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On $^{\star}b$-open sets and $^{\star}b$- sets in nano topological spaces

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Abstract: The aim of this paper is to introduce certain novel sets, $^{\star}b$-set and $^{\star}b$-set. Moreover, we study about some of their properties.

Key words: n-open sets, ns-open sets, nb-open sets and $n^{\star}b$-open sets and $n^{\star}b$-sets.

1. Introduction

M. Lellis Thivagar et al [1] introduced the concept of nano topological spaces which was defined in terms of approximations and boundary region of a subset of a universe U using an equivalence relation on it.

2. Preliminaries

Definition 2.1. [3] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$.

1. The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X) = \bigcup_{x \in U}\{R(x) : R(x) \subseteq X\}$, where $\tau_R(x)$ denotes the equivalence class determined by x.
2. The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is, $U_R(X) = \bigcup_{x \in U}\{R(x) : R(x) \cap X \neq \phi\}$.
3. The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not - X with respect to R and it is denoted by $B_R(X)$. That is, $B_R(X) = U_R(X) - L_R(X)$.

Definition 2.2. [1] Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then $\tau_R(X)$ satisfies the following axioms:

1. U and $\phi \in \tau_R(X)$,
2. The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$,
3. The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

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Thus $\tau_R(X)$ is a topology on U called the nano topology with respect to X and $(U, \tau_R(X))$ is called the nano topological space. The elements of $\tau_R(X)$ are called nano-open sets (briefly n-open sets). The complement of a n-open set is called n-closed.

Through out this paper, we denote a nano topological space by $(U, \mathcal{N})$, where $\mathcal{N} = \tau_R(X)$. The nano-interior and nano-closure of a subset H of U are denoted by $I_n(H)$ and $C_n(H)$, respectively.

Definition 2.3. A subset H of a space $(U, \mathcal{N})$ is called

1. nano semi-open set (resp. ns-open) [1] if $H \subseteq C_n(I_n(H))$.
2. nano pre-open (resp. np-open) [1] if $H \subseteq I_n(C_n(H))$.
3. nano regular-open (resp. nr-open) [1] if $H = I_n(C_n(H))$.
4. nano b-open (resp. nb-open) [2] if $H \subseteq I_n(C_n(H)) \cup C_n(I_n(H))$.

The complements of the above mentioned sets are called their respective closed sets.

3. On nano $^\star b$-open sets and nano $^\star b$-sets

Definition 3.1. Let $(U, \mathcal{N})$ be a nanotopological space and $A \subseteq U$. Then,

1. H is said to be nano $^\star b$-open (resp. $n^\star b$-open) if $H \subseteq C_n(I_n(H)) \cap I_n(C_n(H))$.
   The complement of a $^\star b$-open set is called $^\star b$-closed set.
2. An element $x \in H$ is said to be $n^\star b$-interior point of H if there exits a $n^\star b$-open set K such that $x \in K \subseteq H$. The set of all $n^\star b$-interior points of H is said to be $n^\star b$-interior of H an it is denoted by $^\star b$-$I_n(H)$.

Proposition 3.1. *In a nanotopological spaces $(U, \mathcal{N})$, every n-open set is $n^\star b$-open.*

Proof.

Let H be a n-open set of U. Then, $H \subseteq C_n(I_n(H)) \cap I_n(C_n(H))$, since H is n-open set, then $I_n(H) = H$ and $C_n(H) = U$ or $C_n(H) = Q$ where Q is n-closed set of U. Thus, $H \subseteq C_n(H) \cap I_n(Q)$. Now, $C_n(H) = U$ or $C_n(Q)$ where Q is n-closed set of U and $H \subseteq I_n(Q)$, therefore $I_n(Q) = H$ or $I_n(Q) = S$ where $H \subseteq S \subseteq I_n(Q)$. In consequence, $H \subseteq Q \cap S$, but $H \subseteq Q$ and $H \subseteq S$, this implies that $H \subseteq Q \cap S$.

Hence H is a $n^\star b$-open set of U. □

Remark 3.1. *The next Example shows that the converse of the Proposition 3.1, it is not always true.*

Example 3.1.

Let $U = \{\alpha, \beta, \gamma, \eta, \delta\}$ with $U/R = \{\{\alpha\}, \{\beta, \gamma\}, \{\eta, \delta\}\}$ and $X = \{\alpha, \beta\}$. Then the nano topology $\mathcal{N} = \{\phi, \{\alpha\}, \{\beta, \gamma\}, \{\alpha, \beta, \gamma\}, U\}$. Clear the set $\{\alpha, \beta\}$ is $n^\star b$-open set but n-open.

Theorem 3.1. *Let $(U, \mathcal{N})$ be a nanotopological space and $H \subseteq U$. Then, $^\star b$-$I_n(H) = \bigcup\{K | K$ is $n^\star b$-open and $K \subseteq H\}$.*

Proof.

Let $x \in {}^\star b$-$I_n(H)$. Then, there exits $n^\star b$-open set K such that $x \in K \subseteq H$. Hence, $x \in \bigcup\{K | K$ *is $n^\star b$-open and* $K \subseteq H\}$. Then, $y \subseteq K_0$ for some $n^\star b$-open set $K_0 \subseteq H$. Therefore, $y \in {}^\star b$-$I_n(H)$.

In consequence, $\bigcup\{K|K \text{ is } n^{\star}b\text{-open and } K \subseteq H\} \subseteq {}^{\star}b\text{-}I_n(H)$. □

Theorem 3.2. *Let $(U,\mathcal{N})$ be a nanotopological space and $H \subseteq U$. Then, H is $n^{\star}b$-open $\iff$ $H = {}^{\star}b\text{-}I_n(H)$.*

Proof.

Let H be a $n^{\star}b$-open set. Then, $H \subseteq H$ and this implies that $H \in \{K|K$ is $n^{\star}b$-open and $K \subseteq H\}$. Since union of this collection is in H. Therefore, $H = {}^{\star}b\text{-}I_n(H)$. Conversely, suppose that $H = {}^{\star}b\text{-}I_n(H)$. Hence, H is $n^{\star}b$-open. □

Theorem 3.3. *Let $(U,\mathcal{N})$ be a nanotopological space and $H \subseteq U$. If $n^{\star}b$-open set is ns-open.*

Proof.

Let H be a $n^{\star}b$-open set, then $H \subseteq C_n(I_n(H)) \cap I_n(C_n(H))$; $H \subseteq C_n(I_n(H))$. Therefore, H is a ns-open set. □

Remark 3.2. *The next Example shows that the converse of the Theorem 3.3, it is not always true.*

Example 3.2. *In Example 3.1, the set $\{\alpha,\eta\}$ is ns-open set but not $n^{\star}b$-open set.*

Theorem 3.4. *Let $(U,\mathcal{N})$ be a nanotopological space and $H \subseteq U$. If $n^{\star}b$-open set is np-open.*

Proof.

Let H be a $n^{\star}b$-open set, then $H \subseteq C_n(I_n(H)) \cap I_n(C_n(H))$; $H \subseteq I_n(C_n(H))$. Therefore, H is a np-open set. □

Remark 3.3. *The converse of the Theorem 3.4 is not always true when $C_n(I_n(H)) = \phi$ and $I_n(C_n(H)) = Q$, where $H \subseteq Q$.*

Proposition 3.2. *In a nanotopological spaces, every $n^{\star}b$-open set is nb-open set.*

Proof.

The proof is followed by the Theorem 3.3 and 3.4. □

Theorem 3.5. *The Intersection of a nb-open set and a $n^{\star}b$-open set is a nb-open set.*

Proof.

Let H be a $n^{\star}b$-open set and Q be a nb-open set.

Then $H\cap Q \subseteq [C_n(I_n(H))\cap I_n(C_n(H))]\cap[C_n(I_n(Q))[I_n(C_n(Q))] = [C_n(I_n(H))\cap I_n(C_n(H))\cap C_n(I_n(Q))]\cup [I_n(C_n(H) \cap C_n(I_n(H)) \cap I_n(C_n(Q))] \subseteq [I_n(C_n(H)) \cap C_n(I_n(H \cap Q))] \cup [C_n(I_n(H)) \cap I_n(C_n(H \cap Q))]$.

By The Proposition 3.2, $H \cap Q$ is a nb-open set and let $H \cap Q = S$.

Then $[I_n(C_n(H)) \cap C_n(I_n(S))] \cup [C_n(I_n(H)) \cap I_n(C_n(S))]$.

Now, $I_n(C_n(H)) \cap C_n(I_n(S)) = E$, where $E \neq \phi$ and $C_n(I_n(H)) \cap I_n(C_n(S)) = F$, where $F \neq \phi$ and so. $H \cap Q \subseteq E \cup F$, where $E \cup F$ is a nb-open set.

□

4. On nano $^{\star}b$-sets

Definition 4.1. Let $(U,\mathcal{N})$ be a nanotopological space and $H \subseteq U$. Then, H is said to be nano $^{\star}b$-set (resp. $n^{\star}b$-set) if $H = C_n(I_n(H)) \cap I_n(C_n(H))$.

Remark 4.1. *The next example shows that the union of two $n^{\star}b$-sets not need be a $n^{\star}b$-set.*

Theorem 4.1. *In a nanotopological spaces, the intersection of two $n^{\star}b$-sets is a $n^{\star}b$-set.*

Proof.

Let H and Q two $n^{\star}b$-sets, then $H = C_n(I_n(H)) \cap I_n(C_n(H))$ and $Q = C_n(I_n(Q)) \cap I_n(C_n(Q))$. Now, $H \cap Q = [C_n(I_n(H)) \cap I_n(C_n(H))] \cap [C_n(I_n(Q)) \cap I_n(C_n(Q))] = [C_n(I_n(H)) \cap C_n(I_n(Q))] \cap [I_n(C_n(H)) \cap I_n(C_n(Q))] \subseteq [C_n(I_n(H) \cap I_n(Q)) \cap I_n(C_n(H) \cap C_n(Q)) \subseteq [C_n(I_n(H \cap Q)) \cap I_n(C_n(H \cap Q))$; $[C_n(I_n(H \cap Q)) \cap I_n(C_n(A \cap Q)) \subseteq [C_n(I_n(H)) \cap C_n(I_n(Q))] \cap [I_n(C_n(H)) \cap I_n(C_n(Q))] = H \cap Q$.

Therefore, $H \cap Q$ is a $n^{\star}b$-set. □

Theorem 4.2. *In nanotopological spaces, If H is a $n^{\star}b$-set then H is a $n^{\star}b$-open set.*

Proof.

Let H be a $n^{\star}b$-set, then $H = C_n(I_n(H)) \cap I_n(C_n(H))$; $H \subseteq C_n(I_n(H)) \cap I_n(C_n(H))$. Therefore, H is a $n^{\star}b$-open set. □

Remark 4.2. *The next example shows that the converse of the Theorem 4.2, it is not always true.*

Example 4.1. *In Example 3.1, the set $\{\alpha, \beta\}$ is $n^{\star}b$-open set but $n^{\star}b$-set.*

Proposition 4.1. *Let $(U,\mathcal{N})$ be a nanotopological space and $H \subseteq U$. Then, the following statements hold:*

1. *If H is a $n^{\star}b$-set, then H is a ns-open set.*
2. *If H is a $n^{\star}b$-set, then H is a np-open set.*
3. *If H is a $n^{\star}b$-set, then H is a nb-open set.*

Proof.

1. Let H be a $n^{\star}b$-set, then $H = C_n(I_n(H)) \cap I_n(C_n(H)) \subseteq C_n(I_n(H)) \cap (I_n(C_n(H)) \subseteq C_n(I_n(H))$. Therefore, H is ns-open.
2. Let H be a $n^{\star}b$-set, then $H = C_n(I_n(H)) \cap I_n(C_n(H)) \subseteq C_n(I_n(H)) \cap (I_n(C_n(H)) \subseteq I_n(C_n(H))$. Therefore, H is np-open.
3. Let H be a $n^{\star}b$-set, then $H = C_n(I_n(H)) \cap I_n(C_n(H)) \subseteq C_n(I_n(H)) \cap (I_n(C_n(H)) \subseteq C_n(I_n(H)) \cup I_n(C_n(H))$. Therefore, H is nb-open.

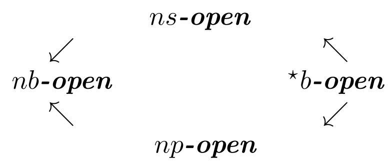
□

Theorem 4.3. *Let $(U,\mathcal{N})$ be a nanotopological space and $H \subseteq U$. If H is a $n^{\star}b$-set and $C_n(I_n(H)) \subsetneq I_n(C_n(H))$, then H is a nr-open set.*

Proof.

Let H be a $n^{\star}b$-set, then $H = C_n(I_n(H)) \cap I_n(C_n(H))$, since $C_n(I_n(H)) \subsetneq I_n(C_n(H))$, then $H \in I_n(C_n(H))$. Therefore, H is a nr-open set. □

Remark 4.3. *These relations are shown in the diagram.*



Example 4.2. *In Example 3.1, the set* $\{\beta\}$ *is* np*-open set but not* $n^{\star}b$*-open.*

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