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On some nano Δ-open set in nanotopological spaces

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Abstract: The object of the article work is define a new class of some nano sets, we study properties of nano Δ-open and semi nano Δ-open and we define the nano Δ-interior, nano Δ-closure, semi nano Δ-interior and semi nano Δ-closure of a set E_1 in a nano topological spaces.

Key words: n-open sets, ns-open sets, nano Δ-open sets, semi nano Δ-open sets.

1. Introduction and Preliminaries

In 2013 M. Lellis Thivagar et al. introduced the concept of nano topological spaces and weak form of nano sets, nano semi-open (or ns-open) sets and nano semi-closed (or ns-closed) sets.

After that many nano topologist paved their own pathway in nano spaces introducing new types of nano sets.

Through out this article, The elements of $\tau_R(X)$ are called nano-open sets (or n-open sets).

The complement of a n-open set is called n-closed. we signify a nano space by $(U, \mathcal{N})$, where $\mathcal{N} = \tau_R(X)$. The nano-interior and nano-closure of a subset E_1 of U are denoted by $n\text{-}I(E_1)$ and $n\text{-}C(E_1)$.

2. On nano Δ-open and semi nano Δ-open sets

Definition 2.1. A subset E_1 of a nano space is called a

1. nano Δ-open $\iff$ $E_1 = (Z_1 - Z_2) \cup (Z_2 - Z_1)$, where Z_1 and Z_2 are n-open subsets in U.
2. semi nano Δ-open $\iff$ $E_1 = (Z_1 - Z_2) \cup (Z_2 - Z_1)$, where Z_1 and Z_2 are ns-open subsets in U.

The complements of the above mentioned sets are called their respective nano closed sets.

Definition 2.2. A subset E_1 of a nano space is called a

1. nano Δ-interior of E_1 is defined as $\cup\{D : D \subseteq E_1$ and D is nano Δ-open$\}$.
2. nano Δ-closure of E_1 is defined as $\cap\{D : E_1 \subseteq D$ and D is nano Δ-closed$\}$.
3. semi nano Δ-interior of E_1 is defined as $\cup\{D : D \subseteq E_1$ and D is semi nano Δ-open$\}$.

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4. semi nano Δ-closure of E_1 is defined as $\cap\{D : E_1 \subseteq D$ and D is semi nano Δ-closed$\}$.

The nano Δ-interior, nano Δ-closure, semi nano Δ-interior and semi nano Δ-closure of a subset E_1 of U are indicated by $n\text{-}\Delta I(E_1)$, $n\text{-}\Delta C(E_1)$, $ns\text{-}\Delta I(E_1)$ and $ns\text{-}\Delta C(E_1)$ respectively.

Remark 2.1. *In a nano space, if E_1 is nano Δ-open set as moreover every nano semi-open set is semi nano Δ-open.*

But the converse implications are not true.

Example 2.1. *Let $U = \{100, 101, 102\}$ with $U/R = \{\{100\}, \{101, 102\}\}$ and $X = \{100\}$. Then $\mathcal{N} = \{\{100\}, \phi, U\}$ is a nano topology on U. Then $\{101\}$ is semi nano Δ-open but is nighter nano Δ-open nor ns-open.*

Theorem 2.1. *If E_1 is a semi nano Δ-open $\Longrightarrow \exists$ a nano Δ-open set $V_1 \ni V_1 \subseteq E_1 \subseteq n\text{-}C(V_1)$.*

Proof.

If E_2 is semi nano Δ-open, there exist ns-open sets Z_1 and Z_2 such that $E_2 = (Z_1 - Z_2) \cup (Z_2 - Z_1))$. Now, Z_1 and Z_2 are ns-open sets, it follows that $\exists$ n-open sets K_1 and K_2 in $U \ni K_1 \subseteq Z_1 \subseteq n\text{-}C(K_1)$ and $K_2 \subseteq Z_2 \subseteq n\text{-}C(K_2)$. This implies that $(R_1 - R_2) \cup (R_2 - R_1) \subseteq (Z_1 - Z_2) \cup (Z_2 - Z_1) \subseteq [n\text{-}C(R_1) - n\text{-}C(R_2)] \cup [n\text{-}C(R_2) - n\text{-}C(R_1)] \subseteq n\text{-}C(R_1 - R_2) \cup n\text{-}C(R_2 - R_1) = n\text{-}C[(R_1 - R_2) \cup (R_2 - R_1)]$.

Theorem 2.2. *If E_1 is n-open and E_2 is semi nano Δ-open in a nano space, then $E_1 \cap E_2$ is semi nano Δ-open in U.*

Proof.

Let $E_1 \cap E_2 = E_1 \cap [(Z_1 - Z_2) \cup (Z_2 - Z_1)]$ where Z_1 and Z_2 are ns-open in U.

$E_1 \cap [(Z_1 - Z_2) \cup (Z_2 - Z_1)] = [E_1 \cap (Z_1 - Z_2)] \cup [E_1 \cap (Z_2 - Z_1)] = [(E_1 \cap Z_1) - (E_1 \cap Z_2)] \cup [(E_1 \cap Z_2) - (E_1 \cap Z_1)]$ which is semi nano Δ-open (since $(E_1 \cap Z_2)$ *and* $(E_1 \cap Z_1)$ are ns-open).

Theorem 2.3. *A subset E_1 of a nano space is semi nano Δ-open $\Longleftrightarrow E_1 \subseteq n\text{-}C(n\text{-}\Delta I(E_1))$.*

Proof.

E_1 be a semi nano Δ-open subset of U. Then $\exists$ a nano Δ-open set $G \ni G \subseteq E_1 \subseteq n\text{-}C(G)$. But $G = n\text{-}\Delta I(G) \subseteq n\text{-}\Delta I(E_1)$ and so $n\text{-}C(G) \subseteq n\text{-}C(n\text{-}\Delta I(E_1))$. Thus $E_1 \subseteq n\text{-}C(G) \subseteq n\text{-}C(n\text{-}\Delta I(E_1))$. Now, suppose $E_1 \subseteq n\text{-}C(n\text{-}\Delta I(E_1))$. Put $G = n\text{-}\Delta I(E_1)$. Thus G is nano Δ-open with $G \subseteq E_1 \subseteq n\text{-}C(n\text{-}\Delta I(E_1))$. Hence $G \subseteq E_1 \subseteq n\text{-}C(G)$ and E_1 is semi nano Δ-open.

Theorem 2.4. *If $\big\{\{E_1\}_i : i \in K\big\}$ is a family of semi nano Δ-open subsets, then $\bigcup_{i\in K}\{E_1\}_i$ is semi nano Δ-open.*

Proof.

For each $i \in K$, $\exists$ a semi nano Δ-open set G_i such that $G_i \subseteq \{E_1\}_i \subseteq n\text{-}C(G_i)$. Now $\bigcup_{i\in E_1}\{E_1\}_i \subseteq \bigcup_{i\in K}\{E_1\}_i \subseteq \bigcup_{i\in K} n\text{-}C(G_i) \subseteq n\text{-}C\big(\bigcup_{i\in K} G_i\big)$. Then $\bigcup_{i\in K}\{E_1\}_i$ is semi nano Δ-open.

Theorem 2.5. *Let E_1 be a subset of a nano space. Then $ns\text{-}\Delta I(E_1) = E_1 \cap n\text{-}C(n\text{-}\Delta I(E_1))$.*

Proof.

Let $E_1 \cap n\text{-}C(n\text{-}\Delta I(E_1)) \subseteq n\text{-}C[n\text{-}\Delta I(E_1 \cap n\text{-}I(E_1))] \subseteq n\text{-}C[n\text{-}\Delta I(E_1 \cap n\text{-}C(n\text{-}\Delta I(E_1))]$. Thus $E_1 \cap n\text{-}C(n\text{-}\Delta I(E_1))$ is semi nano Δ-open set contained in E_1. Hence $E_1 \cap n\text{-}C(\Delta I(E_1)) \subseteq ns\text{-}\Delta I(E_1))$.

On the other hand, since $ns\text{-}\Delta I(E_1)$ is a semi nano Δ-open set, we have $ns\text{-}\Delta I(E_1) \subseteq n\text{-}C(n\text{-}\Delta I(ns\text{-}\Delta I(E_1)) \subseteq n\text{-}C(ns\text{-}\Delta I(E_1))$. Hence $ns\text{-}\Delta I(E_1) = E_1 \cap n\text{-}C(n\text{-}\Delta I(E_1))$.

Theorem 2.6. *Let $(V, \mathcal{N})$ be a subspace of a space $(U, \mathcal{N})$ and let $E_1 \subseteq V$. If E_1 is semi nano Δ-open in U, then E_1 is semi nano Δ-open in V.*

Proof.

Let E_1 is semi nano Δ-open in U, there exist a nano Δ-open set K in U such that $K \subseteq E_1 \subseteq n\text{-}C(K)$. Then $K = K \cap V \subseteq E_1 \subseteq n\text{-}C(K) \cap V = n\text{-}C_V(K)$. Thus E_1 is semi nano Δ-open in V.

Theorem 2.7. *Let V be a nano Δ-open set in a space $(U, \mathcal{N})$. If $E_1 \subseteq V$ and E_1 is semi nano Δ-open in V, then E_1 is semi nano Δ-open in U.*

Proof.

Let E_1 be semi nano Δ-open in V. Then $\exists$ a nano Δ-open subset K of $(V, \mathcal{N}) \ni K \subseteq E_1 \subseteq n\text{-}C_V(K)$. Since V is nano Δ-open in U, therefore, K is nano Δ-open in U and $K \subseteq E_1 \subseteq n\text{-}C_V(K) \subseteq n\text{-}C(K)$. Hence E_1 is semi nano Δ-open in U.

Theorem 2.8. *A subset E_1 of a space $(U, \mathcal{N})$ is semi nano Δ-closed $\iff n\text{-}I(n\text{-}\Delta C(E_1)) \subseteq E_1$.*

Proof.

Obvious.

Theorem 2.9. *If E_1 is a subspace of a space $(U, \mathcal{N})$, then $ns\text{-}\Delta C(E_1) = E_1 \cup n\text{-}I(n\text{-}\Delta C(E_1))$.*

Proof.

Let $n\text{-}I[n\text{-}\Delta C(E_1 \cup n\text{-}I(n\text{-}\Delta C(E_1))] \subseteq n\text{-}I[n\text{-}\Delta C(E_1 \cup n\text{-}C(E_1))] = n\text{-}I[n\text{-}\Delta C(E_1)] \subseteq E_1 \cup n\text{-}I[n\text{-}\Delta C(E_1))]$.Thus by Theorem 2.8 is $E_1 \cup n\text{-}I(n\text{-}\Delta C(E_1))$ is a semi nano Δ-closed set containing E_1 and so $ns\text{-}\Delta C(E_1) \subseteq E_1 \cup n\text{-}I(n\text{-}\Delta C(E_1))$.

On the other hand, since $ns\text{-}\Delta C(E_1)$ is semi nano Δ-closed, therefore,

$n\text{-}I[n\text{-}\Delta C(ns\text{-}\Delta C(E_1) \subseteq ns\text{-}\Delta C(E_1)]$.

Hence $n\text{-}I[n\text{-}\Delta C(E_1)] \subseteq n\text{-}I[n\text{-}\Delta C(ns\text{-}\Delta C(E_1))] \subseteq ns\text{-}\Delta C(E_1)$ and consequently $E_1 \cup n\text{-}I(n\text{-}\Delta C(E_1)) \subseteq ns\text{-}\Delta C(E_1)$. Thus $ns\text{-}\Delta C(E_1) = E_1 \cup n\text{-}I(n\text{-}\Delta(E_1))$.

Theorem 2.10. *Let E_1 is a subset of $(U, \mathcal{N})$. Then $ns\text{-}\Delta C(E_1) \subseteq ns\text{-}C(E_1) \cap n\text{-}\Delta C(E_1)$*

Proof.

Let $ns\text{-}\Delta C(E_1) = E_1 \cup n\text{-}I(n\text{-}\Delta C(E_1)) \subseteq n\text{-}I(n\text{-}C(E_1)) = ns\text{-}C(E_1)$.

Also, $ns\text{-}\Delta C(E_1) \subseteq E_1 \cup n\text{-}\Delta C(E_1) = n\text{-}\Delta C(E_1)$. Therefore, $ns\text{-}\Delta C(E_1) \subseteq ns\text{-}C(E_1) \cap \Delta n\text{-}C(E_1)$.

Theorem 2.11. *If K is n-closed and M is semi nano Δ-closed in a space $(U, \mathcal{N})$, then $K \cup M$ is semi nano Δ-closed.*

Proof.

Let $(U-K)$ is n-open and $(U-M)$ is semi nano Δ-open. Then by Theorem 2.2 $(U-K)\cap(U-M)$ is semi nano Δ-open. That is $U-(K\cup M)$ is semi nano Δ-open. Hence $K\cup M$ is semi nano Δ-closed.

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