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Split domination of splitted graphs

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Abstract: In this paper, we introduce and investigate some new splitted graphs called $S(P_l), S(H_l), S(P_l^+), S(P_loNK_1)$. Also we discuss some splitted graphs and it's properties are obtained.

Key words: Domination, split domination number and splitted graph

1. Introduction

The concepts of graph theory instead of [7]. By a graph, we mean a finite simple and undirected graph. The vertex set and edge set of a graph G denoted are by $V(G)$ and $E(G)$ respectively. Let G be a graph. For each vertex v of a graph G, take a new vertex u. Join u to those vertices of G adjacent to v. The graph thus obtained is called the splitting graph of G. It is denoted by $S(G)$. For a graph G, the splitting graph S of G is obtained by adding a new vertex v corresponding to each vertex u of G such that $N(u) = N(v)$ and it is denoted by $S(G)$. The notions of splitting graphs were studied and investigated of many more authors for Example [1], [11], [12], [13], [14] and [15]. A dominating set D of a graph $S(G)$ is said to be a split dominating set of $S(G)$ if the subgraph induced by $V - D$ is disconnected. The minimum cardinality among all split dominating sets is called the split domination number of $S(G)$ and is denoted as $\gamma_s[S(G)]$. In this paper deal some splitted graphs namely $S(P_l), S(H_l), S(P_l^+), S(P_loNK_1)$.

2. Preliminaries

[10] A set D of vertices in a splitted graph $S(G) = (V, E)$ is called a dominating set if every vertex in $V - D$ is adjacent to some vertex in D. [9] A dominating set D of a graph $S(G)$ is said to be a split dominating set of $S(G)$ if the subgraph induced by $V - D$ is disconnected. The minimum cardinality among all split dominating sets is called the split domination number of $S(G)$ and is denoted as γ_s [S(G)]. Esakkimuthu [4, 5] was introduced and studied the idea of path related graphs and connected domination graphs. The Recently, the idea of the Domination and independent domination graphs are studied by several researchers for Example [2], [3], [6], [8], [9], [10] and [16] so on.

3. γ_s-Main Results

Theorem 3.1. $\gamma_s S(P_l) = \ulcorner l/2 \urcorner$ *when* $l \equiv 1(mod2)$.

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Proof. Let $V[S(P_l)] = \{c_h, d_h : 1 \leq h \leq l\}$.

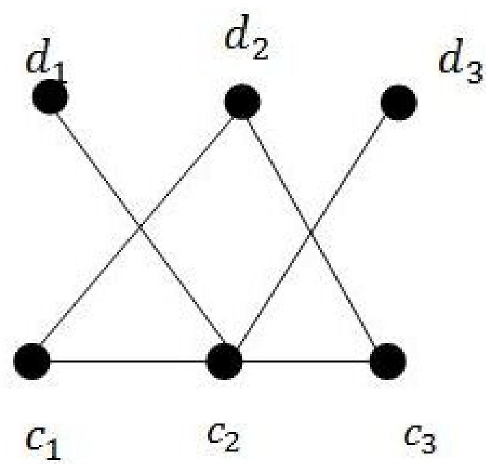
Let $E[S(P_l)] = \{(c_h c_{h+1}) \cup (d_h c_{h+1}) \cup (c_h d_{h+1}); 1 \leq h \leq l-1\}$. In P_l, vertex c_{h+1} is adjacent to c_{h+1}, d_{h+2}, c_h, d_h where as c_{h+2} is adjacent to c_{h+3}, d_{h+3}, c_{h+1}, d_{h+1}.

By definition c_{h+1} and c_{h+2} are adjacent and in every fourth of c_{h+1} and c_{h+2} the center vertices satisfy the γ_s-ing set condition and c_{i-1} is adjacent to c_{i-2}, c_{i-2}, c_i, d_i.

Hence, $\gamma_s[S(P_1)] = \ulcorner l/2 \urcorner$.

For example, $\gamma_s[S(P_3)] = \{c_2, c_3\} = 2$ as seen from the following fig-1. □

Figure 1. $S(P_3)$.



Theorem 3.2. $\gamma_s S(P_l) = 2\ulcorner l/4 \urcorner$ *when* $l \equiv$ *0 (mod 2)*

Proof. Let $V[S(P_l)] = \{c_h, d_h : 1 \leq h \leq l\}$.

Let $E[S(P_l)] = \{(c_h c_{h+1}) \cup (d_h c_{h+1}) \cup (c_h d_{h+1}); 1 \leq h \leq l-1\}$.

In P_l, vertex c_{h+1} is adjacent to $c_{h+1}, d_{h+2}, c_h, d_h$ where as c_{h+2} is adjacent to $ch+3, d_{h+3}, c_{h+1}, d_{h+1}$.

By definition c_{h+1} and c_{h+2} are adjacent and in every fourth of c_{h+1} and c_{h+2} the center vertices satisfy the γ - ing set condition and c_{i-1} is adjacent to $c_{i-2}, c_{i-2}, c_i, d_i$.

Hence, $\gamma_s[S(P_1)] = 2\ulcorner l/4 \urcorner$ For example, $\gamma_s[S(P_4)] = \{c_2, c_3\} = 2$ as seen from the following fig-2. □

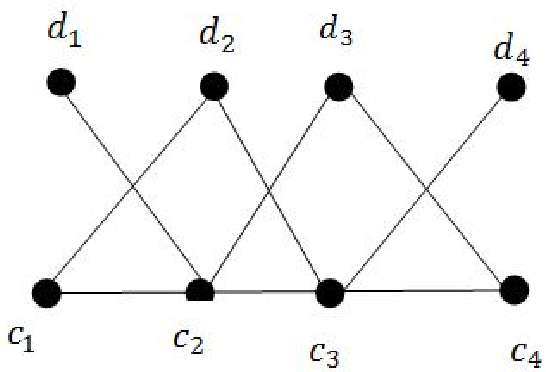
Theorem 3.3.

$$\gamma_s S(H_l) = \begin{cases} l+1 & \textit{if } l \equiv \textit{1 (mod4)} \\ 2 & \textit{if } l = 3. \end{cases}$$

Proof. **Case I**. Let $V[S(H_l)] = \{c_h, d_h, c_h^{'}, d_h^{'} : 1 \leq h \leq l\}$. Let $E[S(H_l)] = \{(c_h c_{h+1}) \cup (d_h c_{h+1}) \cup (c_h d_{h+1}) \cup (c_{h+1/2} c^{'}_{h+1/2}) \cup (c_{h+1/2} d^{'}_{h+1/2}) \cup (d_{h+1/2} c^{'}_{h+1/2}) \cup (c^{'}_h c^{'}_{h+1}) \cup (c^{'}_h d^{'}_{h+1}) \cup (d^{'}_h d^{'}_{h+1}); 1 \leq h \leq l-1\}$.

For $1 \leq h \leq l+1/2$.In H_l vertex c_{h+1} is adjacent to $c_{h+1}, d_{h+2}, c_h, d_h$ where as c_{h+2} is adjacent to $c_{h+3}, d_{h+3}, c_{h+1}, d_{h+1}$ by definition c_{h+1} and c_{h+2} are adjacent and in every fourth of c_{h+1} and c_{h+2} the center vertices satisfy the γ_s-ing set condition.

Figure 2. $S(P_4)$



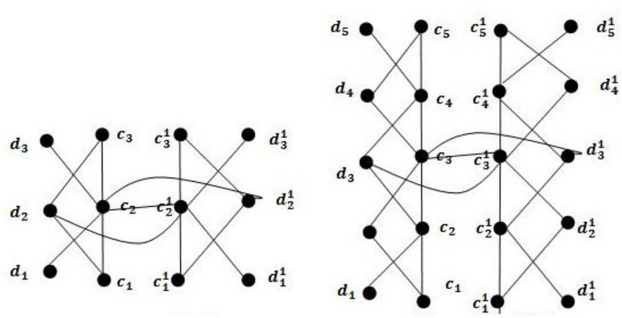
The vertex $c_{l+1/2}$ and $c^{'}_{l+1/2}$ Split dominating any other vertex in the graph from the remaining vertex.

Consider, for $(l+3)/2 \leq h \leq l$, vertex c_{h+1} is adjacent to $c_{h+1}, d_{h+2}, c_h, d_h$ where as c_{h+2} is adjacent to $ch+3, d_{h+3}, c_{h+1}, d_{h+1}$ by definition c_{h+1} and c_{h+2} are adjacent and in every fourth of c_{h+1} and c_{h+2} the center vertices satisfy the γ_s-ing set condition, and c_{l-1} is adjacent to c_{l-2}, d_{l-2}, c_l, d_l. The same process applicable for the set of vertex $c^{'}_{h+1}$ and $c^{'}_{h+2}$.

Case II. When l = 3, Let $V[S(H_l)] = \{c_h, d_h, c^{'}_h, d^{'}_h : 1 \leq h \leq l\}$, Let $E[S(H_l)] = \{(c_h c_{h+1}) \cup (d_h c_{h+1}) \cup (c_h d_{h+1}) \cup (c_{h+1/2} c^{'}_{h+1/2}) \cup (c_{h+1/2} d^{'}_{h+1/2}) \cup (d_{h+1/2} c^{'}_{h+1/2}) \cup (c^{'}_h c^{'}_{h+1}) \cup (c^{'}_h d^{'}_{h+1}) \cup (d^{'}_h d^{'}_{h+1}); 1 \leq h \leq l-1\}$. In H_l, The vertex $c_{l+1/2}$ and $c^{'}_{l+1/2}$ γ_s-ing set condition. $\square$

For example, $\gamma_s[S(H_5)] = \{c_2, c_3, c_4, c^{'}_2, c^{'}_3, c^{'}_4\} = 6$ and $\gamma_s[S(H_3)] = \{c_2, c^{'}_2\} = 2$ as seen from the following fig-3.

Figure 3. $S(H_3)$ and $S(H_5)$



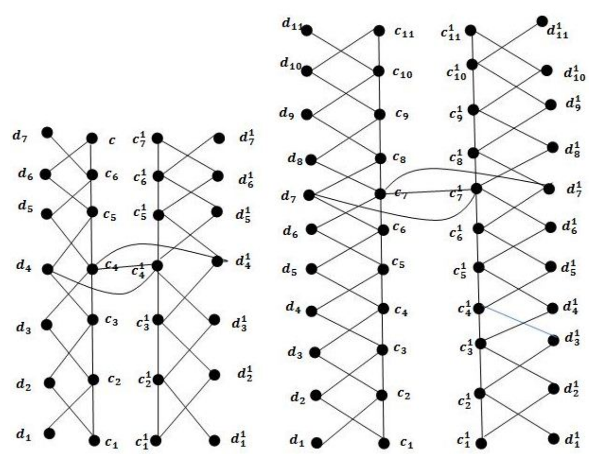
Theorem 3.4.

$$\gamma_s S(H_l) = \begin{cases} l+1 & \textit{if } l \equiv 7 \ (mod8) \\ l-1 & \textit{otherwise.} \end{cases}$$

Proof. **Case I**. When $l \equiv 7(mod8)$,Let $V[S(H_l)] = \{c_h, d_h, c_h^{'}, d_h^{'} : 1 \leq h \leq l\}$, Let $E[S(H_l)]=\{(c_hc_{h+1}) \cup (d_hc_{h+1}) \cup (c_hd_{h+1}) \cup (c_{h+1/2}c^{'}_{h+1/2}) \cup (c_{h+1/2}d^{'}_{h+1/2}) \cup (d_{h+1/2}c^{'}_{h+1/2}) \cup (c^{'}_hc^{'}_{h+1}) \cup (c^{'}_hd^{'}_{h+1}) \cup (d^{'}_hd^{'}_{h+1}); 1 \leq h \leq l-1\}$. For $1 \leq h \leq l+1/2$, In H_l, vertex c_{h+1} is adjacent to $c_{h+1}, d_{h+2}, c_h, d_h$ where as c_{h+2} is adjacent to $c_{h+3}, d_{h+3}, c_{h+1}, d_{h+1}$ by definition c_{h+1} and c_{h+2} are adjacent and in every fourth of c_{h+1} and c_{h+2} the center vertices satisfy the γ - ing set condition, The vertex $c_{l+1/2}$ and $c^{'}_{l+1/2}$ Split dominating any other vertex in the graph from the remaining vertex. Consider, for $(l+3)/2 \leq h \leq l$,vertex c_{h+1} is adjacent to $c_{h+1}, d_{h+2}, c_h, d_h$ where as c_{h+2} is adjacent to $ch+3, d_{h+3}, c_{h+1}, d_{h+1}$ by definition c_{h+1} and c_{h+2} are adjacent and in every fourth of c_{h+1} and c_{h+2} the center vertices satisfy the γ - ing set condition, and c_{l-1} is adjacent to c_{l-2}, d_{l-2}, c_l, d_l. The same process applicable for the set of vertex $c^{'}_{h+1}$ and $c^{'}_{h+2}$.

Case II. Let $V[S(H_l)] = \{c_h, d_h, c_h^{'}, d_h^{'} : 1 \leq h \leq l\}$. Let $E[S(H_l)]=\{(c_hc_{h+1}) \cup (d_hc_{h+1}) \cup (c_hd_{h+1}) \cup (c_{h+1/2}c^{'}_{h+1/2}) \cup (c_{h+1/2}d^{'}_{h+1/2}) \cup (d_{h+1/2}c^{'}_{h+1/2}) \cup (c^{'}_hc^{'}_{h+1}) \cup (c^{'}_hd^{'}_{h+1}) \cup (d^{'}_hd^{'}_{h+1}); 1 \leq h \leq l-1\}$.For $1 \leq h \leq l+1/2$. In H_l, vertex c_{h+1} is adjacent to $c_{h+1}, d_{h+2}, c_h, d_h$ where as c_{h+2} is adjacent to c_{h+3}, d_{h+3}, c_{h+1}, d_{h+1} by definition c_{h+1} and c_{h+2} are adjacent and in every fourth of c_{h+1} and c_{h+2} the center vertices satisfy the γ_s-ing set condition, The vertex $c_{l+1/2}$ and $c^{'}_{l+1/2}$ Split dominating any other vertex in the graph from the remaining vertex. Consider, for $(l+3)/2 \leq h \leq l$,vertex c_{h+1} is adjacent to $c_{h+1}, d_{h+2}, c_h, d_h$ where as c_{h+2} is adjacent to $ch+3, d_{h+3}, c_{h+1}, d_{h+1}$ by definition c_{h+1} and c_{h+2} are adjacent and in every fourth of c_{h+1} and c_{h+2} the center vertices satisfy the γ - ing set condition, and c_{l-1} is adjacent to c_{l-2}, d_{l-2}, c_l, d_l. The same process applicable for the set of vertex $c^{'}_{h+1}$ and $c^{'}_{h+2}$. Hence $\gamma_s S(H_l) = l+1$ if $l \equiv 7(mod8)$ and $l-1$ otherwise. For example,$\gamma_s[S(H_{11})] = \{c_2, c_3, c_6, c_9, c_{10}, c^{'}_2, c^{'}_3, c^{'}_6, c^{'}_9, c^{'}_{10}\} = 10$ and $\gamma_s[S(H_7)] = \{c_2, c_3, c_6, d_6, c^{'}_2, c^{'}_3, c^{'}_6, d^{'}_6\} = 8$ as seen from the following fig-4. □

Figure 4. $S(H_7)$ and $S(H_{11})$.

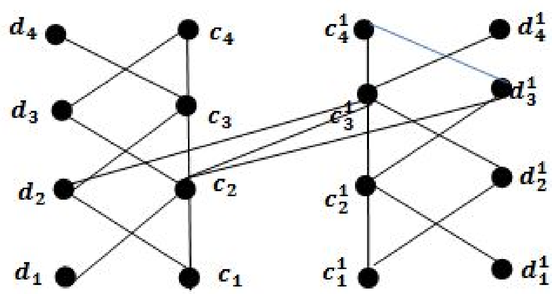


Theorem 3.5. *$\gamma_s S(H_l) = l$ when $l \equiv 0$ (mod4)*

Proof. Let $V[S(H_l)] = \{c_h, d_h, c_h^{'}, d_h^{'} : 1 \leq h \leq l\}$, Let $E[S(H_l)]=\{(c_h c_{h+1}) \cup (d_h c_{h+1}) \cup (c_h d_{h+1}) \cup (c_{h/2} c^{'}_{(h/2)+1}) \cup (c_{h/2} d^{'}_{(h/2)+1}) \cup (d_{h/2} c^{'}_{(h/2)+1}) \cup (c^{'}_h c^{'}_{h+1}) \cup (c^{'}_h d^{'}_{h+1}) \cup (d^{'}_h d^{'}_{h+1}); 1 \leq h \leq l-1\}$. In H_l, vertex c_{h+1} is adjacent to $c_{h+1}, d_{h+2}, c_h, d_h$ where as c_{h+2} is adjacent to $c_{h+3}, d_{h+3}, c_{h+1}, d_{h+1}$ by definition c_{h+1} and c_{h+2} are adjacent and in every fourth of c_{h+1} and c_{h+2} the center vertices satisfy the γ_s - ing set condition,The same process applicable for the set of vertex $c^{'}_{h+1}$ and $c^{'}_{h+2}$. Hence, $\gamma_s S(H_l) = l$ when $l \equiv 0(mod4)$. For example, $\gamma_s[S(H_4)] = \{c_2, c_3, c^{'}_2, c^{'}_3\} = 4$ as seen from the following fig-5.

□

Figure 5. $S(H_4)$.



Theorem 3.6.

$$\gamma_s S(H_l) = \begin{cases} l+2 & \text{if } l \equiv 6 \text{ (mod8)} \\ l & \text{if } l \equiv 6 \text{ (mod8).} \end{cases}$$

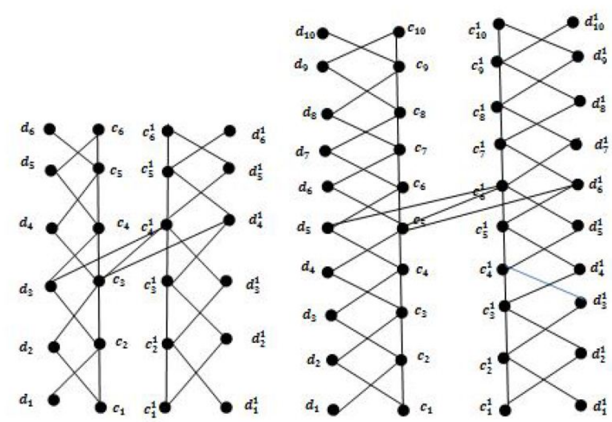
Proof. **Case I**. $l \equiv 6$ (mod8), Let $V[S(H_l)] = \{c_h, d_h, c^{'}_h, d^{'}_h : 1 \leq h \leq l\}$, Let $E[S(H_l)]=\{(c_h c_{h+1}) \cup (d_h c_{h+1}) \cup (c_h d_{h+1}) \cup (c_{h/2} c^{'}_{(h/2)+1}) \cup (c_{h/2} d^{'}_{(h/2)+1}) \cup (d_{h/2} c^{'}_{(h/2)+1}) \cup (c^{'}_h c^{'}_{h+1}) \cup (c^{'}_h d^{'}_{h+1}) \cup (d^{'}_h d^{'}_{h+1}); 1 \leq h \leq l-1\}$. In H_l, vertex c_{h+1} is adjacent to $c_{h+1}, d_{h+2}, c_h, d_h$ where as c_{h+2} is adjacent to $c_{h+3}, d_{h+3}, c_{h+1}, d_{h+1}$ by definition c_{h+1} and c_{h+2} are adjacent and in every fourth of c_{h+1} and c_{h+2} the center vertices satisfy the γ_s - ing set condition,The same process applicable for the set of vertex $c^{'}_{h+1}$ and $c^{'}_{h+2}$. Hence, $\gamma_s S(H_l) = l+2$ when $l \equiv 6(mod8)$.

Case II. $l \equiv 2$ (mod8), Let $V[S(H_l)] = \{c_h, d_h, c^{'}_h, d^{'}_h : 1 \leq h \leq l\}$, Let $E[S(H_l)]=\{(c_h c_{h+1}) \cup (d_h c_{h+1}) \cup (c_h d_{h+1}) \cup (c_{h/2} c^{'}_{(h/2)+1}) \cup (c_{h/2} d^{'}_{(h/2)+1}) \cup (d_{h/2} c^{'}_{(h/2)+1}) \cup (c^{'}_h c^{'}_{h+1}) \cup (c^{'}_h d^{'}_{h+1}) \cup (d^{'}_h d^{'}_{h+1}); 1 \leq h \leq l-1\}$. The vertex $c_{l/2}$ and $c^{'}_{l+1/2}$ Split dominating set any other vertex in the graph from the remaining vertex. For $1 \leq h \leq l/2$, In H_l, vertex c_{h+1} is adjacent to $c_{h+1}, d_{h+2}, c_h, d_h$ where as c_{h+2} is adjacent to $c_{h+3}, d_{h+3}, c_{h+1}, d_{h+1}$ by definition c_{h+1} and c_{h+2} are adjacent and in every fourth of c_{h+1} and c_{h+2} the center vertices satisfy the

γ_s - ing set condition, Consider, for $l/2 \leq h \leq l$ vertex c_{h+1} is adjacent to $c_{h+1}, d_{h+2}, c_h, d_h$ where as c_{h+2} is adjacent to $c_{h+3}, d_{h+3}, c_{h+1}, d_{h+1}$ by definition c_{h+1} and c_{h+2} are adjacent and in every fourth of c_{h+1} and c_{h+2} the center vertices satisfy the γ_s - ing set condition, The same process applicable for the set of vertex $c^{'}_{h+1}$ and $c^{'}_{h+2}$. Hence, $\gamma_s S(H_l) = l$ example, $\gamma_s[S(H_{10})] = \{c_2, c_5, c_8, c_9, d_2, c^{'}_2, c^{'}_5, c^{'}_8, c^{'}_9, d^{'}_2\} = 10$ and $\gamma_s[S(H_6)] = \{c_2, c_5, d_2, d_5, c^{'}_2, c^{'}_5, d^{'}_2, d^{'}_5\} = 8$ as seen from the following fig-6.

□

Figure 6. $S(H_6)$ and $S(H_10)$.



Theorem 3.7. $\gamma_s S(P^+_l) = l$

Proof. Let $V[S(P^+_l) {=} \{c_h, d_h, c^{'}_h, d^{'}_h : 1 \leq h \leq l\}$.

Let $E[S(P^+_l)] {=} \{[(c_h c_{h+1}) 1 \leq h \leq l] \cup [(c_h c^{'}_h) \cup (c_h d^{'}_h); 1 \leq h \leq l] \cup [(d_h c_{h-1}) \cup (d_h c^{'}_h) \cup (d_h c_{h+1}); 2 \leq h \leq l-1] \cup (d_1 c^{'}_1) \cup (d_1 c_2) \cup (d_l c^{'}_l) \cup (d_l c_{l-1})\}$. In P^+_l, For n ¿ 2, vertex c_h is adjacent to c_{h+1}, d_{h+1} for all $1 \leq h \leq l-1$ where as c_h is adjacent to $c^{'}_h$, $d^{'}_h$ for all $1 \leq h \leq l$ and vertex c_{h+1} is adjacent to c_h, d_h for all $1 \leq h \leq l$ by defn c_h and c_{h+1} are adjacent and in every c_h and c_{h+1} $1 \leq h \leq l-1$ vertices satisfy the γ_s - ing set condition. Hence $\gamma_s S(P^+_l) = l$ example, $\gamma_s S(P^+_5) {=} \{c_1, c_2, c_3, c_4, c_5\}$ as seen from the following fig-7. □

Theorem 3.8. $\gamma_s[S(P_l o N K_1)] = l$

Proof. Let $V[S(P_l o N K_1)] {=} \{c_h, d_h, c_{hg}, d_{hg} : 1 \leq h \leq l, 1 \leq g \leq N\}$.

Let $E[S(P_l o N K_1] {=} \{[(c_h c_{h+1}) 1 \leq h \leq l-1] \cup [(c_h c_{hg} \cup (d_h c_{hg}) \cup (c_h d_{hg}); 1 \leq h \leq l, 1 \leq g \leq N] \cup [(d_h c_{h-1}) \cup (d_h c_{h+1}); 2 \leq h \leq l-1] \cup (d_1 c_2) \cup (d_l c_{l-1})\}$.

In $S(P_l o N K_1)$, Vertex c_h is adjacent to c_{hg}, d_{hg} for all $1 \leq h \leq l, 1 \leq g \leq N$, where as c_h is adjacent to $c_{h-1}, d_{h-1}, c_{h+1}, d_{h+1}$ for all $2 \leq h \leq l-1$.

Hence $\gamma_s[S(P_l o N K_1)] = l$ example, $\gamma_s[S(P_4 o N K_1)] = \{c_1, c_2, c_3, c_4\} = 4$, as seen from the following fig 8. □

Figure 7. $S(P_5^+)$.

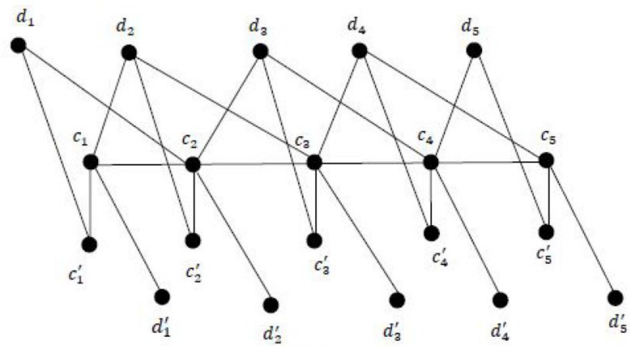
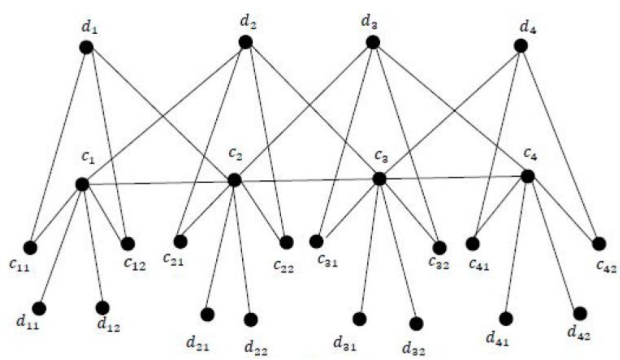


Figure 8. $S(P_4 oNK_1)$



Theorem 3.9. *Let G be a path splitted graph. Let k be maximum degree. Then $G-k$ is γ_s-ing set.*

Proof. Let G be a path splitted graph. Let k be maximum degree. Now, a cut vertices k of path splitted graph G, k is a point whose removal increases the number of components. So $G-k$ is disconnected graph. Then $G-k$ is γ_s-ing set. □

Acknowledgment

Split dominating sets of splitted graphs are useful for the communication and modeling network graphs

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