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On decomposition of $n\ddot{g}$-continuity in nano Topological spaces

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Abstract: This article focuses on decomposition of a weaker form of nano continuity, namely $n\ddot{g}$-continuity, by providing the concepts of $n\ddot{g}_t$-sets, $n\ddot{g}_{\alpha}*$-sets, $n\ddot{g}_t$-continuity and $n\ddot{g}_{\alpha}*$-continuity.

Key words: $n\ddot{g}_{\alpha}$-closed, $n\ddot{g}_p$-closed, $n\ddot{g}_t$s, $n\ddot{g}_{\alpha}*$s

1. Introduction

Various interesting problems arise when one considers nano continuous (in short, n-cts) and nano generalized continuous (in shor, ng-cts). Decomposition of n-cts are obtained by many mathematician with the help of ng-cts maps in nano topological spaces (in short, ntss)([5, 6, 19]). This article, we obtained decomposition of $n\ddot{g}$-continuous (in short, $n\ddot{g}$-cts) in ntss using $n\ddot{g}_p$-continuous (in short, $n\ddot{g}_p$-cts) [20], $n\ddot{g}_{\alpha}$-continuous (in short, $n\ddot{g}_{\alpha}$-cts) [20], $n\ddot{g}_t$-continuous (in short, $n\ddot{g}_t$-cts) and $n\ddot{g}_{\alpha}*$-continuous (in short, $n\ddot{g}_{\alpha}*$-cts).

2. Preliminaries

Definition 2.1. [11] If (K, $\tau_R(P)$) is the ntss with respect to P where P $\subseteq$ K and if S $\subseteq$ K, then

1. The nano interior of the set S is defined as the union of all nano open(in short, no) subsets contained in S and it is denoted by ninte(S). That is, ninte(S) is the largest no subset of S.
2. The nano closure of the set S is defined as the intersection of all nano closed (in short, nc)sets containing S and it is denoted by nclo(S). That is, nclo(S) is the smallest nc set containing S.

Definition 2.2. [11] A subset S of a ntss (K, $\tau_R(P)$) is said to be

1. nano α-closed (in short, $n\alpha$c) if nclo(ninte(nclo(S))) $\subseteq$ S.
2. nano semi-closed (in short, nsc) if ninte(nclo(S)) $\subseteq$ S.
3. nano pre-closed (in short, npc) if nclo(ninte(S)) $\subseteq$ S.

The complements of the above mentioned nc are called their respective no respectively.

The nano α-closure [18] (respectively, nano semi-closure [2, 3], nano pre-closure [1]) of a subset S of K, denoted by $n\alpha$clo(S) (respectively. nsclo(S), npclo(S)) is defined to be the intersection of all $n\alpha$c (respectively. nsc, npc) sets of (K, $\tau_R(P)$) containing S.

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The nano α-interior [18] (respectively, nano semi-interior [2, 3], nano pre-interior[1]) of a subset S of K, denoted by $n\alpha$inte(S) (respectively. nsinte(S), npinte(S)) is defined to be the union of all $n\alpha$o (respectively. nso, npo) sets of (K, $\tau_R(P)$) containing S.

Definition 2.3. A subset S of a ntss (K, $\tau_R(P)$) is called :

1. nsg-open (in short, nsgo) [2] if T $\subseteq$ nsinte(S) whenever T $\subseteq$ S and T is nano semi-closed in (K, $\tau_R(P)$).
2. $n\ddot{g}$-open (in short, $n\ddot{g}$o) [18] if T $\subseteq$ ninte(S) whenever T $\subseteq$ S and T is nsg-closed in (K, $\tau_R(P)$).
3. $n\ddot{g}_\alpha$-open (in short, $n\ddot{g}_\alpha$o) [18] if T $\subseteq$ $n\alpha$inte(S) whenever T $\subseteq$ S and T is nsg-closed in (K, $\tau_R(P)$).
4. $n\ddot{g}_p$-open (in short, $n\ddot{g}_p$o) [20] if T $\subseteq$ npinte(S) whenever T $\subseteq$ S and T is nsg-closed in (K, $\tau_R(P)$).
   The complement of above mentioned no-sets are called their respective nc-sets.

Definition 2.4. A subset S of a ntss (K, $\tau_R(P)$) is said to be

1. nt-set (in short, nts) [10] if ninte(S) = ninte(nclo(S)).
2. $n\alpha$*-set (in short, $n\alpha$*s) [16] if ninte(S)= ninte(nclo(inte(S))).
3. an $n\eta$-set [8] if S = I $\cap$ J where I is no & J is a $n\alpha$c.
4. $n\eta^\sharp$-set (in short, $n\eta^\sharp$s) [19] if S = I $\cap$ J, where I is nsgo & J is $n\alpha$c in (K, $\tau_R(P)$).
5. $n\eta^{\sharp\sharp}$-set (in short, $n\eta^{\sharp\sharp}$s) [19] if S = I $\cap$ J, where I is $n\ddot{g}_\alpha$o & J is nts in (K, $\tau_R(P)$).
6. $n\ddot{g}$lc*-set (in short, $n\ddot{g}$lc*s) [20] if S = I $\cap$ J, where I is nsgo & J is nc (K, $\tau_R(P)$)

Collection of all $n\eta$s (respectively, $n\eta^\sharp$s, $n\eta^{\sharp\sharp}$s) in a ntss (K, $\tau_R(P)$) is denoted by $n\eta$(K, $\tau_R(P)$) (respectively, $n\eta^\sharp$(K, $\tau_R(P)$), $n\eta^{\sharp\sharp}$(K, $\tau_R(P)$)).

Remark 2.1. *$n\alpha$c sets and $n\ddot{g}$c sets are independent.*

Example 2.1. 1. *Let K = {11, 12, 13} with K / R = {{13}, {11, 12} {12, 11}} and P = {11, 12}, $\tau_R(P)$ = {ϕ, {11, 12}, K}. Here, the set {12, 13} is $n\ddot{g}$c but not $n\alpha$c.*

2. *Let K = {11, 12, 13} with K / R = {{11}, {12, 13}} and P = {11}, $\tau_R(P)$ = {ϕ, {11}, K}. Here, the set {12} is $n\alpha$c but not $n\ddot{g}$c.*

Remark 2.2. *[16]*

1. *Every nts is an $n\alpha$*s but not conversely.*
2. *Union of 2 $n\alpha$*s need not be an $n\alpha$*s.*
3. *Intersection of 2 $n\alpha$*s is an $n\alpha$*s.*

Definition 2.5. A map f : (K, $\tau_R(P)$) $\rightarrow$ (L, $\sigma_R(Q)$) is called:

1. n-cts [12] if f^{-1}(D) is a no in (K, $\tau_R(P)$), for every no set D of (L, $\sigma_R(Q)$).
2. $n\alpha$-continuous (in short, $n\alpha$-cts) [13] if f^{-1}(D) is $n\alpha$o in (K, $\tau_R(P)$), for every no set D of (L, $\sigma_R(Q)$).

3. $n\ddot{g}$-cts [20] if f^{-1}(D) is $n\ddot{g}$o in (K, $\tau_R(P)$), for every no set D of (L, $\sigma_R(Q)$).
4. $n\ddot{g}_{\alpha}$-cts [20](respectively, $n\ddot{g}_{p}$-cts [20]) if f^{-1}(D) is $n\ddot{g}_{\alpha}$o (respectively. $n\ddot{g}_{p}$o) set in (K, $\tau_R(P)$), for every no set D of (L, $\sigma_R(Q)$).
5. $n\ddot{g}$lc*-continuous (in short, $n\ddot{g}$lc*-cts)[20] if f^{-1}(D) $\in$ $n\ddot{g}$lc*(K, $\tau_R(P)$), for every no set D of (L, $\sigma_R(Q)$).
6. $n\eta^{\sharp}$-continuous (in short, $n\eta^{\sharp}$-cts) [18] if f^{-1}(D) $\in$ $n\eta^{\sharp}$(K, $\tau_R(P)$), for every no set D of (L, $\sigma_R(Q)$).
7. $n\eta^{\sharp\sharp}$-continuous (in short, $n\eta^{\sharp\sharp}$-cts) [18] if f^{-1}(D) $\in$ $n\eta^{\sharp\sharp}$(K, $\tau_R(P)$), for every no set D of (L, $\sigma_R(Q)$).

Theorem 2.1. *[20] A map f : (K, $\tau_R(P)$) $\to$ (L, $\sigma_R(Q)$) is n-cts iff it is both $n\ddot{g}$-cts and $n\ddot{g}$lc*-cts.*

Theorem 2.2. *[19]*

1. *A map f : (K, $\tau_R(P)$) $\to$ (L, $\sigma_R(Q)$) is $n\alpha$-cts iff it is both $n\ddot{g}_{\alpha}$-cts and $n\eta^{\sharp}$-cts.*
2. *A map f : (K, $\tau_R(P)$) $\to$ (L, $\sigma_R(Q)$) is $n\ddot{g}_{\alpha}$-cts iff it is both $n\ddot{g}_{p}$-cts and $n\eta^{\sharp\sharp}$-cts.*

3. $n\ddot{g}_{t}$s & $n\ddot{g}_{\alpha}*$s

Definition 3.1. A subset H of a ntss (K, $\tau_R(P)$) is said to be

1. an $n\ddot{g}_{t}$-sets (in short, $n\ddot{g}_{t}$s) if H = I $\cap$ J, where I is $n\ddot{g}$o in K & J is a nts in K.
2. an $n\ddot{g}_{\alpha}*$-sets (in short, $n\ddot{g}_{\alpha}*$s) if H = I $\cap$ J, where I is $n\ddot{g}$o in K & J is a $n\alpha^*$s in K.

Collection of all $n\ddot{g}_{t}$s (respectively, $n\ddot{g}_{\alpha}*$s) in a ntss (K, $\tau_R(P)$) is denoted by $n\ddot{g}_{t}$(K, $\tau_R(P)$) (respectively, $n\ddot{g}_{\alpha}$*(K, $\tau_R(P)$)).

Proposition 3.1. *Let H be a subset of K. Then*

1. *if H is a nts, then H $\in$ $n\ddot{g}_{t}$(K, $\tau_R(P)$).*
2. *if H is an $n\alpha^*$s, then H $\in$ $n\ddot{g}_{\alpha}$*(K, $\tau_R(P)$).*
3. *if H is an $n\ddot{g}$o set in K, then H $\in$ $n\ddot{g}_{t}$(K, $\tau_R(P)$) & H $\in n\ddot{g}_{\alpha}$*(K, $\tau_R(P)$).*

Proof. The proof is straightforward from the definitions. □

Proposition 3.2. *In a ntss K, every $n\ddot{g}_{t}$s is an $n\ddot{g}_{\alpha}*$s but not conversely.*

Proof. The proof is straightforward from the definitions. □

Example 3.1. *Let K and $\tau_R(P)$ in the Example 2.1(1). Here, the set {11, 13} is $n\ddot{g}_{\alpha}*$s but not $n\ddot{g}_{t}$s.*

Remark 3.1. *The following examples show that*

1. *the converse of Proposition 3.1 need not be true.*
2. *$n\ddot{g}_{t}$s & $n\ddot{g}_{p}$o sets are independent.*
3. *$n\ddot{g}_{\alpha}*$s & $n\ddot{g}_{\alpha}$o sets are independent.*

Example 3.2. *Let K & $\tau_R(P)$ in the Example 2.1(1). Here, the set $\{11\}$ is $n\ddot{g}_t s$ but not a nts & the set $\{11, 12\}$ is $n\ddot{g}_\alpha{}^*s$ but not $n\alpha{}^*s$.*

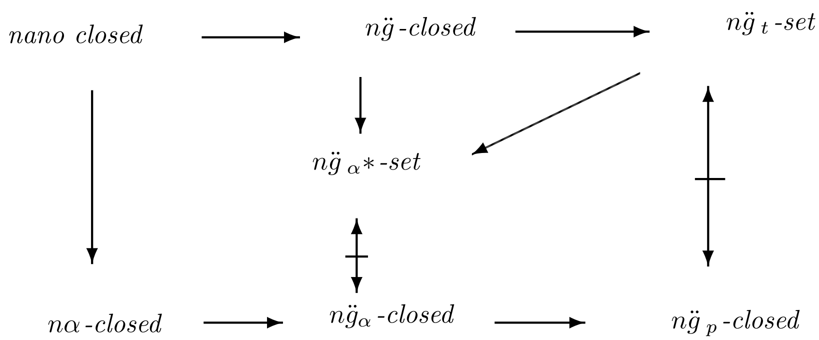
Example 3.3. *Let K & $\tau_R(P)$ in the Example 2.1(1). Here, the set $\{13\}$ is both $n\ddot{g}_t s$ & $n\ddot{g}_\alpha{}^*s$ but not $n\ddot{g}o$ set.*

Example 3.4. *Let K & $\tau_R(P)$ in the Example 2.1(1). Here, (a) the set $\{13\}$ is $n\ddot{g}_t s$ but not a $n\ddot{g}_p o$, (b) the set $\{12, 13\}$ is a $n\ddot{g}_p o$ set but not $n\ddot{g}_t s$.*

Example 3.5. *Let $K = \{11, 12, 13, 14\}$ with $K / R = \{\{11\}, \{12, 13\}, \{12, 14\}\}$ and $P = \{11\}$, $\tau_R(P) = \{\phi, \{11\}, K\}$. Here, (a) the set $\{12\}$ is $n\ddot{g}_\alpha{*}s$ but not $n\ddot{g}_\alpha o$, (b) the set $\{11, 13\}$ is $n\ddot{g}_\alpha o$ but not $n\ddot{g}_\alpha{*}s$.*

Example 3.6. *Let K & $\tau_R(P)$ in the Example 2.1(1). Here, the set $\{11\}$ is $n\ddot{g}_\alpha{}^*s$ and $n\ddot{g}_t s$ but it is not $n\ddot{g}c$.*

Remark 3.2. *From the above discussions and known results in [[14], [18], [20]], we obtained the following diagram where $A \rightarrow B$ represents A implies B, but not conversely.*



Remark 3.3. ,

1. *Union of 2 $n\ddot{g}_t s$ need not be an $n\ddot{g}_t s$.*
2. *Union of 2 $n\ddot{g}_\alpha{*}s$ need not be an $n\ddot{g}_\alpha{*}s$.*

Example 3.7. ,

1. *Let K and $\tau_R(P)$ in the Example 2.1(1). Here, the sets are $\{12\}$ and $\{13\}$ are $n\ddot{g}_t$-sets but $\{12\} \cup \{13\} = \{12, 13\}$ is not an $n\ddot{g}_t$-set.*
2. *Let K and $\tau_R(P)$ in the Example 3.5. Here, the sets are $\{11\}$ and $\{12\}$ are $n\ddot{g}_\alpha{*}$-sets but $\{11\} \cup \{12\} = \{11, 12\}$ is not an $n\ddot{g}_\alpha{*}s$.*

Remark 3.4. ,

1. *Intersection of any 2 $n\ddot{g}_t s$ belongs to $n\ddot{g}_t$ $(K, \tau_R(P))$.*
2. *Intersection of any 2 of $n\ddot{g}_\alpha{*}s$ belongs to $n\ddot{g}_\alpha{*}$ $(K, \tau_R(P))$.*

Lemma 3.1. ,

1. *A subset H of (K, $\tau_R(P)$) is $n\ddot{g}o$ [18] iff F ⊆ ninte(H) whenever F ⊆ H and F is nsgc in K.*

2. *A subset H of (K, $\tau_R(P)$) is $n\ddot{g}_\alpha o$ [20] iff F ⊆ $n\alpha$inte(H) whenever F ⊆ H and F is nsgc in K.*

3. *A subset H of (K, $\tau_R(P)$) is $n\ddot{g}_p o$ [20] iff F ⊆ npinte(H) whenever F ⊆ H and F is nsgc in K.*

Theorem 3.1. *A subset H is $n\ddot{g}o$ in (K, $\tau_R(P)$) iff it is both $n\ddot{g}_\alpha o$ and $n\ddot{g}_{\alpha *}s$ in (K, $\tau_R(P)$).*

Proof. Necessity. Obvious.

Sufficiency. Suppose H be both $n\ddot{g}_\alpha$o set and $n\ddot{g}_\alpha$*s. Since H is an $n\ddot{g}_\alpha$*s, H = I ∩ J, where I is $n\ddot{g}$o and J is an $n\alpha$*s. Assume that F ⊆ H, where F is nsgc in K. Since I is $n\ddot{g}$o, by Lemma 3.1 (1), F ⊆ ninte(H). Since H is $n\ddot{g}_\alpha$o in K, by Lemma 3.1 (2), F ⊆ $n\alpha$inte(H) = H ∩ ninte(nclo(ninte(H))) = (I ∩ J) ∩ ninte(nclo(ninte(I ∩ J))) ⊆ I ∩ J ∩ ninte(nclo(ninte(I))) ∩ ninte(nclo(ninte(J))) = I ∩ J ∩ ninte(nclo(ninte(I))) ∩ ninte(J) ⊆ ninte(J). Therefore, we obtained F ⊆ ninte(J) and hence F ⊆ ninte(I) ∩ ninte(J) = ninte(H). Hence H is nsgo, by Lemma 3.1 (1). □

Theorem 3.2. *A subset H is $n\ddot{g}o$ in (K, $\tau_R(P)$) iff it is both $n\ddot{g}_p o$ and $n\ddot{g}_t s$ in (K, $\tau_R(P)$).*

Proof. The proof is similar to theorem 3.1. □

Definition 3.2. A map f : (K, $\tau_R(P)$) → (L, $\sigma_R(Q)$) is said to be

1. $n\ddot{g}_t$-cts if f^{-1}(D) ∈ $n\ddot{g}_t$(K, $\tau_R(P)$), for every no set D of (L, $\sigma_R(Q)$).

2. $n\ddot{g}_{\alpha}*$-cts if f^{-1}(D) ∈ $n\ddot{g}_{\alpha}*$(K, $\tau_R(P)$), for every no set D of (L, $\sigma_R(Q)$).

Theorem 3.3. *For a map f : (K, $\tau_R(P)$) → (L, $\sigma_R(Q)$), the following implications hold:*

1. *$n\ddot{g}$-cts ⇒ $n\ddot{g}_t$-cts.*

2. *$n\ddot{g}$-cts ⇒ $n\ddot{g}_\alpha *$-cts.*

3. *$n\ddot{g}_t$-cts is an $n\ddot{g}_\alpha *$-cts.*

4. *$n\ddot{g}$-cts ⇒ $n\ddot{g}_\alpha$-cts ⇒ $n\ddot{g}_p$-cts. [20]*

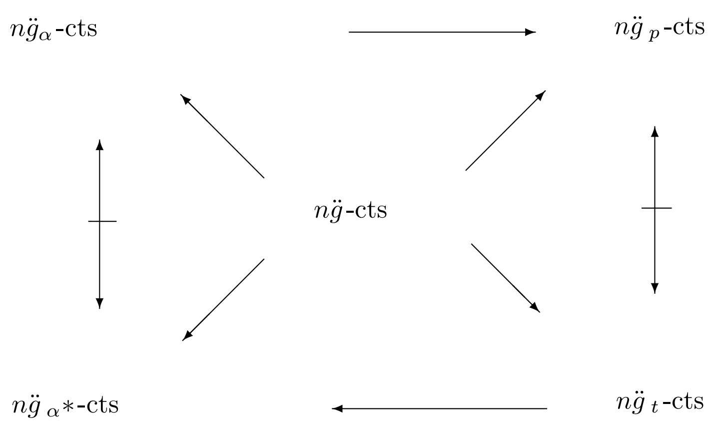
Proof. (1) and (2). The proof is straightforward from the proposition 3.1.
(3). The proof is straightforward from the proposition 3.2. □

Remark 3.5. ,

1. *$n\ddot{g}_t$-cts and $n\ddot{g}_p$-cts are independent.*

2. *$n\ddot{g}_\alpha *$-cts and $n\ddot{g}_\alpha$-cts are independent.*

Remark 3.6.



We obtained some dcomposition of $n\ddot{g}$-cts

Theorem 3.4. *A map $f: (K, \tau_R(P)) \to (L, \sigma_R(Q))$ is $n\ddot{g}$-cts iff it is both $n\ddot{g}_{\alpha}$-cts and $n\ddot{g}_{\ \alpha}*$-cts.*

Proof. The proof is straightforward from theorem 3.1. □

Theorem 3.5. *A map $f: (K, \tau_R(P)) \to (L, \sigma_R(Q))$ is $n\ddot{g}$-cts iff it is both $n\ddot{g}_{\ p}$-cts, $n\eta^{\sharp\sharp}$-cts and $n\ddot{g}_{\ \alpha}*$-cts.*

Proof. The proof is straightforward from theorem 2.2 (2) and theorem 3.4. □

Theorem 3.6. *A map $f: (K, \tau_R(P)) \to (L, \sigma_R(Q))$ is $n\ddot{g}$-cts iff it is both $n\ddot{g}_{\ p}$-cts and $n\ddot{g}_{\ t}$-cts.*

Proof. The proof is straightforward from theorem 3.2. □

Conclusion

We obtained decomposition of $n\ddot{g}$-cts in ntss using $n\ddot{g}_{\ p}$-cts, $n\ddot{g}_{\alpha}$-cts, $n\ddot{g}_{\ t}$-cts and $n\ddot{g}_{\ \alpha}*$-cts. The results of this study may be help to many researches.

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