Asia Mathematika
Volume: 5 Issue: 3 , (2021) Pages: 14 – 21
Available online at **www.asiamath.org**

Characterization of near-ring by interval valued Picture fuzzy ideals

V.S.Subha[1]*and P.Dhanalakshmi[2]
[1]Assistant Professor(Deputed), PG and Research Department of Mathematics, Govt. Arts. College, C. Mutlur, Chidambaram. ORCID: 0000-0002-1227-8232
[2]Research Scholar, Department of Mathematics, Annamalai University, Chidambaram. ORCID: 0000-0002-1583-8047

Received: 16 May 2021 • **Accepted:** 17 Sep 2021 • **Published Online:** 30 Dec 2021

Abstract: The aim of this paper is to introduce a concept of interval valued picture fuzzy ideals in near rings. Also we investigate the union and intersection of two interval valued picture fuzzy ideals.Moreover the union and intersection of interval valued picture fuzzy ideals is also an interval valued picture fuzzy ideal. We illustrate direct product of two interval valued picture fuzzy ideals. Furthermore we prove the image and pre-image of an interval valued picture fuzzy ideal is also an interval valued picture fuzzy ideal.

Key words: Interval valued fuzzy set, Picture fuzzy set, Picture fuzy ideal, Near ring, Interval valued picture fuzzy set

1. Introduction

Cuong [5–7] introduced the concept of a picture fuzzy set is an extension of Atanassov[3] intutionistic fuzzy set, containing the grades of truth, abstinence, falsity and refusal, whose sum is belonging to a unit interval. To deal real life problem, picture fuzzy set is more effctive than the intutionistic fuzzy set and fuzzy set. The concept of interval-valued picture fuzzy sets was also proposed in Cuong[5]. In interval-valued picture fuzzy sets, the degrees of membership, abstinence and non-membership are given in closed sub-intervals of $[0, 1]$ and have a condition that the sum of the supremum of all three subintervals must belong to a closed unit interval. Obviously, interval-valued picture fuzzy sets can describe fuzzy information more easily than fuzzy sets, intutionistic fuzzy sets, interval valued intutionistic fuzzy sets.

The summery of this manuscript is as follows: In section 2 we review some basic concepts related to this article. In section 3 we study the notion of interval valued picture fuzzy ideals in near rings. Section 4 deals with homomorphism of interval picture fuzzy ideals in near rings. In section 5 we discuss about the direct product of interval valued picture fuzzy ideals in near-rings.

2. Preliminaries

This section deals with the basic concepts related to this article. Also for basic results refer [1], [4],[5], [6], and [7].

Definition 2.1. A IPF $\wp$ on the universe G defined by

$$\wp = (\langle j, t_\wp(j), i_\wp(j), f_\wp(j)\rangle\, j \in G)$$

where $t_\wp, i_\wp, f_\wp : G \longrightarrow [0, 1]$ and $sup(t_\wp) + sup(i_\wp) + sup(f_\wp) \leq 1$
here $t_\wp$ is truth membership function, $i_\wp$ is an indeterminacy function and $f_\wp$ is a falsity membership function.

©Asia Mathematika, DOI: 10.5281/zenodo.5809160
*Correspondence: vpdhanam83@gmail.com

3. Interval valued Picture fuzzy Ideals(IPFIs) of Near-rings

In this section we studied the notion of $IPFI$ of NR G. Also we discuss the intersection and union of $IPFI$ is also an $IPFI$ of G.

Definition 3.1. An IPF set $\wp$ in a NR G is called an $IPFNR$ of G if for all $e, q \in G$

1. $t_\wp(e-q) \geq t_\wp(e) \wedge t_\wp(q)$
   $i_\wp(e-q) \leq i_\wp(e) \vee i_\wp(q)$
   $f_\wp(e-q) \leq f_\wp(e) \vee f_\wp(q)$

2. $t_\wp(eq) \geq t_\wp(e) \wedge t_\wp(q)$
   $i_\wp(eq) \leq i_\wp(e) \vee i_\wp(q)$
   $f_\wp(eq) \leq f_\wp(e) \vee f_\wp(q)$

Definition 3.2. An IPF left ideal of G is defined as follows: if for all $s, q \in G$.

1. $t_\wp(s-q) \geq t_\wp(s) \wedge t_\wp(q)$
   $i_\wp(s-q) \leq i_\wp(s) \vee i_\wp(q)$
   $f_\wp(s-q) \leq f_\wp(s) \vee f_\wp(q)$

2. $t_\wp(s+q-s) \geq t_\wp(q)$
   $i_\wp(s+q-s) \leq i_\wp(q)$
   $f_\wp(s+q-s) \leq f_\wp(q)$

3. $t_\wp(sq) \geq t_\wp(q)$
   $i_\wp(sq) \leq i_\wp(q)$
   $f_\wp(sq) \leq f_\wp(q)$

Definition 3.3. An IPF right ideal of G is defined as follows: if for all $l, q, k \in G$.

1. $t_\wp(l-q) \geq t_\wp(l) \wedge t_\wp(q)$
   $i_\wp(l-q) \leq i_\wp(l) \vee i_\wp(q)$
   $f_\wp(l-q) \leq f_\wp(l) \vee f_\wp(q)$

2. $t_\wp(l+q-l) \geq t_\wp(q)$
   $i_\wp(l+q-l) \leq i_\wp(q)$
   $f_\wp(l+q-l) \leq f_\wp(q)$

3. $t_\wp((l+q)k-lk) \geq t_\wp(q)$
   $i_\wp((l+q)k-lk) \leq i_\wp(q)$
   $f_\wp((l+q)k-lk) \leq f_\wp(q)$

Theorem 3.1. *Let Φ and Ψ be two $IPFIs$ of G. If $\Phi \subset \Psi$ then $\Phi \cup \Psi$.*

Proof. Let Φ and Ψ be an $IPFIs$ of G. Let $p, q, e \in G$ then

$$t_{\Phi\cup\Psi}(p-q) = t_\Phi(p-q) \vee t_\Psi(p-q)$$

$$\geq \vee \{(t_\Phi(p) \wedge t_\Phi(q)), (t_\Psi(p) \wedge t_\Psi(q))\}$$
$$= \wedge \{(t_\Phi(p) \vee t_\Psi(p)), (t_\Phi(q) \vee t_\Psi(q))\}$$
$$= t_{\Phi\cup\Psi}(p) \wedge t_{\Phi\cup\Psi}(q).$$

Equivalently we can prove for other cases

$$i_{\Phi\cup\Psi}(p-q) \leq i_{\Phi\cup\Psi}(p) \vee i_{\Phi\cup\Psi}(q) \text{ and}$$
$$f_{\Phi\cup\Psi}(p-q) \leq f_{\Phi\cup\Psi}(p) \vee f_{\Phi\cup\Psi}(q).$$

Next ,

$$t_{\Phi\cup\Psi}(p+q-p) = t_\Phi(p+q-p) \vee t_\Psi(p+q-p)$$
$$\geq t_\Phi(q) \vee t_\Psi(q)$$
$$= t_{\Phi\cup\Psi}(q)$$

In similar way we can prove

$$i_{\Phi\cup\Psi}(p+q-p) \leq i_{\Phi\cup\Psi}(q)$$
$$f_{\Phi\cup\Psi}(p+q-p) \leq f_{\Phi\cup\Psi}(q)$$

Furthermore we deduce that

$$t_{\Phi\cup\Psi}(pq) = t_\Phi(pq) \vee t_\Psi(pq) \geq t_\Phi(q) \vee t_\Psi(q) = t_{\Phi\cup\Psi}(q)$$
$$i_{\Phi\cup\Psi}(pq) = i_\Phi(pq) \wedge i_\Psi(pq) \leq i_\Phi(q) \wedge i_\Psi(q) = i_{\Phi\cup\Psi}(q)$$
$$f_{\Phi\cup\Psi}(lk) = f_\Phi(lk) \wedge f_\Psi(lk) \leq f_\Phi(k) \wedge f_\Psi(k) = f_{\Phi\cup\Psi}(k)$$

At last

$$t_{\Phi\cup\Psi}((p+q)e-pe) = t_\Phi((p+q)e-pe) \vee t_\Psi((p+q)e-pe) \geq t_\Phi(e) \vee t_\Psi(e) = t_{\Phi\cup\Psi}(e)$$
$$i_{\Phi\cup\Psi}((p+q)e-pe) = i_\Phi((p+q)e-pe) \wedge i_\Psi((p+q)e-pe) \leq i_\Phi(e) \wedge i_\Psi(e) = i_{\Phi\cup\Psi}(e)$$
$$f_{\Phi\cup\Psi}((p+q)e-pe) = f_\Phi((p+q)e-pe) \wedge f_\Psi((p+q)e-pe) \leq f_\Phi(e) \wedge f_\Psi(e) = f_{\Phi\cup\Psi}(e)$$

Hence $\Phi \cup \Psi$ is an $IPFI$ of G. □

By the Similar argument we prove the following theorem.

Theorem 3.2. *Let Φ and Ψ be two $IPFIs$ of G. If $\Phi \subset \Psi$ then $\Phi \cap \Psi$.*

Corollary 3.1. *If $(\wp_1, \wp_2, \wp_3.....\wp_j)$ are $IPFIs$ of G then $\wp = \bigcap_{i=1}^{j} \wp_j$ is an $IPFI$ of G.*

Note: For all $l, k \in G$ and n is any positive integer if $l = k$, then

- $l^n \leq k^n$
- $[min(l,k)]^n = min(l^n, k^n)$
- $[max(l,k)]^n = max(l^n, k^n)$.

Theorem 3.3. *Let $\wp$ be an $IPFI$ of G. Then $\wp^n = \langle e, t_{\wp^n}(e), i_{\wp^n}(e), f_{\wp^n}(e) : e \in G\rangle$ is an $IPFI$ of G, where n is a positive integer $t_{\wp^n}(e) = (t_\wp(e))^n$, $i_{\wp^n}(e) = (i_\wp(e))^n$ and $f_{\wp^n}(e) = (f_\wp(e))^n$.*

Proof. Since $\wp$ is an $IPFI$ of G. Let $e, p, s \in G$. Then

$$t_{\wp^n}(e-p) = (t_\wp(e-p))^n$$
$$\geq (min(t_\wp(e), t_\wp(p)))^n$$
$$= min((t_\wp(e))^n, (t_\wp(p))^n)$$
$$= min(t_{\wp^n}(e), t_{\wp^n}(p))$$

$$\begin{aligned} i_{\wp^n}(e-p) &= (i_{\wp}(e-p))^n \\ &\leq (max\,(i_{\wp}(e), i_{\wp}(p)))^n \\ &= max\,((i_{\wp}(e))^n, (i_{\wp}(p))^n) \\ &= max\,(i_{\wp^n}(e), i_{\wp^n}(p)) \end{aligned}$$

$$\begin{aligned} f_{\wp^n}(e-p) &= (f_{\wp}(e-p))^n \\ &\leq (max\,(f_{\wp}(e), f_{\wp}(p)))^n \\ &= max\,((f_{\wp}(e))^n, (f_{\wp}(p))^n) \\ &= max\,(f_{\wp^n}(e), f_{\wp^n}(p)) \end{aligned}$$

Next

$$t_{\wp^n}(e+p-e) = (t_{\wp}(e+p-e))^n \geq (t_{\wp}(e))^n = t_{\wp^n}(e)$$
$$i_{\wp^n}(e+p-e) = (i_{\wp}(e+p-e))^n \leq (i_{\wp}(e))^n = i_{\wp^n}(e)$$
$$f_{\wp^n}(e+p-e) = (f_{\wp}(e+p-e))^n \leq (f_{\wp}(e))^n = f_{\wp^n}(e)$$

Also,

$$t_{\wp^n}(ep) = (t_{\wp}(ep))^n \geq (t_{\wp}(p))^n = t_{\wp^n}(p)$$
$$i_{\wp^n}(ep) = (i_{\wp}(ep))^n \leq (i_{\wp}(p))^n = i_{\wp^n}(p)$$
$$f_{\wp^n}(ep) = (f_{\wp}(ep))^n \leq (z_{\wp}(p))^n = f_{\wp^n}(p)$$

At last,

$$t_{\wp^n}((e+p)s-es) = (t_{\wp}((e+p)s-es))^n \geq (t_{\wp}(p))^n = t_{\wp^n}(p)$$
$$i_{\wp^n}((e+p)s-es) = (i_{\wp}((e+p)s-es))^n \leq (t_{\wp}(p))^n = i_{\wp^n}(p)$$
$$f_{\wp^n}((e+p)s-es) = (f_{\wp}((e+p)s-es))^n \leq (f_{\wp}(p))^n = f_{\wp^n}(p)$$

Hence the theorem. □

4. Homomorphism of Interval Valued Picture Fuzzy (IPF) sets

This section explore the homomorphism of IPF sets. Also we have proved the image and pre image of an $IPFI$ is also an $IPFI$.

Definition 4.1. Let G and H be two near-rings. Then the mapping $M : G \longrightarrow H$ is called a near-ring homomorphism if for all $r, s \in G$ then

(1) $M(r+s) = M(r) + M(s)$

(2) $M(rs) = M(r) + M(s)$

Definition 4.2. Let $\wp$ and $\Re$ be two nonempty sets and $M : \wp \longrightarrow \Re$ be function

1. If Q is an IPF set in $\Re$, then $M^{-1}(Q)$ is the IPF in $\wp$ defined by
   $M^{-1}(Q) = \left\{ \left(s, M^{-1}(t_Q(s)), M^{-1}(i_Q(s)), M^{-1}(f_Q(s))\right) : s \in \wp \right\}$,
   where $M^{-1}(t_Q(s)) = t_Q(M(s))$, $M^{-1}(i_Q(s)) = i_Q(M(s))$ and $M^{-1}(f_Q(s)) = f_Q(M(s))$.

2. If P is an IPF set in $\wp$ then $M(P)$ is IPF in $\Re$ defined by
   $M(P) = \{(s, M(t_P(s)), M(i_P(s)), M(f_P(s))) : s \in \wp\}$

where

$$M(t_\wp(s)) = \begin{cases} \sup\limits_{h\in M^{-1}(s)} t_\wp(h), if \quad M^{-1}(s) \neq 0 \\ 0, otherwise \end{cases}$$

$$M(i_\wp(s)) = \begin{cases} \sup\limits_{h\in M^{-1}(s)} i_\wp(h), if \quad M^{-1}(s) \neq 0 \\ 0, otherwise \end{cases}$$

$$M(f_\wp(s)) = \begin{cases} \sup\limits_{h\in M^{-1}(s)} f_\wp(h), if \quad M^{-1}(s) \neq 0 \\ 0, otherwise \end{cases}$$

where $M(t_\wp(s)) = (1 - M(1 - t_\wp))(s)$.

Theorem 4.1. *Let G and H be two NRs and M be a homomorphism of G onto H. If $\wp$ is an $IPFI$ of H then $M^{-1}(\wp)$ is an $IPFI$ of G.*

Proof. Let $e, p, s \in G$. Then

$$\begin{aligned} M^{-1}(t_\wp)(e-p) &= t_\wp(M(e-p)) \\ &= t_\wp(M(e) - M(p)) \\ &\geq t_\wp(M(e)) \wedge t_\wp(M(p)) \\ &= M^{-1}(t_\wp)(e) \wedge M^{-1}(t_\wp)(p) \end{aligned}$$

Furthermore we can prove

$$M^{-1}(i_\wp)(e-p) \leq M^{-1}(i_\wp)(e) \wedge M^{-1}(i_\wp)(p)$$
$$M^{-1}(f_\wp)(e-p) \leq M^{-1}(f_\wp)(e) \wedge M^{-1}(f_\wp)(p)$$

Next,

$$\begin{aligned} M^{-1}(t_\wp)(e+p-e) &= t_\wp(M(e+p-e)) \\ &= t_\wp(M(e) + M(p) - M(e)) \\ &\geq t_\wp(M(p)) \\ &= M^{-1}(t_\wp)(p) \end{aligned}$$

$M^{-1}(i_\wp)(e+p-e) \leq M^{-1}(i_\wp)(p)$ and $M^{-1}(f_\wp)(e+p-e) \leq M^{-1}(f_\wp)(p)$
And

$$\begin{aligned} M^{-1}(t_\wp)(ep) &= t_\wp(M(ep)) \\ &= t_\wp(M(e)M(p)) \\ &\geq t_\wp(M(p)) \\ &= M^{-1}(t_\wp)(p) \end{aligned}$$

Correspondingly we can prove for
$M^{-1}(i_\wp)(ep) \leq M^{-1}(i_\wp)(p)$ and $M^{-1}(f_\wp)(ep) \leq M^{-1}(f_\wp)(p)$
Finally

$$\begin{aligned} M^{-1}(t_\wp)((e+p)s - es) &= t_\wp(M((e+p)s - es) \\ &= t_\wp([M(e) + M(p)]M(s) - M(e)M(s)) \end{aligned}$$

$$\geq t_{\wp}(M(p))$$
$$= M^{-1}(t_{\wp})(p)$$

Similarly we can prove
$M^{-1}(i_{\wp})((e+p)s-es) \leq M^{-1}(i_{\wp})(p)$ and $M^{-1}(f_{\wp})((e+p)s-es) \leq M^{-1}(f_{\wp})(p)$
Hence $M^{-1}(\wp)$ is an $IPFI$ of G. □

Theorem 4.2. *Let G and H be two near-rings and M be a homomorphism of G onto H. If $\wp_1$ is an $IPFI$ of G then $M(\wp_1)$ is an $IPFI$ of H.*

Proof. Let $l_1, k_1, t_1 \in G$ and $l_2, k_2, t_2 \in H$. Then,

$$\begin{aligned} M(t_{\wp_1}(l_2-k_2)) &= \sup_{l_1,k_1\in M^{-1}(H)} t_{\wp_1}(l_1-k_1) \\ &\geq \sup_{l_1,k_1\in M^{-1}(H)} (t_{\wp_1}(l_1)\wedge t_{\wp_1}(k_1)) \\ &= \left(\sup_{l_1\in M^{-1}(H)} (t_{\wp_1}(l_1))\right) \wedge \left(\sup_{k_1\in M^{-1}(H)} (t_{\wp_1}(k_1))\right) \\ &= M(t_{\wp_1}(l_2)) \wedge M(t_{\wp_1}(k_2)) \end{aligned}$$

Similarly, we can prove for other case
$M(i_{\wp_1}(l_2-k_2)) \leq M(i_{\wp_1}(l_2)) \vee M(i_{\wp_1}(k_2))$ and $M(f_{\wp_1}(l_2-k_2)) \leq M(f_{\wp_1}(l_2)) \vee M(f_{\wp_1}(k_2))$
Also

$$\begin{aligned} M(t_{\wp_1}(l_2+k_2-l_2)) &= \sup_{l_1,k_1\in M^{-1}(H)} t_{\wp_1}(l_1+k_1-l_1) \\ &\geq \sup_{l_1\in M^{-1}(H)} (t_{\wp_1}(l_1)) \\ &= M(t_{\wp_1}(l_2)) \end{aligned}$$

In similar way we prove
$M(i_{\wp_1}(l_2+k_2-l_2)) \leq M(i_{\wp_1}(l_2))$ and $M(f_{\wp_1}(l_2+k_2-l_2)) \leq M(f_{\wp_1}(l_2))$
Also,

$$\begin{aligned} M(t_{\wp_1}((l_2+t_2)k_2-l_2k_2) &= \sup_{l_1,k_1,t_1\in M^{-1}(H)} t_{\wp_1}((l_1+t_1)k_1+l_1k_1) \\ &\geq \sup_{t_1\in M^{-1}(H)} t_{\wp_1}(t_1) = M(t_{\wp_1}(t_2)) \end{aligned}$$

In same way we prove
$M(i_{\wp_1}((l_2+t_2)k_2-l_2k_2) \leq M(i_{\wp_1}(t_2))$ and $M(f_{\wp_1}((l_2+t_2)k_2-l_2k_2) \leq M(f_{\wp_1}(t_2))$
Hence $M(\wp_1)$ is an $IPFI$ of H. □

5. Direct Product of IPF sets

In this section we define the direct product of an IPF sets. Also we prove direct product of an $IPFI$ is also an $IPFI$.

Definition 5.1. The direct product of two IPF sets A and B of near-rings G and H is defined by $A\times B : G\times H \longrightarrow [0,1]$ such that
$A\times B = \{\langle (e,s), t_{A\times B}(e,s), i_{A\times B}(e,s), f_{A\times B}(e,s)\rangle : e\in A, s\in B\}$
where
$t_{A\times B}(e,s) = t_A(e)\wedge t_B(s)$

$i_{A\times B}(e,s) = i_A(e) \vee i_B(s)$
$f_{A\times B}(e,s) = f_A(e) \vee f_B(s)$

Definition 5.2. The direct product of two $IPFI$ A and B of G and H is defined by: if for all $(e,s),(e_0,s_0),(e_1,s_1) \in G\times H$ the following conditions are satisfied,
$t_{A\times B}((e,s)-(e_0,s_0)) \geq (t_{A\times B}(e,s)) \wedge (t_{A\times B}(e_0,s_0))$
$i_{A\times B}((e,s)-(e_0,s_0)) \leq (i_{A\times B}(e,s)) \vee (i_{A\times B}(e_0,s_0))$
$f_{A\times B}((e,s)-(e_0,s_0)) \leq (f_{A\times B}(e,s)) \vee (f_{A\times B}(e_0,s_0))$
$t_{A\times B}((e,s)+(e_0,s_0)-(e,s)) \geq t_{A\times B}(e_0,s_0)$
$i_{A\times B}((e,s)+(e_0,s_0)-(e,s)) \leq i_{A\times B}(e_0,s_0)$
$f_{A\times B}((e,s)+(e_0,s_0)-(e,s)) \leq f_{A\times B}(e_0,s_0)$
$t_{A\times B}((e,s)(e_0,s_0)) \geq t_{A\times B}(e_0,s_0)$
$i_{A\times B}((e,s)(e_0,s_0)) \leq i_{A\times B}(e_0,s_0)$
$f_{A\times B}((e,s)(e_0,s_0)) \leq f_{A\times B}(e_0,s_0)$
$t_{A\times B}[(e,s)+(e_1,s_1)](e_0,s_0)-((e,s)(e_0,s_0))) \geq (t_{A\times B}(e_1,s_1))$
$i_{A\times B}[(e,s)+(e_1,s_1)](e_0,s_0)-((e,s)(e_0,s_0))) \leq (i_{A\times B}(e_1,s_1))$
$f_{A\times B}[(e,s)+(e_1,s_1)](e_0,s_0)-((e,s)(e_0,s_0))) \leq (f_{A\times B}(e_1,s_1))$

Theorem 5.1. *Let A and B be an $IPFIs$ of G and H respectively. Then $A\times B$ is an $IPFIs$ of $G\times H$.*

Proof. Since A and B are $IPFIs$ of G and H respectively.
Let $(e_1,e_2),(p_1,p_2),(s_1,s_2) \in G\times H$. Then

$$\begin{aligned} t_{A\times B}((e_1,e_2)-(p_1,p_2)) &= t_{A\times B}(e_1-p_1, e_2-p_2) \\ &= (t_A(e_1-p_1)) \bigwedge (t_B(e_2-p_2)) \\ &\geq (t_A(e_1) \wedge t_A(p_1) \bigwedge (t_B(e_2) \wedge t_B(p_2) \\ &= (t_A(e_1) \wedge t_B(e_2) \bigwedge (t_A(p_1) \wedge t_B(p_2) \\ &= t_{A\times B}(e_1,e_2) \bigwedge t_{A\times B}(p_1,p_2) \end{aligned}$$

Similarly we can prove for the other case
$i_{A\times B}((e_1,e_2)-(p_1,p_2)) \leq i_{A\times B}(e_1,e_2) \bigvee i_{A\times B}(p_1,p_2)$
$f_{A\times B}((e_1,e_2)-(p_1,p_2)) \leq f_{A\times B}(e_1,e_2) \bigvee f_{A\times B}(p_1,p_2)$
Also

$$\begin{aligned} t_{A\times B}((e_1,e_2)+(p_1,p_2)-(e_1,e_2)) &= t_{A\times B}(e_1+p_1-e_1, e_2+p_2-e_2) \\ &= (t_A(e_1+p_1-e_1)) \bigwedge (t_B(e_2+p_2-e_2)) \\ &\geq (t_A(p_1) \wedge t_B(p_2)) \\ &= t_{A\times B}(p_1,p_2) \end{aligned}$$

In similar way we can prove the other case
$i_{A\times B}((e_1,e_2)+(p_1,p_2)-(e_1,e_2)) \leq i_{A\times B}(p_1,p_2)$
$f_{A\times B}((e_1,e_2)+(p_1,p_2)-(e_1,e_2)) \leq f_{A\times B}(p_1,p_2)$
Moreover

$$\begin{aligned} t_{A\times B}((e_1,e_2)(p_1,p_2)) &= t_{A\times B}(e_1p_1, e_2p_2) \\ &= (t_A(e_1p_1)) \bigwedge (t_B(e_2p_2)) \\ &\geq (t_A(p_1) \wedge t_B(p_2) \end{aligned}$$

$= t_{A\times B}(p_1, p_2)$

Similarly we can prove for the other case

$i_{A\times B}((e_1, e_2)(p_1, p_2)) \leq i_{A\times B}(p_1, p_2)$

$f_{A\times B}((e_1, e_2)(p_1, p_2)) \leq f_{A\times B}(p_1, p_2)$

Finally we prove

$t_{A\times B}[(e_1, e_2) + (s_1, s_2)](p_1, p_2) - ((e_1, e_2)(p_1, p_2))$

$= t_{A\times B}([e_1 + s_1]p_1 - (e_1p_1), [e_2 + s_2]p_2 - (e_2p_2))$

$= (t_A([e_1 + s_1]p_1 - (e_1p_1))) \bigvee (i_B([e_2 + s_2]p_2 - (e_2p_2)))$

$\leq t_A(s_1) \bigvee t_B(s_2)$

$= t_{A\times B}(s_1, s_2)$

Similarly

$i_{A\times B}[(e_1, e_2) + (s_1, s_2)](p_1, p_2) - ((e_1, e_2)(p_1, p_2)) \geq i_{A\times B}(s_1, s_2)$

$f_{A\times B}[(e_1, e_2) + (s_1, s_2)](p_1, p_2) - ((e_1, e_2)(p_1, p_2)) \geq f_{A\times B}(s_1, s_2)$

Hence direct product of an $IPFI$ is also an $IPFI$. □

6. Conclusion

In this paper we characterized algebraic properties of an interval valued picture fuzzy sets in near-rings. Moreover some important possessions are discussed. In sequel this concept can be extended to more real life problems. Also this algebraic properties can be extend to many domains such as gamma-near-rings, semihyperrings etc.

References

[1] S. Abou Zaid, n fuzzy sub near-rings and ideals, Fuzzy sets and Sys. 44 (1991),139-146.

[2] M. Anitha and M.Latha, A study on T-fuzzysoft subhemirings of a hemiring, Asia mathematika, 2017, 1(1), 1-6.

[3] K. Attanassov, Intuitionistic fuzzy sets, Fuzzy sets and system. 20 (1986) 87-96.

[4] Awais Asif, Hassan Aydi, Muhammad Arshad, Abdul Rehman and Usman Tariq, Picture fuzzy ideals of Near-rings, Hindwai Journal of Mathematics, 2020.

[5] Cuong, B.C. Picture Fuzzy Sets-First Results. Part 2, Seminar Neuro-Fuzzy Systems with Applications; Institute of Mathematics: Hanoi, Vietnam, 2013.

[6] Cuong, B.C. Picture Fuzzy Sets-First Results. Part 1, Seminar Neuro-Fuzzy Systems with Applications; Institute of Mathematics: Hanoi, Vietnam, 2013.

[7] Coung, B.C. Picture fuzzy sets. J. Comput. Sci. Cybern. 30(2014), 409–420.

[8] Shahad T. Almohammadi and Cenep Ozel, Rough approximations in topological vector spaces, Asia mathematika, 2019, 3(2), 19-25.

[9] L. A. Zadeh, Fuzzy sets. Inform and control. 8(1965) 338-353.