On decomposition of $n\tilde{g}$-continuity in nano Topological spaces

Selvaraj Ganesan
PG & Research Department of Mathematics,
Raja Doraisingham Government Arts College, Sivagangai-630561, Tamil Nadu, India.
(Affiliated to Alagappa University, Karaikudi, Tamil Nadu, India). Orchid iD: 0000-0002-7728-8941

Abstract: This article focuses on decomposition of a weaker form of nano continuity, namely $n\tilde{g}$-continuity, by providing the concepts of $n\tilde{g}\alpha$-sets, $n\tilde{g}\alpha*$-sets, $n\tilde{g}t$-continuity and $n\tilde{g}\alpha*$-continuity.

Key words: $n\tilde{g}\alpha$-closed, $n\tilde{g}p$-closed, $n\tilde{g}t$s, $n\tilde{g}\alpha*$

1. Introduction
Various interesting problems arise when one considers nano continuous (in short, n-cts) and nano generalized continuous (in shor, ng-cts). Decomposition of n-cts are obtained by many mathematician with the help of ng-cts maps in nano topological spaces (in short, ntss)([5, 6, 19]). This article, we obtained decomposition of $n\tilde{g}$-continuous (in short, $n\tilde{g}$-cts) in ntss using $n\tilde{g}p$-continuous (in short, $n\tilde{g}p$-cts) [20], $n\tilde{g}\alpha$-continuous (in short, $n\tilde{g}\alpha$-cts) [20], $n\tilde{g}t$-continuous (in short, $n\tilde{g}t$-cts) and $n\tilde{g}\alpha*$-continuous (in short, $n\tilde{g}\alpha*$-cts).

2. Preliminaries
Definition 2.1. [11] If $(K, \tau_R(P))$ is the ntss with respect to P where $P \subseteq K$ and if $S \subseteq K$, then

1. The nano interior of the set $S$ is defined as the union of all nano open (in short, no) subsets contained in $S$ and it is denoted by $ninte(S)$. That is, $ninte(S)$ is the largest no subset of $S$.
2. The nano closure of the set $S$ is defined as the intersection of all nano closed (in short, nc) sets containing $S$ and it is denoted by $nclo(S)$. That is, $nclo(S)$ is the smallest nc set containing $S$.

Definition 2.2. [11] A subset $S$ of a ntss $(K, \tau_R(P))$ is said to be

1. nano $\alpha$-closed (in short, $n\alpha c$) if $nclo(nclo(nclo(S))) \subseteq S$.
2. nano semi-closed (in short, nsc) if $ninte(nclo(S)) \subseteq S$.
3. nano pre-closed (in short, npc) if $nclo(ninte(S)) \subseteq S$.

The complements of the above mentioned nc are called their respective no respectively.

The nano $\alpha$-closure [18] (respectively, nano semi-closure [2, 3], nano pre-closure [1]) of a subset $S$ of $K$, denoted by $n\alpha clo(S)$ (respectively, $nsclo(S)$, $npclo(S)$) is defined to be the intersection of all $n\alpha c$ (respectively, nsc, npc) sets of $(K, \tau_R(P))$ containing $S$. 

©Asia Mathematika, DOI: 10.5281/zenodo.5253049
*Correspondence: sgsgsgsgsg77@gmail.com
The nano $\alpha$-interior [18] (respectively, nano semi-interior [2, 3], nano pre-interior[1]) of a subset $S$ of $K$, denoted by $\ nano $\alpha$-interior \[18\] (respectively, $\ nano$ semi-interior $[2, 3]$, nano pre-interior$[1]$) of a subset $S$ of $K$, denoted by $\ nano$ $\alpha$ (respectively, $\ nano$ semi, $\ nano$ pre) sets of $(K, \tau_R(P))$ containing $S$.

**Definition 2.3.** A subset $S$ of a ntss $(K, \tau_R(P))$ is called:

1. nsg-open (in short, nsgo) [2] if $T \subseteq nsinte(S)$ whenever $T \subseteq S$ and $T$ is nano semi-closed in $(K, \tau_R(P))$.
2. n$\tilde{g}$-open (in short, n$\tilde{g}$o) [18] if $T \subseteq ninte(S)$ whenever $T \subseteq S$ and $T$ is nsg-closed in $(K, \tau_R(P))$.
3. n$\tilde{g}$-$\alpha$-open (in short, n$\tilde{g}$-$\alpha$o) [18] if $T \subseteq ninte(S)$ whenever $T \subseteq S$ and $T$ is nsg-closed in $(K, \tau_R(P))$.
4. n$\tilde{g}$-$p$-open (in short, n$\tilde{g}$-$p$o) [20] if $T \subseteq npinte(S)$ whenever $T \subseteq S$ and $T$ is nsg-closed in $(K, \tau_R(P))$.

The complement of above mentioned no-sets are called their respective nc-sets.

**Definition 2.4.** A subset $S$ of a ntss $(K, \tau_R(P))$ is said to be:

1. nt-set (in short, nts) [10] if $ninte(S) = ninte(nclo(S))$.
2. n$\alpha$-*set (in short, n$\alpha$*-s) [16] if $ninte(S) = ninte(nclo(ninte(S)))$.
3. an $\eta$-set if $S = I \cap J$ where $I$ is no & $J$ is a n$\alpha$c.
4. $\eta^s$-set (in short, $\eta^s$s) [19] if $S = I \cap J$, where $I$ is nsgo & $J$ is n$\alpha$c in $(K, \tau_R(P))$.
5. $\eta^d$-set (in short, $\eta^d$s) [19] if $S = I \cap J$, where $I$ is n$\tilde{g}$-$\alpha$o & $J$ is nts in $(K, \tau_R(P))$.
6. n$\tilde{g}$lc*-set (in short, n$\tilde{g}$lc*-s) [20] if $S = I \cap J$, where $I$ is nsgo & $J$ is nc $(K, \tau_R(P))$.

Collection of all $\eta$s (respectively, $\eta^s$s, $\eta^d$s) in a ntss $(K, \tau_R(P))$ is denoted by $\eta(K, \tau_R(P))$ (respectively, $\eta^s(K, \tau_R(P))$, $\eta^d(K, \tau_R(P))$).

**Remark 2.1.** n$\alpha$c sets and n$\tilde{g}$c sets are independent.

**Example 2.1.** Let $K = \{11, 12, 13\}$ with $K / R = \{\{13\}, \{11, 12\}, \{12, 11\}\}$ and $P = \{11, 12\}$, $\tau_R(P) = \{\phi, \{11, 12\}, K\}$. Here, the set $\{12, 13\}$ is n$\tilde{g}$c but not n$\alpha$c.

2. Let $K = \{11, 12, 13\}$ with $K / R = \{\{11\}, \{12, 13\}\}$ and $P = \{11\}$, $\tau_R(P) = \{\phi, \{11\}, K\}$. Here, the set $\{12\}$ is n$\alpha$c but not n$\tilde{g}$c.

**Remark 2.2.** [16]

1. Every nts is an $\alpha$-*s but not conversely.
2. Union of 2 $\alpha$-*s need not be an $\alpha$-*s.
3. Intersection of 2 $\alpha$-*s is an $\alpha$-*s.

**Definition 2.5.** A map $f : (K, \tau_R(P)) \to (L, \sigma_R(Q))$ is called:

1. n-cts [12] if $f^{-1}(D)$ is a n in $(K, \tau_R(P))$, for every no set D of $(L, \sigma_R(Q))$.
2. n$\alpha$-continuous (in short, n$\alpha$-cts) [13] if $f^{-1}(D)$ is n$\alpha$o in $(K, \tau_R(P))$, for every no set D of $(L, \sigma_R(Q))$. 

2
3. $n\gamma$-cts [20] if $f^{-1}(D)$ is $n\gamma o$ in $(K, \tau_R(P))$, for every no set $D$ of $(L, \sigma_R(Q))$.

4. $n\gamma_{\alpha}$-cts [20] (respectively, $n\gamma_{\rho}$-cts [20]) if $f^{-1}(D)$ is $n\gamma_{\alpha} o$ (respectively, $n\gamma_{\rho} o$) set in $(K, \tau_R(P))$, for every no set $D$ of $(L, \sigma_R(Q))$.

5. $n\gamma lc^*$-continuous (in short, $n\gamma lc^*$-cts) [20] if $f^{-1}(D) \in n\gamma lc^*(K, \tau_R(P))$, for every no set $D$ of $(L, \sigma_R(Q))$.

6. $n\eta^\sharp$-continuous (in short, $n\eta^\sharp$-cts) [18] if $f^{-1}(D) \in n\eta^\sharp(K, \tau_R(P))$, for every no set $D$ of $(L, \sigma_R(Q))$.

7. $n\eta^\sharp$-continuous (in short, $n\eta^\sharp$-cts) [18] if $f^{-1}(D) \in n\eta^\sharp(K, \tau_R(P))$, for every no set $D$ of $(L, \sigma_R(Q))$.

**Theorem 2.1.** [20] A map $f : (K, \tau_R(P)) \to (L, \sigma_R(Q))$ is $n$-cts iff it is both $n\gamma$-cts and $n\gamma lc^*$-cts.

**Theorem 2.2.** [19]

1. A map $f : (K, \tau_R(P)) \to (L, \sigma_R(Q))$ is $n\alpha$-cts iff it is both $n\gamma_{\alpha}$-cts and $n\eta^\sharp$-cts.

2. A map $f : (K, \tau_R(P)) \to (L, \sigma_R(Q))$ is $n\gamma_{\alpha}$-cts iff it is both $n\gamma_{\rho}$-cts and $n\eta^\sharp$-cts.

3. $n\gamma_{\iota}$s & $n\gamma_{\alpha}$s

**Definition 3.1.** A subset $H$ of a ntss $(K, \tau_R(P))$ is said to be

1. an $n\gamma_{\iota}$-sets (in short, $n\gamma_{\iota}$s) if $H = I \cap J$, where $I$ is $n\gamma o$ in $K$ & $J$ is a $nts$ in $K$.

2. an $n\gamma_{\alpha}$s-sets (in short, $n\gamma_{\alpha}$s) if $H = I \cap J$, where $I$ is $n\gamma o$ in $K$ & $J$ is a $n\alpha$-s in $K$.

Collection of all $n\gamma_{\iota}$s (respectively, $n\gamma_{\alpha}$s) in a ntss $(K, \tau_R(P))$ is denoted by $n\gamma_{\iota}(K, \tau_R(P))$ (respectively, $n\gamma_{\alpha}*(K, \tau_R(P))$).

**Proposition 3.1.** Let $H$ be a subset of $K$. Then

1. If $H$ is a $nts$, then $H \in n\gamma_{\iota}(K, \tau_R(P))$.

2. If $H$ is an $n\alpha$-s, then $H \in n\gamma_{\alpha}*(K, \tau_R(P))$.

3. If $H$ is an $n\gamma o$ set in $K$, then $H \in n\gamma_{\iota}(K, \tau_R(P))$ & $H \in n\gamma_{\alpha}*(K, \tau_R(P))$.

**Proof.** The proof is straightforward from the definitions.

**Proposition 3.2.** In a ntss $K$, every $n\gamma_{\iota}$s is an $n\gamma_{\alpha}$s but not conversely.

**Proof.** The proof is straightforward from the definitions.

**Example 3.1.** Let $K$ and $\tau_R(P)$ in the Example 2.1(1). Here, the set $\{11, 13\}$ is $n\gamma_{\alpha}$s but not $n\gamma_{\iota}$s.

**Remark 3.1.** The following examples show that

1. the converse of Proposition 3.1 need not be true.

2. $n\gamma_{\iota}$s & $n\gamma_{\rho} o$ sets are independent.

3. $n\gamma_{\alpha}$s & $n\gamma_{\rho} o$ sets are independent.
Example 3.2. Let $K \& \tau_R(P)$ in the Example 2.1(1). Here, the set $\{11\}$ is $\text{n} \text{g} \text{t}$ but not an $\text{n} \text{g}$ $\text{a}$ *s but not $\text{n} \text{g}$ o $\text{s}$.

Example 3.3. Let $K \& \tau_R(P)$ in the Example 2.1(1). Here, the set $\{13\}$ is both $\text{n} \text{g} \text{t}$ and $\text{n} \text{g}$ $\text{a}$ *s but not $\text{n} \text{g}$ o set.

Example 3.4. Let $K \& \tau_R(P)$ in the Example 2.1(1). Here, (a) the set $\{13\}$ is $\text{n} \text{g} \text{t}$ but not a $\text{n} \text{g}$ $\text{p}$ o, (b) the set $\{12, 13\}$ is a $\text{n} \text{g}$ $\text{p}$ o set but not $\text{n} \text{g}$ t $\text{s}$.

Example 3.5. Let $K = \{11, 12, 13, 14\}$ with $K / R = \{\{11\}, \{12, 13\}, \{12, 14\}\}$ and $P = \{11\}$, $\tau_R(P) = \{\phi, \{11\}, K\}$. Here, (a) the set $\{12\}$ is $\text{n} \text{g}$ $\text{a}$ *s but not $\text{n} \text{g}$ a o, (b) the set $\{11, 13\}$ is $\text{n} \text{g}$ a o but not $\text{n} \text{g}$ a *s.

Example 3.6. Let $K \& \tau_R(P)$ in the Example 2.1(1). Here, the set $\{11\}$ is $\text{n} \text{g}$ $\text{a}$ *s and $\text{n} \text{g}$ t $\text{s}$ but it is not $\text{n} \text{g}$ c.

Remark 3.2. From the above discussions and known results in [14, 18, 20], we obtained the following diagram where $A \rightarrow B$ represents $A$ implies $B$, but not conversely.

```
\begin{array}{ccc}
nano closed & \rightarrow & \text{n} \text{g}-closed \\
\downarrow & & \downarrow \\
n\text{a}-closed & \rightarrow & \text{n} \text{g} \text{a}*-closed
\end{array}
```

Remark 3.3.
1. Union of $2$ $\text{n} \text{g}$ t $\text{s}$ need not be an $\text{n} \text{g}$ t $\text{s}$.
2. Union of $2$ $\text{n} \text{g}$ $\text{a}$ *s need not be an $\text{n} \text{g}$ $\text{a}$ *s.

Example 3.7.
1. Let $K$ and $\tau_R(P)$ in the Example 2.1(1). Here, the sets are $\{12\}$ and $\{13\}$ are $\text{n} \text{g}$ t $\text{-sets}$ but $\{12\} \cup \{13\} = \{12, 13\}$ is not an $\text{n} \text{g}$ t $\text{-set}$.
2. Let $K$ and $\tau_R(P)$ in the Example 3.5. Here, the sets are $\{11\}$ and $\{12\}$ are $\text{n} \text{g}$ $\text{a}$ *$\text{-sets}$ but $\{11\} \cup \{12\} = \{11, 12\}$ is not an $\text{n} \text{g}$ $\text{a}$ *$\text{s}$.

Remark 3.4.
1. Intersection of any $2$ $\text{n} \text{g}$ t $\text{s}$ belongs to $\text{n} \text{g}$ t ($K, \tau_R(P)$).
2. Intersection of any $2$ of $\text{n} \text{g}$ $\text{a}$ *$\text{s}$ belongs to $\text{n} \text{g}$ $\text{a}$ * ($K, \tau_R(P)$).

Lemma 3.1.
1. A subset $H$ of $(K, \tau_{R}(P))$ is $n\tilde{g}o$ [18] iff $F \subseteq \text{ninte}(H)$ whenever $F \subseteq H$ and $F$ is nsgc in $K$.

2. A subset $H$ of $(K, \tau_{R}(P))$ is $n\tilde{g}\alpha o$ [20] iff $F \subseteq \text{nalpha}(H)$ whenever $F \subseteq H$ and $F$ is nsgc in $K$.

3. A subset $H$ of $(K, \tau_{R}(P))$ is $n\tilde{g}\beta o$ [20] iff $F \subseteq \text{npinte}(H)$ whenever $F \subseteq H$ and $F$ is nsgc in $K$.

**Theorem 3.1.** A subset $H$ is $n\tilde{g}o$ in $(K, \tau_{R}(P))$ iff it is both $n\tilde{g}\alpha o$ and $n\tilde{g}\alpha s$ in $(K, \tau_{R}(P))$.

**Proof.** Necessity. Obvious.

Sufficiency. Suppose $H$ be both $n\tilde{g}\alpha o$ set and $n\tilde{g}\alpha s$. Since $H$ is an $n\tilde{g}\alpha s$, $H = I \cap J$, where $I$ is $n\tilde{g}o$ and $J$ is an $nalpha$s. Assume that $F \subseteq H$, where $F$ is nsgc in $K$. Since $I$ is $n\tilde{g}o$, by Lemma 3.1 (1), $F \subseteq \text{ninte}(H)$. Since $H$ is $n\tilde{g}o$ in $K$, by Lemma 3.1 (2), $F \subseteq \text{nalpha}(H) = H \cap \text{ninte}(\text{nclo}(\text{ninte}(H))) = (I \cap J) \cap \text{ninte}(\text{nclo}(\text{ninte}(I \cap J))) \subseteq I \cap J \cap \text{ninte}(\text{nclo}(\text{ninte}(I))) \subseteq \text{ninte}(J)$. Therefore, we obtained $F \subseteq \text{ninte}(J)$ and hence $F \subseteq \text{ninte}(I) \cap \text{ninte}(J) = \text{ninte}(H)$. Hence $H$ is nsgo, by Lemma 3.1 (1).

**Theorem 3.2.** A subset $H$ is $n\tilde{g}o$ in $(K, \tau_{R}(P))$ iff it is both $n\tilde{g}\beta o$ and $n\tilde{g}\beta s$ in $(K, \tau_{R}(P))$.

**Proof.** The proof is similar to theorem 3.1.

**Definition 3.2.** A map $f : (K, \tau_{R}(P)) \to (L, \sigma_{R}(Q))$ is said to be

1. $n\tilde{g}\iota$-cts if $f^{-1}(D) \in n\tilde{g}\iota(K, \tau_{R}(P))$, for every no set $D$ of $(L, \sigma_{R}(Q))$.

2. $n\tilde{g}\alpha*$-cts if $f^{-1}(D) \in n\tilde{g}\alpha*(K, \tau_{R}(P))$, for every no set $D$ of $(L, \sigma_{R}(Q))$.

**Theorem 3.3.** For a map $f : (K, \tau_{R}(P)) \to (L, \sigma_{R}(Q))$, the following implications hold:

1. $n\tilde{g}$-cts $\Rightarrow$ $n\tilde{g}\iota$-cts.
2. $n\tilde{g}$-cts $\Rightarrow$ $n\tilde{g}\alpha*$-cts.
3. $n\tilde{g}\iota$-cts is an $n\tilde{g}\alpha*$-cts.
4. $n\tilde{g}$-cts $\Rightarrow$ $n\tilde{g}\alpha$-cts $\Rightarrow$ $n\tilde{g}\beta$-cts. [20]

**Proof.** (1) and (2). The proof is straightforward from the proposition 3.1.

(3). The proof is straightforward from the proposition 3.2.

**Remark 3.5.**

1. $n\tilde{g}\iota$-cts and $n\tilde{g}\beta$-cts are independent.

2. $n\tilde{g}\alpha*$-cts and $n\tilde{g}\alpha$-cts are independent.

**Remark 3.6.**
Selvaraj Ganesan

We obtained some decomposition of $n\check{g}$-cts

**Theorem 3.4.** A map $f : (K, \tau_R(P)) \to (L, \sigma_R(Q))$ is $n\check{g}$-cts iff it is both $n\check{g}_\alpha$-cts and $n\check{g}_\alpha^*$-cts.

**Proof.** The proof is straightforward from theorem 3.1.

**Theorem 3.5.** A map $f : (K, \tau_R(P)) \to (L, \sigma_R(Q))$ is $n\check{g}$-cts iff it is both $n\check{g}_\mu$-cts, $n\check{g}_\#^*$-cts and $n\check{g}_\alpha^*$-cts.

**Proof.** The proof is straightforward from theorem 2.2 (2) and theorem 3.4.

**Theorem 3.6.** A map $f : (K, \tau_R(P)) \to (L, \sigma_R(Q))$ is $n\check{g}$-cts iff it is both $n\check{g}_\mu$-cts and $n\check{g}_\check{t}$-cts.

**Proof.** The proof is straightforward from theorem 3.2.

**Conclusion**

We obtained decomposition of $n\check{g}$-cts in ntss using $n\check{g}_\mu$-cts, $n\check{g}_\alpha$-cts, $n\check{g}_\check{t}$-cts and $n\check{g}_\alpha^*$-cts. The results of this study may be help to many researches.

**References**


