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Weak forms of strongly nano open sets in ideal nanotopological spaces

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Abstract: The object of the paperwork is to define new concepts called strongly α-nI-open sets, strongly pre-nI-open sets, strongly b-nI-open sets and strongly β-nI-open sets, which are weak forms of nano open sets in ideal nanotopological spaces. Also we characterize the relations between them and the related properties.

Key words: Strongly α-nI-open sets, strongly pre-nI-open sets, strongly b-nI-open sets, strongly β-nI-open sets

1. Introduction

An ideal I [13] on a topological space (X, τ) is a non-empty collection of subsets of X which satisfies the following conditions.

1. $A \in I$ and $B \subset A$ imply $B \in I$ and
2. $A \in I$ and $B \in I$ imply $A \cup B \in I$.

Given a topological space (X, τ) with an ideal I on X. If $\wp(X)$ is the family of all subsets of X, a set operator $(.)^\star : \wp(X) \to \wp(X)$, called a local function of A with respect to τ and I is defined as follows: for $A \subset X$, $A^\star(I,\tau) = \{x \in X : U \cap A \notin I \text{ for every } U \in \tau(x)\}$ where $\tau(x) = \{U \in \tau : x \in U\}$ [1]. The closure operator defined by $cl^\star(A) = A \cup A^\star(I,\tau)$ [12] is a Kuratowski closure operator which generates a topology $\tau^\star(I,\tau)$ called the $\star$-topology finer than τ. The topological space together with an ideal on X is called an ideal topological space or an ideal space denoted by (X,τ,I). We will simply write $A^\star$ for $A^\star(I,\tau)$ and $\tau^\star$ for $\tau^\star(I,\tau)$.

Some types of new notions in the concept of ideal nanotopological spaces were introduced by Parimala et al. [3, 4] and Rajasekaran et al. [7, 8, 10].

In this paper work is define a new concepts called strongly α-nI-open sets, strongly pre-nI-open sets, strongly b-nI-open sets and strongly β-nI-open sets, which are weak forms of nano open sets in ideal nanotopological spaces. Also we characterize the relations between them and the related properties.

2. Preliminaries

Definition 2.1. [6] Let U be a non-empty finite set of objects called the universe and R be an equivalence

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relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$.

1. The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X) = \bigcup_{x\in U}\{R(x) : R(x) \subseteq X\}$, where R(x) denotes the equivalence class determined by x.
2. The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is, $U_R(X) = \bigcup_{x\in U}\{R(x) : R(x) \cap X \neq \phi\}$.
3. The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not - X with respect to R and it is denoted by $B_R(X)$. That is, $B_R(X) = U_R(X) - L_R(X)$.

Definition 2.2. [2] Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then $\tau_R(X)$ satisfies the following axioms:

1. U and $\phi \in \tau_R(X)$,
2. The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$,
3. The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

Thus $\tau_R(X)$ is a topology on U called the nano topology with respect to X and $(U, \tau_R(X))$ is called the nano topological space. The elements of $\tau_R(X)$ are called nano-open sets (briefly n-open sets). The complement of a n-open set is called n-closed.

Through out this paper, we denote a nano topological space by $(U, \mathcal{N})$, where $\mathcal{N} = \tau_R(X)$. The nano-interior and nano-closure of a subset A of U are denoted by $I_n(O)$ and $C_n(O)$, respectively.

Definition 2.3. A subset A of a space $(U, \mathcal{N})$ is called a

1. nano α-open (resp. $n\alpha$-open) [2] if $O \subseteq I_n(C_n(I_n(O)))$.
2. nano semi-open (resp. ns-open) [2] if $O \subseteq C_n(I_n(O))$.
3. nano pre-open (resp. np-open) [2] if $O \subseteq I_n(C_n(O))$.
4. nano b-open (resp. nb-open) [5] if $O \subseteq I_n(C_n(O)) \cup C_n(I_n(O))$.
5. nano β-open (resp. $n\beta$-open) [11] if $O \subseteq C_n(I_n(C_n(O)))$.

The complements of the above mentioned sets are called their respective closed sets.

A nano topological space $(U, \mathcal{N})$ with an ideal I on U is called [3] an ideal nano topological space and is denoted by $(U, \mathcal{N}, I)$. $G_n(x) = \{G_n \mid x \in G_n, G_n \in \mathcal{N}\}$, denotes [3] the family of nano open sets containing x.

In future an ideal nano topological spaces $(U, \mathcal{N}, I)$ is referred as a space.

Definition 2.4. [3] Let $(U, \mathcal{N}, I)$ be a space with an ideal I on U. Let $(.)^\star_n$ be a set operator from $\wp(U)$ to $\wp(U)$ ($\wp(U)$ is the set of all subsets of U). For a subset $A \subseteq U$, $A^\star_n(I, \mathcal{N}) = \{x \in U : G_n \cap A \notin I$, for every $G_n \in G_n(x)\}$ is called the nano local function (briefly, n-local function) of A with respect to I and $\mathcal{N}$. We will simply write $A^\star_n$ for $A^\star_n(I, \mathcal{N})$.

Theorem 2.1. *[3] Let $(U,\mathcal{N},I)$ be a space and A and B be subsets of U. Then*

1. $A \subseteq B \Rightarrow A_n^\star \subseteq B_n^\star$,
2. $A_n^\star = C_n(A_n^\star) \subseteq C_n(A)$ ($A_n^\star$ *is a* n*-closed subset of* $C_n(A)$),
3. $(A_n^\star)_n^\star \subseteq A_n^\star$,
4. $(A \cup B)_n^\star = A_n^\star \cup B_n^\star$,
5. $V \in \mathcal{N} \Rightarrow V \cap A_n^\star = V \cap (V \cap A)_n^\star \subseteq (V \cap A)_n^\star$,
6. $J \in I \Rightarrow (A \cup J)_n^\star = A_n^\star = (A - J)_n^\star$.

Theorem 2.2. *[3] Let $(U,\mathcal{N},I)$ be a space with an ideal I and $A \subseteq A_n^\star$, then $A_n^\star = C_n(A_n^\star) = C_n(A)$.*

Definition 2.5. [3] Let $(U,\mathcal{N},I)$ be a space. The set operator $C_n^\star$ called a nano $\star$-closure is defined by $C_n^\star(A) = A \cup A_n^\star$ for $A \subseteq X$.

It can be easily observed that $C_n^\star(A) \subseteq C_n(A)$.

Theorem 2.3. *[4] In a space $(U,\mathcal{N},I)$, if A and B are subsets of U, then the following results are true for the set operator $C_n^\star$.*

1. $A \subseteq C_n^\star(A)$,
2. $C_n^\star(\phi) = \phi$ *and* $C_n^\star(U) = U$,
3. *If* $A \subset B$, *then* $C_n^\star(A) \subseteq C_n^\star(B)$,
4. $C_n^\star(A) \cup C_n^\star(B) = C_n^\star(A \cup B)$,
5. $C_n^\star(C_n^\star(A)) = C_n^\star(A)$.

Definition 2.6. [4]

A subset O of a space $(U,\mathcal{N},I)$ is said to be nano-I-open (resp. nI-open) if $O \subseteq I_n(O_n^\star)$.

Definition 2.7. [7] A subset O of space $(U,\mathcal{N},I)$ is said to be a

1. nano α-I-open (resp. α-nI-open) if $O \subseteq I_n(C_n^\star(I_n(O)))$,
2. nano semi-I-open (resp. semi-nI-open) if $O \subseteq C_n^\star(I_n(O))$,
3. nano pre-I-open (resp. pre-nI-open) if $O \subseteq I_n(C_n^\star(O))$,
4. nano b-I-open (resp. b-nI-open) if $O \subseteq I_n(C_n^\star(O)) \cup C_n^\star(I_n(O))$,
5. nano β-I-open (resp. β-nI-open) if $O \subset C_n(I_n(C_n^\star(O)))$.

Remark 2.1. *[7] These relations are shown in the diagram.*

$$\begin{array}{ccccc}
n\text{-}\textbf{open} & \longrightarrow & \alpha\text{-}nI\text{-}\textbf{open} & \longrightarrow & \textbf{pre-}nI\text{-}\textbf{open} \\
 & & \downarrow & & \downarrow \\
 & & \textbf{semi-}nI\text{-}\textbf{open} & \longrightarrow & \textbf{b-}nI\text{-}\textbf{open} \\
 & & & & \downarrow \\
 & & & & \beta\text{-}nI\text{-}\textbf{open}
\end{array}$$

Lemma 2.1. *[7] Let $(U, \mathcal{N}, I)$ be a space and O a subset of U. If H is n-open in $(U, \mathcal{N}, I)$, then $H \cap C_n^\star(O) \subseteq C_n^\star(H \cap O)$.*

Definition 2.8. [10] A subset O of a space $(U, \mathcal{N}, I)$ is called a

1. nano t-I-set (resp. t-nI-set) if $I_n(O) = I_n(C_n^\star(O))$,
2. nano t$_\alpha$-I-set (resp. t$_\alpha$-nI-set) if $I_n(O) = I_n(C_n^\star(I_n(O)))$,
3. nano $\mathcal{R}$-I-set (resp. $\mathcal{R}$-nI-set) if $O = O_1 \cap O_2$, where O_1 is n-open and O_2 is t-nI-set,
4. nano $\mathcal{R}_\alpha$-I-set (resp. $\mathcal{R}_\alpha$-nI-set) if $O = O_1 \cap O_2$, where O_1 is n-open and O_2 is t$_\alpha$-nI-set.

Definition 2.9. [8]

A subset O of a space $(U, \mathcal{N}, I)$ is called a strong nano $\mathcal{R}$-I-set (resp. strong $\mathcal{R}$-nI-set) if $O = O_1 \cap O_2$, where O_1 is n-open & O_2 is t-nI-set and $I_n(C_n^\star(O_2)) = C_n^\star(I_n(O_2))$.

3. Weak forms of strongly nano open sets in ideal nano spaces

Definition 3.1. A subset O of an ideal nano space $(U, \mathcal{N}, I)$ is said to be

1. strongly nano α-I-open (resp. of $S\alpha$-nI-open) if O is b-nI-open as well as $\mathcal{R}$-nI-set.
2. strongly nano pre-I-open (resp. SP-nI-open) if if O is pre-nI-open as well as $\mathcal{R}_\alpha$-nI-set.
3. strongly nano b-I-open (resp. Sb-nI-open) if O is b-nI-open as well as $\mathcal{R}_\alpha$-nI-set
4. strongly nano β-I-open (resp. $S\beta$-nI-open) if $O \subseteq C_n^\star(I_n(C_n^\star(O)))$.

The complements of the above mentioned sets are called their respective closed sets.

Theorem 3.1. *In a ideal nano space, every $S\alpha$-nI-open set is SP-nI-open*

Proof.

It follows from the fact that every α-nI-open set is P-nI-open and let O be a $\mathcal{R}$-nI-set. Then $O = O_1 \cap O_2$, where O_1 is n-open and O_2 is t-nI -set. Then $I_n(O_2) = I_n(C_n^\star(O_2)) \supseteq I_n(C_n^\star(I_n(O_2))) \supseteq I_n(O_2)$ and hence $I_n(O_2) = I_n(C_n^\star(I_n(O_2)))$. This shows that O is a $\mathcal{R}_\alpha$-nI-set. Therefore O is SP-nI-open set. □

Theorem 3.2. *In a ideal nano space, every $S\alpha$-nI-open set is Sb-nI-open.*

Proof.

It follows from Theorem 3.1 and Remark 2.1.

□

Theorem 3.3. *In a ideal nano space, every SP-nI-open set is Sb-nI-open.*

Proof.

It follows from Remark 2.1 □

Definition 3.2. A subset O of an ideal nano space is said to be a

1. nano $\mathcal{O}_I$-set if $O = O_3 \cap O_4$, where O_3 is n-open and $O_4 = (I_n(O_4))_n^\star$.

2. nano I-locally closed set if $O = O_3 \cap O_4$, where O_3 is n-open and $O_4 = (O_4)^\star_n$.

3. almost strong nano I-open (resp. almost SnI-set) if $O \subseteq C^\star_n(I_n(O^\star_n))$.

Proposition 3.1. *Let $(U,\mathcal{N},I)$ be an ideal nano space. A subset O of U is nano I-locally closed set if O is both n-open and nano $\mathcal{O}_I$-set.*

Proof.

Let O be a n-open and nano $\mathcal{O}_I$-set, then $O = O_1 \cap O_2$, where O_1 is n-open and $O_2 = (I_n(O_2))^\star_n = (O_2)^\star_n$. This show that O is nano I-locally closed set.

Observe that if O_2 is rare, then O is empty. □

Theorem 3.4. *If $(U,\mathcal{N},I)$ is any ideal nano space and $O \subseteq U$, then the following are conditions are equivalent.*

1. *O is n-open.*
2. *O is both Sb-nI-open and strong $\mathcal{R}$-nI-set.*

Proof.

(1) $\Longrightarrow$ (2) is Obvious.

(2) $\Longrightarrow$ (1) Suppose O is Sb-nI-open and also a strong $\mathcal{R}$-nI-set.

$O \subseteq I_n(C^\star_n(O)) \cup C^\star_n(I_n(O)) = I_n(C^\star_n(O_2 \cap O_3)) \cup C^\star_n(I(O_2 \cap O_3))$ where O_2 is n-open and $I_n(C^\star_n(O_3)) = C^\star_n(I_n(O_3))$. Hence $O \subseteq I_n(C^\star_n(O_2)) \cap I_n(C^\star_n(O_3)) \cup C^\star_n(I_n(O_2)) \cap C^\star_n(I_n(O_3)) \subseteq O_2 \cap (I_n(C^\star_n(O_3)) \cup C^\star_n(I_n(O_3)) \subseteq O_2 \cap (I_n(C^\star_n(O_3)) \subseteq O_2 \cap I_n(O_3) = I_n(O)$. This implies O is n-open.

□

Theorem 3.5. *If $(U,\mathcal{N},I)$ is any ideal nano space and $O \subseteq U$, then the following are conditions are equivalent.*

1. *O is b-nI-open and nano $\mathcal{O}_I$-set.*
2. *O is Sb-nI-open and nano $\mathcal{O}_I$-set.*

Proof.

(1) $\Longrightarrow$ (2) If O is nano $\mathcal{O}_I$-set, then $O = O_2 \cap O_3$ where O_2 is n-open and $O_3 = (I_n(O_3))^\star_n$. Now $I_n(C^\star_n(I_n(O_3))) = I_n(I_n(O_3) \cup (I_n(O_3))^\star_n) = I_n(I_n(O_3) \cup O_3) = I_n(O_3)$. It follows that O is Sb-nI-open.

(2) $\Longrightarrow$ (1) Follows from the fact that every Sb-nI-open set is b-nI-open. □

Theorem 3.6. *Let $(U,\mathcal{N},I)$ ba an ideal nano space. A subset O of $(U,\mathcal{N},I)$ is pre-nI-open and $\mathcal{R}$-nI-set is O is $S\alpha$-nI-open.*

Proof.

Let O be $S\alpha$-nI-open set. Since every α-nI-open set is pre-nI-open, then O is pre-nI-open and $\mathcal{R}$-nI-set. □

Theorem 3.7. *Let $(U,\mathcal{N},I)$ ba an ideal nano space. A subset O of $(U,\mathcal{N},I)$ is $S\alpha$-nI-open $\Longleftrightarrow$ it is semi-nI-open, pre-nI-open and $\mathcal{R}$-nI-set.*

Proof.

Necessity : It follows from the fact that every α-nI-open set is semi-nI-open and pre-nI-open.

Sufficiency : Let O be semi-nI-open, pre-nI-open and $\mathcal{R}$-nI-set. Then, we have $O \subseteq I_n(C_n^\star(O)) \subseteq I_n(C_n^\star(C_n^\star(I_n(O)))) = I_n(C_n^\star(I_n(O)))$. This show that O is α-nI-open set and also O is $\mathcal{R}$-nI-set. Therefore O is a $S\alpha$-nI-open set.

□

Theorem 3.8. *In an ideal nano space, every b-nI-open set is $S\beta$-nI-open.*

Proof.

Let O is b-nI-open. Then $O \subseteq I_n(C_n^\star(O)) \cup C_n^\star(I_n(O)) \subseteq C_n^\star(I_n(C_n^\star(O))) \cup C_n^\star(I_n(C_n^\star(O))) = C_n^\star(I_n(C_n^\star(O)))$. This show that O is $S\beta$-nI-open.

□

Proposition 3.2. *For a subset of an ideal nano space, the following condition hold.*

1. *every nI-open set is $S\beta$-nI-open.*
2. *every pre-nI-open set is $S\beta$-nI-open.*
3. *every $S\beta$-nI-open set is β-nI-open.*
4. *every semi-nI-open set is $S\beta$-nI-open.*

Proof.

1. Let O is nI-open. Then $O \subseteq I_n(O_n^\star) \subseteq C_n^\star(I_n(C_n^\star(O)))$. This show that O is $S\beta$-nI-open.
2. Let O be a pre-nI-open set. Then $O \subseteq I_n(C^\star(O)) \subseteq I_n(C_n^\star(O)) \cup (I_n(C_n^\star(O)))_n^\star = C_n^\star(I_n(C_n^\star(O)))$. This show that O is $S\beta$-nI-open.
3. Let O is $S\beta$-nI-open. Then $O \subseteq C_n^\star(I_n(C_n^\star(O))) \subseteq C_n(I_n(C_n^\star(O)))$. This show that O is β-nI-open.
4. Let O be a semi-nI-open set. Then $O \subseteq C_n^\star(I_n(O)) \subseteq C_n^\star(I_n(C_n^\star(O)))$. This show that O is $S\beta$-nI-open.

□

Proposition 3.3. *In an ideal nano space, evey $S\beta$-nI-open set is $n\beta$-open.*

Proof.

Let O be a $S\beta$-nI-open set. Then $O \subset C_n^\star(I_n(C_n^\star(O))) \subset C_n(I_n(C_n(O)))$. This shows that O is $n\beta$-open. □

Example 3.1. *Let $U = \{100, 101, 102, 103\}$ with $U/R = \{\{100\}, \{102\}, \{101, 103\}\}$ and $X = \{100, 101\}$. Then $\mathcal{N} = \{\phi, U, \{100\}, \{101, 103\}, \{100, 101, 103\}\}$. Let the ideal be $I = \{\phi, \{100\}\}$. Clear the set $\{100, 102\}$ is $n\beta$-open but not $S\beta$-nI-open.*

Proposition 3.4. *The intersection of a $S\beta$-nI-open set and n-open set is $S\beta$-nI-open.*

Proof.

Let O be $S\beta$-nI-open and O_1 be n-open. Then $O \subset C_n^\star(I_n(C_n^\star(O)))$ and $O_1 \cap O \subset O_1 \cap C_n^\star(I_n(C_n^\star(O))) \subset C_n^\star(O_1 \cap I_n(C_n^\star(O))) \subset C_n^\star(I_n(O_1) \cap I_n(C_n^\star(O))) = C_n^\star(I_n(O_1 \cap C_n^\star(O))) \subset C_n^\star(I_n(C_n^\star(O_1 \cap O)))$ by Lemma 2.1. This shows that $O_1 \cap O$ is $S\beta$-nI-open. □

Remark 3.1. *The intersection of two $S\beta$-nI-open sets but not be a $S\beta$-nI-open.*

Example 3.2. *In Example 3.1, $Q_1 = \{101, 102\}$ and $Q_2 = \{102, 103\}$ are $S\beta$-nI-open. But $Q_1 \cap Q_2 = \{102\}$ is not $S\beta$-nI-open.*

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