] studied the total hop dominating sets under binary operations like join, corona and lexicographic product of graphs.

Theorem 2.50. *[31] Let G be a nontrivial graph and H be any graph. Then $\gamma_{tht}(G\square H) = \gamma_{th}(G)$, where $G\square H$ denotes corona product of G and H.*

Farhadi [8] showed that for any integer $k \geq 2$, the decision problems for the k -step dominating set and k -hop dominating set are NP-complete for planar bipartite graphs and planar chordal graphs.

A Hop Roman dominating function(HRDF)[41] on G is a labeling $f : V(G) \longrightarrow \{0, 1, 2\}$ such that for every vertex $v \in V$ with $f(v) = 0$, there is a vertex u with $f(u) = 2$ and $d(u, v) = 2$. The weight of a HRDF is the value $f(V) = \sum_{v\in V} f(v)$. The minimum weight of a HRDF called the hop Roman domination number of G and is denoted by $\gamma_{hR}(G)$. Shabani et al.[42] found the following bounds for $\gamma_{hR}(G)$.

Proposition 2.3. *[42] For any graph G of order n and with $\Delta_h(G) \geq 1$, $\gamma_{hR}(G) \geq \frac{2n}{\Delta_h(G)+1}$.*

Proposition 2.4. *[42] For any graph G of order n and with $\Delta_h(G) \geq 1$, $\gamma_{hR}(G) \leq n - \Delta_h(G) + 1$.*

Mahadevan et al. [23] introduced the concept of Non split hop dominating set of a graph. A set $S \subseteq V$ is a Non split hop dominating set(NSHD) of G if S is a hop dominating set and $< V - S >$ is connected. Also, Mahadevan et al. [24] found a result for cartesian product of two cycles $C_m \times C_n$ as

Theorem 2.51. *[24]* $NSHD(C_m \times C_n) = \begin{cases} \frac{4n}{6}, & n \equiv 0(mod\,6); \\ \frac{4n-1}{6} + 1, & n \equiv 1(mod\,6); \\ \frac{4n-2}{6} + 2, & n \equiv 2(mod\,6); \\ \frac{4n-3}{6} + 3, & n \equiv 3(mod\,6); m=4 \\ \frac{4n-3}{6} + 4, & n \equiv 3(mod\,6); m=5 \\ \frac{4(n+2)}{6}, & n \equiv 4(mod\,6); \\ \frac{4(n+1)}{6}, & n \equiv 5(mod\,6). \end{cases}$

Natarajan et al. [30] computed the hop domination number of some special families of graphs such as central graph $C(G)$, middle graph $M(G)$ and total graph $T(G)$ of a star graph $K_{1,n}$ and path P_n.

A hop dominating set $S \subseteq V(G)$ is called a connected hop dominating set of G if the induced subgraph $G[S]$ of S is connected. The smallest cardinality of a connected hop dominating set of G, denoted by $\gamma_h^c(G)$, is called the connected hop domination number of G. Pabilona et al.[32] characterized the connected hop dominating sets.

Proposition 2.5. *[32] Let G be a connected graph of order n. Then $1 \leq \gamma_h^c(G) \leq n$. Moreover,*

(i) if G is a complete graph, then $\gamma_h^c(G) = n$; and

(ii) $\gamma_h^c(G) = 2$ if and only if there exist two adjacent vertices $x, y \in V(G)$, where $e(x) \leq 3$, $e(y) \leq 3$ and $N_G(x) \cap N_G(y) \neq \emptyset$.

A total hop dominating set $D \subseteq V(G)$ is called a connected total hop dominating set of G if the induced subgraph $G[D]$ of D is connected. The connected total hop domination number of G, denoted by $\gamma_{th}^c(G)$, is the minimum cardinality of a connected total hop dominating set of G. Vinolin et al.[50] stated the existence of a connected hop dominating set in an arbitrary graph and obtained some bounds for hop domination number.

Theorem 2.52. *[50] If G is a connected graph with girth g, then $\gamma_h(G) \geq \lceil \frac{g+1}{3} \rceil$.*

Theorem 2.53. *[50] Let $G = (V, E)$ be a connected graph. Let $G_1, G_2, \ldots, G_s (s \geq 2)$ be connected proper subgraphs of G with connected hop dominating sets $D_1, D_2, \ldots, D_s$ respectively.*

If $\bigcup_{i=1}^{s} V(G_i) = V$, then there exists a connected hop dominating set D of G such that $D \subseteq \bigcup_{i=1}^{s} D_i$ and $|D| \leq \sum_{i=1}^{s} |D_i| + 2s$.

Let $G = (V(G), E(G))$ be a simple graph. A set $S \subseteq V(G)$ is a perfect hop dominating set of G if for every $v \in V(G) - S$, there is exactly one vertex $u \in S$ such that $d(u, v) = 2$. The smallest cardinality of a perfect hop dominating set of G is called the perfect hop domination number of G, denoted by $\gamma_{ph}(G)$.

Rakim et al.[35] studied perfect hop domination in graphs and discussed the perfect hop domination number for certain binary operations like join, corona, lexicographic and cartersian products of graphs.

Theorem 2.54. *[35] Let G and H be graphs. Then $S \subseteq V(G + H)$ is a perfect hop dominating set of $G + H$ if and only if $S = S_G \cup S_H$ where S_G and S_H are perfect point-wise non-dominating sets of G and H, respectively.*

Theorem 2.55. *[35] Let G be a nontrivial connected graph whose total perfect hop dominating set exists and H a nontrivial connected graph with $\gamma(H) = 1$. A nonempty proper subset $C = \cup_{x \in S}[x \times T_x]$ of $V(G[H])$ where $S \subseteq V(G)$ and $T_x \subseteq V(H)$ for all $x \in S$, is a perfect hop dominating set of $G[H]$ if and only if S is a total perfect hop dominating set of G and T_x is a γ-set of H for every $x \in S$.*

Jafari Rad [27]gave an algorithm that decides whether $\gamma_{hR}(T) = 2\gamma_h^c(T)$ for a given tree T.

Input: A tree T.

Output: The set of all inner vertices of T, i.e., $I(T)$

1 $I(T) = \emptyset$.

2 for each $v \in T$, Compute T_v.

3 diam(T)=4 then

4 $I(T) = I(T) \cup \{v\}$

5 end

6 return $I(T)$

A subset S of vertices of a graph G is a hop independent dominating set (HIDS) if S is a Hop Dominating Set and for any pair $v, w \in S, d(v, w) \neq 2$. N. Jafari Rad et al.[28] studied the complexity of the hop independent dominating problem(HIDP) and the hop Roman domination function problem (HRDFP)[**?**] and show that the decision problem is NP-complete even when restricted to planar bipartite graphs or planar chordal graphs.

Theorem 2.56. *[28] HIDP is NP-complete for planar bipartite graphs and planar chordal graphs.*

Theorem 2.57. *[28] HRDFP is NP-complete for planar bipartite graphs and planar chordal graphs.*

More Recently, Sergioet al. [41] related hop dominating set with point-wise non-dominating sets, which is defined as, A set $S \subseteq V(G)$ is a point-wise non-dominating set of G if for each $v \in V(G) \setminus S$, there exists $u \in S$ such that $v \notin N_G(u)$. Also they applied hop dominating sets to various Graph operations.

In 2019, Sambhu Charan Barman[38] discussed an optimal algorithm to find a minimum k-hop dominating set of interval graphs with n vertices which runs in $O(n)$ time and its complexity. Recently, Henning et al.[16] obtained some new results on Hop dominating sets and also presented an algorithm for computing a minimum hop dominating set in bipartite permutation graphs whose running time is $O(n+m)$. A graph $G = (V, E)$ is a permutation graph if there exists a one to one correspondence between $V(G)$ and a set of line segments between two parallel lines such that two vertices are adjacent if and only if their corresponding line segments intersect. A bipartite permutation graph is a graph that is both a bipartite graph and a permutation graph.

Theorem 2.58. *[16] A minimum hop dominating set of a given bipartite permutation graph can be computed in linear time.*

There are several other distance based domination parameters like k-distance paired domination number, inverse distance-2 domination number, 2-step perfecr domination number etc.,([22, 34, 37]) which are to be explored by interested researchers.

Joanna Raczek[34] studied a generalization of the paired domination number. A set $D \subseteq V(G)$ is a k-distance paired dominating set of G if D is a k-distance dominating set of G and the induced subgraph $G[D]$ has a perfect matching. The k-distance paired domination number $\gamma_p^k(G)$ is the cardinality of a smallest k-distance paired dominating set and Raczek[34] proved that the decision problem k-PDS for $k > 1$ still NP-complete, even when restricted to bipartite graphs and obtained some bounds for $\gamma_p^k(G)$.

Theorem 2.59. *[34] If G has no isolated vertex and $\Delta = \Delta(G) \geq 3$, then $\gamma_p^k(G) \geq \frac{p(\Delta-2)}{(\Delta-1)^{k+1}-1}$.*

Proposition 2.6. *[34] If G is a graph with no isolated vertex, then $\gamma_p^k(G) = n(G)$ if and only if each component of G is K_2.*

Later, Kan Yu Mei Lu [20] obtained some bounds for distance-2 paired domination number.

Theorem 2.60. *[20] Let $G = (V, E)$ be a connected graph with $n \geq 9$ and $\delta(G) \geq 2$. Then $\gamma_p^2(G) \leq \lfloor \frac{n-1}{4} \rfloor$.*

Theorem 2.61. *[20] Let G be a connected graph of order n with girth g and $\delta(G) \geq 3$. Then $\gamma_p^2(G) \leq 2\lfloor \frac{n}{4} - \frac{g-7}{12} \rfloor$.*

Another such distance based domination parameter is inverse distance-2 domination number which is defined as follows: Let $D \subseteq G$ be the non-empty subset of G such that D is the minimum distance-2 dominating set in the graph $G = (V, E)$. If $V \setminus D$ contains a distance-2 dominating set D' of G, then D' is called an inverse distance - 2 dominating set with respect to D. The cardinality of a minimum inverse distance-2 dominating set of G is inverse distance-2 domination number denote $\gamma_{\leq 2}^{-1}(G)$. Lakshmi[22] obtained some bounds on inverse distance-2 domination number.

Theorem 2.62. *[22] For any graph $G, \gamma_{\leq 2}^{-1}(G) \leq n - \Delta(G)$.*

Theorem 2.63. *[22] For any graph G with no isolated vertices, $\gamma_{\leq 2}^{-1}(G) \leq ef(G)$, where $ef(G)$ denotes the maximum number of pendent edges in a spanning forest of G.*

Recently, Sahaya Jasmine et al.[37] introduced a new domination parameter called 2-step perfect domination. A dominating set S is a 2-step perfect dominating set, if for every vertex not in S, there exists one vertex such that $u \in S$ and $d(u,v) = 2$ and $|N(v) \cap S| = 1$. The minimum cardinality of 2-step perfect dominating set is denoted by $\gamma_{2sp}(G)$.

Theorem 2.64. *[37] For any cycle $C_m (m \geq 4)$, then*

$$\gamma_{2sp}(C_m) = \begin{cases} \frac{m}{3} & m = 3n, n \geq 2 \\ \lceil \frac{m}{3} \rceil & m = 3n, n \geq 1 \\ \lfloor \frac{m}{2} \rfloor & m = 3n+2, n \geq 2. \end{cases}$$

Theorem 2.65. *[37] For the corona graph of cycle C_m and path $P_n (n \geq 4)$, 2-step perfect domination number is exactly m.*

3. Summary and Conclusion

Haynes et al.([11, 12]) made an excellent survey of domination in graphs based on research papers published till 1998. In this survey, we have made an attempt to include some selected results thereafter on distance domination in graphs, which we realise to be one of the important parameters in theory of domination in graphs with reference to growing applications in various fields of Engineering.

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