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# New properties of $gb^*-$closed map and $gb^*$-open map in topological spaces

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**Abstract:** In this paper, we some of their properties of semi-open, $\alpha$-open, pre-open, $\alpha$-open sets to induced topology, we compare with the semi-open or ( $b$-open, pre-open, $\alpha$-open ) set if for the topological space and the topological space induced, finally we inntroduce a new properties for $gb^*$-closed maps and $gb^*$-open map in topological space induced.

**Key words:** semi-open set, $\alpha$-open set, $gb^*-$closed map, $gb^*$-open map.

## 1. Introduction and preliminaries

Different types of closed and open were studied by various researchers. The notion of $b$-open sets was introduced by D. Andrijevic [4] in the year 1968. A. A. Omari et al [1] made an analytical study and result the concepts of generalized $b$-closed sets in topological spaces. The idea of introduced regular generalized $b$-closed map in topological space by S. Sekar and K. Mariappa [13] in 2013. A new class of generalised $b$ star- closed map in Topological Spaces was introduced in 2017 by S. Sekar et al [14].

The aim of this paper is to continue the study of new properties of $b$ star-closed maps in topological spaces ome of its properties are based on the induced topology and compares the state of a set with respect to the topological subspace and the total topological space in the following positions. Throughout the present paper, $(\mathcal{E},\mathcal{T})$ and $(\mathcal{F},\Sigma)$ will denote topological spaces with no separation properties assumed. For a subset $\Omega$ of a topological space $(\mathcal{E},\mathcal{T})$, $cl(\Omega)$ and $int(\Omega)$ will denote the closure and interior of $\Omega$ in $(\mathcal{E},\mathcal{T})$, respectively.

We recall the following definitions which are useful in the sequel.

**Definition 1.1.** Let a subset $\Omega$ of a topological space $(\mathcal{E},\mathcal{T})$ is called

(1) a pre-open set [2], if $\Omega \subseteq int(cl(\Omega))$.

(2) a semi-open set [10], if $\Omega \subseteq cl(int(\Omega))$.

(3) a $\alpha$-open set [2], if $\Omega \subseteq int(cl(int(\Omega)))$.

(4) a $b$-open set [4], if $\Omega \subseteq cl(int(\Omega)) \cup int(cl(\Omega))$.

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**Remark 1.1.** *The $b$-closed (resp. semi-closed, pre-closed, $\alpha$-closed) of a subset $\Omega$ of a space $(\mathcal{E},\mathcal{T})$ is the intersection of all $b$-closure (resp. semi-closed, pre-closed, $\alpha$-closed) sets that contain $\Omega$ and is denoted by $bcl(\Omega)$ (resp. $scl(\Omega)$, $pcl(\Omega)$, $\alpha cl(\Omega)$).*

**Remark 1.2.** *The $b$-open (resp. semi-open, pre-open, $\alpha$-open) of a contained in $\Omega$ of a space $(\mathcal{E},\mathcal{T})$ is the union of all $b$-interior (resp. semi-interior, pre-interior, $\alpha$-interior) sets that Contained in $\Omega$ and is denoted by $b$-$int(\Omega)$ (resp. $s$-$int(\Omega)$, $p$-$int(\Omega)$, $\alpha$-$int(\Omega)$).*

**Definition 1.2.** Let a subset $\Omega$ of a topological space $(\mathcal{E},\mathcal{T})$ is called

(1) a generalized closed set (briefly $g$-closed) [9], if $cl(\Omega)\subseteq\theta$ whenever $\Omega\subseteq\theta$ and $\theta$ is open in $\mathcal{E}$.

(2) a generalized $b$-closed set (briefly $gb$-closed) [1], if $bcl(\Omega)\subseteq\theta$ whenever $\Omega\subseteq\theta$ and $\theta$ is open in $\mathcal{E}$.

(3) a $\alpha$ generalized $*$-closed set (briefly $\alpha g^*$-closed) [8], if $cl(\Omega)\subseteq int(\theta)$ whenever $\Omega\subseteq\theta$ and $\theta$ is $\alpha$-open in $\mathcal{E}$.

(4) a $g*s$-closed set (briefly $g*s$-closed) [3], if $scl(\Omega)\subseteq\theta$ whenever $\Omega\subseteq\theta$ and $\theta$ is $gs$-open in $\mathcal{E}$.

(5) a regular generalized $b-$closed set (briefly $rgb$-closed) [5] if $scl(\Omega)\subseteq\theta$ whenever $\Omega\subseteq\theta$ and $\theta$ is regular open in $\mathcal{E}$.

**Definition 1.3.** [14] Let $\mathcal{E}$ and $\mathcal{F}$ be topological spaces. A map $\phi:(\mathcal{E},\mathcal{T})\to(\mathcal{F},\mathcal{T})$ is called generalized $b$ star-closed (briefly, $gb^*$- closed map) if the image of every closed set in $\mathcal{E}$ is $gb^*$-closed in $\mathcal{F}$.

**Definition 1.4.** [14]Let $\mathcal{E}$ and $\mathcal{F}$ be topological spaces. A map $\phi:(\mathcal{E},\mathcal{T})\to(\mathcal{F},\mathcal{T})$ is called generalized $b$ star open (briefly, $gb^*$-open) if the image of every open set in $\mathcal{E}$ is $gb^*$-open in $\mathcal{F}$.

## 2. Main Results

In this section, we study the behavior of pre-open sets, $b$-open sets, $\alpha$-open sets and $b$-closed sets and $g$-closed sets in a sub-space. For simplification, we introduce the following Remark and Notation.

**Notation 2.1.** *Let $\mathcal{E}_0$ be a nonempty set endowed with the induced topology of $(\mathcal{E},\mathcal{T})$, we denote $(\mathcal{E}_0,\mathcal{T}_0)$.*

**Notation 2.2.** *Let a subset $\Omega$ of a topological space $(\mathcal{E}_0,\mathcal{T}_0)$, we give the following set $cl_{\mathcal{E}_0}(\Omega)$, $int_{\mathcal{E}_0}(\Omega)$, $bcl_{\mathcal{E}_0}(\Omega)$, $b$-$int_{\mathcal{E}_0}(\Omega)$ and $scl_{\mathcal{E}_0}(\Omega)$ according to their definitions but in relation to the induced topology.*

**Remark 2.1.** *Let a subset $\Omega$ of a topological space $(\mathcal{E}_0,\mathcal{T}_0)$, we have*

$$cl_{\mathcal{E}_0}(\Omega)=cl(\Omega)\cap\mathcal{E}_0,\qquad int(\Omega)\subseteq int_{\mathcal{E}_0}(\Omega). \tag{1}$$

The principle of the following theorem, we study the behavior of pre-open sets a sub-space.

**Theorem 2.1.** *Let a subset $\Omega$ of a topological space $\mathcal{E}_0$, such as $cl(\Omega)\subset\mathcal{E}_0\subset\mathcal{E}$ and $\Omega$ is pre-open in $(\mathcal{E},\mathcal{T})$. Then $\Omega$ is pre-open in $(\mathcal{E}_0,\mathcal{T}_0)$, but not conversely.*

*Proof.* Let $\Omega$ is pre-open in $(\mathcal{E},\mathcal{T})$, by (1), we get that $\Omega\subseteq int(cl(\Omega))\subseteq int_{\mathcal{E}_0}(cl(\Omega))$, since $cl_{\mathcal{E}_0}(\Omega)=cl(\Omega)\cap\mathcal{E}_0=cl(\Omega)$, then $\Omega\subseteq int_{\mathcal{E}_0}(cl(\Omega)\cap\mathcal{E}_0)=int_{\mathcal{E}_0}(cl_{\mathcal{E}_0}(\Omega))$. Hence $\Omega$ is pre-open in $(\mathcal{E}_0,\mathcal{T}_0)$. □

**Remark 2.2.** *If $(\mathbb{R},\mathcal{T}_u)$ endowed with the usual topology, then $(\mathbb{N},\mathcal{P}(\mathbb{N}))$ is the induced topology.*

**Example 2.1.** *Let $\mathcal{E} = \mathbb{R}$, $\mathcal{E}_0 = \mathbb{N}$ and $\Omega = \{1, 2\}$, we have $cl(\Omega) = \Omega$, $int(cl(\Omega)) = \emptyset$ and $int_{\mathcal{E}_0}(cl_{\mathcal{E}_0}(\Omega)) = \Omega$. Hence $\Omega$ is pre-open in the topology for $\mathbb{N}$ and $\Omega$ is not pre-open in $\mathbb{R}$.*

The following result is a direct consequence of Theorem 2.1.

**Corollary 2.1.** *Let a subset $\Omega$ of a topological space $\mathcal{E}_0$, such as $cl(\Omega) \subset \mathcal{E}_0 \subset \mathcal{E}$, then $p\text{-}int(\Omega) \subseteq pint_{\mathcal{E}_0}(\Omega)$.*

*Proof.* Let a subset $\Omega$ of a topological space $\mathcal{E}_0$, we note $\Upsilon_0 = \{\theta \subset \Omega : \theta \text{ is pre-open in } (\mathcal{E}_0, \mathcal{T}_0)\}$ and $\Upsilon = \{\theta \subset \Omega : \theta \text{ is pre-open in } (\mathcal{E}, \mathcal{T})\}$. By Theorem 2.1, we get $\Upsilon \subset \Upsilon_0$, which implies that

$$p\text{-}int(\Omega) = \underset{\theta \in \Upsilon}{\cup} \theta \subseteq \underset{\theta \in \Upsilon_0}{\cup} \theta = p\text{-}int_{\mathcal{E}_0}(\Omega).$$

□

The principle of the following theorem, we study the behavior of $b$-open sets a sub-space.

**Theorem 2.2.** *Let a subset $\Omega$ of a topological space $\mathcal{E}_0$, such as $cl(\Omega) \subset \mathcal{E}_0 \subset \mathcal{E}$ and $\Omega$ is $b$-open in $(\mathcal{E}_0, \mathcal{T}_0)$. Then $\Omega$ is $b$-open in $(\mathcal{E}, \mathcal{T})$, but not conversely.*

*Proof.* Let a subset $\Omega$ of a topological space $\mathcal{E}_0$, by (1), we have

$$\begin{aligned} \Omega &\subseteq cl_{\mathcal{E}_0}(int_{\mathcal{E}_0}(\Omega)) \cup int_{\mathcal{E}_0}(cl_{\mathcal{E}_0}(\Omega)) \subseteq (cl(int_{\mathcal{E}_0}(\Omega)) \cap \mathcal{E}_0) \cup int_{\mathcal{E}_0}(cl(\Omega) \cap \mathcal{E}_0) \\ &\subseteq (cl(int(\Omega)) \cap \mathcal{E}_0) \cup int(cl(\Omega) \cap \mathcal{E}_0) \subseteq (cl(int(\Omega)) \cap \mathcal{E}_0) \cup int(cl(\Omega) \cap \mathcal{E}_0). \end{aligned}$$

Hence, $cl(int(\Omega)) \cap \mathcal{E}_0 \subset cl(int(\Omega))$ and $int(cl(\Omega) \cap \mathcal{E}_0) \subset int(cl(\Omega))$, which implies that $\Omega \subseteq cl(int(\Omega)) \cup int(cl(\Omega))$, then $\Omega$ is semi-open in $(\mathcal{E}, \mathcal{T})$. □

**Example 2.2.** *Let $\mathcal{E} = \mathbb{R}$, $\mathcal{E}_0 = \mathbb{N}$ and $\Omega = \{1, 2\}$, then $int(cl(\Omega)) = cl(int(\Omega)) = \emptyset$ and $cl_{\mathcal{E}_0}(int_{\mathcal{E}_0}(\Omega)) = int_{\mathcal{E}_0}(cl_{\mathcal{E}_0}(\Omega)) = \Omega$. Hence $\Omega$ is $b$-open in $(\mathbb{N}, \mathcal{P}(\mathbb{N}))$ and $\Omega$ is not $b$-open in $(\mathbb{R}, \mathcal{T}_u)$.*

**Corollary 2.2.** *Let a subset $\Omega$ of a topological space $\mathcal{E}_0$, such as $cl(\Omega) \subset \mathcal{E}_0 \subset \mathcal{E}$, then $b\text{-}int_{\mathcal{E}_0}(\Omega) \subseteq b\text{-}int(\Omega)$.*

*Proof.* Let $\Omega$ be set in $\mathcal{E}_0$, we note $\Theta_0 = \{\theta \subset \Omega : \theta \text{ is } b\text{-open in } (\mathcal{E}_0, \mathcal{T}_0)\}$ and $\Theta = \{\theta \subset \Omega : \theta \text{ is } b\text{-open in } (\mathcal{E}, \mathcal{T})\}$. By Theorem 2.2, we get $\Theta_0 \subset \Theta$, which implies that

$$b\text{-}int_{\mathcal{E}_0}(\Omega) = \underset{\theta \in \Theta_0}{\cup} \theta \subseteq \underset{\theta \in \Theta}{\cup} \theta = b\text{-}int(\Omega).$$

□

The principle of the following theorem, we study the behavior of $\alpha$-open sets a sub-space.

**Theorem 2.3.** *Let a subset $\Omega$ of a topological space $\mathcal{E}_0$, such as $cl(\Omega) \subset \mathcal{E}_0 \subset \mathcal{E}$ and $\Omega$ is $\alpha$-open in $(\mathcal{E}, \mathcal{T})$. Then $\Omega$ is $\alpha$-open in $(\mathcal{E}_0, \mathcal{T}_0)$, but not conversely.*

*Proof.* By remark 2.1, we have

$$cl(int(\Omega)) \subseteq cl(int_{\mathcal{E}_0}(\Omega)) \subseteq cl(int_{\mathcal{E}_0}(\Omega)) \cap \mathcal{E}_0 = cl_{\mathcal{E}_0}(int_{\mathcal{E}_0}(\Omega)).$$

Then,

$$int(cl(int(\Omega))) \subseteq int(cl_{\mathcal{E}_0}(int_{\mathcal{E}_0}(\Omega))) \subseteq int_{\mathcal{E}_0}(cl_{\mathcal{E}_0}(int_{\mathcal{E}_0}(\Omega))).$$

Since $\Omega$ is $\alpha$-open in $(\mathcal{E}, \mathcal{T})$, then $\Omega \subseteq int_{\mathcal{E}_0}(cl_{\mathcal{E}_0}(int_{\mathcal{E}_0}(\Omega)))$. Hence $\Omega$ is $\alpha$-open in $(\mathcal{E}_0, \mathcal{T}_0)$. □

**Example 2.3.** *Let* $\mathcal{E} = \mathbb{R}$, $\mathcal{E}_0 = \mathbb{N}$ *and* $\Omega = \{1, 2\}$, *then* $int\left(cl\left(int\left(\Omega\right)\right)\right) = \emptyset$ *and* $int_{\mathcal{E}_0}\left(cl_{\mathcal{E}_0}\left(int_{\mathcal{E}_0}\left(\Omega\right)\right)\right) = \Omega$. *Hence* $\Omega$ *is* $\alpha$*-open in the topology for* $\mathbb{N}$ *and* $\Omega$ *is not* $\alpha$*-open in the topology for* $\mathbb{R}$.

**Corollary 2.3.** *Let a subset* $\Omega$ *of a topological space* $\mathcal{E}_0$, *such as* $cl\left(\Omega\right) \subset \mathcal{E}_0 \subset \mathcal{E}$, *then* $\alpha\text{-}int\left(\Omega\right) \subseteq \alpha\text{-}int_{\mathcal{E}_0}\left(\Omega\right)$.

*Proof.* Let $\Omega$ be set in $\mathcal{E}_0$, we note $\Xi_0 = \{\theta \subset \Omega : \theta \text{ is } \alpha\text{-open in } (\mathcal{E}_0, \mathcal{T}_0)\}$ and $\Xi = \{\theta \subset \Omega : \theta \text{ is } \alpha\text{-open in } (\mathcal{E}, \mathcal{T})\}$. By Theorem 2.3, we get $\Xi \subset \Xi_0$, which implies that

$$\alpha\text{-}int\left(\Omega\right) = \underset{\theta \in \Xi}{\cup} \theta \subseteq \underset{\theta \in \Xi_0}{\cup} \theta = \alpha\text{-}int_{\mathcal{E}_0}\left(\Omega\right).$$

□

The principle of the following theorem, we study the behavior of $g$-closed sets a sub-space.

**Theorem 2.4.** *Let a subset* $\Omega$ *of a topological space* $\mathcal{E}_0$, *such as* $\Omega \subset \mathcal{E}_0 \subset \mathcal{E}$. *If* $\Omega$ *is* $g$*-closed in* $(\mathcal{E}_0, \mathcal{T}_0)$, *then* $\Omega$ *is* $g$*-closed in* $(\mathcal{E}, \mathcal{T})$.

*Proof.* Assume $\Omega$ is $g$-closed in $(\mathcal{E}_0, \mathcal{T}_0)$. Let $\theta$ is open in $(\mathcal{E}, \mathcal{T})$ and $\Omega \subseteq \theta$, with $\Omega \subseteq \theta \cap \mathcal{E}_0$, since $\theta \cap \mathcal{E}_0$ is open in $(\mathcal{E}_0, \mathcal{T}_0)$, then $cl\left(\Omega\right) \subseteq \theta \cap \mathcal{E}_0 \subseteq \theta$. Hence $\Omega$ is $g$-closed in $\mathcal{E}$. □

**Theorem 2.5.** *Let a subset* $\Omega$ *of a topological space* $\mathcal{E}_0$, *such as* $\mathcal{E}_0$ *is open in* $(\mathcal{E}, \mathcal{T})$, *then* $\Omega$ *is* $g$*-closed in* $(\mathcal{E}, \mathcal{T})$, *if and only if* $\Omega$ *is* $g$*-closed in* $(\mathcal{E}_0, \mathcal{T}_0)$.

*Proof.* By Theorem 2.4, just show that, $\Omega$ is $g$-closed in $(\mathcal{E}, \mathcal{T})$ implies that $\Omega$ is $g$-closed in $(\mathcal{E}_0, \mathcal{T}_0)$. Indeed, let $\theta_0$ is open in $\mathcal{E}_0$, there is an open $\theta$ in $\mathcal{E}$, such that $\theta_0 = \theta \cap \mathcal{E}_0$, if $\Omega \subseteq \theta_0 = \theta \cap \mathcal{E}_0$, since $\theta \cap \mathcal{E}_0$ is open in $(\mathcal{E}, \mathcal{T})$, give us $cl\left(\Omega\right) \subseteq \theta_0$ and $cl_{\mathcal{E}_0}\left(\Omega\right) \subseteq cl\left(\Omega\right)$, then $\Omega$ is $g$-closed in $(\mathcal{E}_0, \mathcal{T}_0)$. □

The following result is some properties of $gb^*$-closed maps.

**Theorem 2.6.** *Let* $\mathcal{E}_0$ *be a closed set and let* $\phi : (\mathcal{E}, \mathcal{T}) \to (\mathcal{F}, \Sigma)$ *is* $gb^*$*-closed map, then* $\phi : (\mathcal{E}_0, \mathcal{T}_0) \to (\mathcal{F}, \Sigma)$ *is* $gb^*$*- closed map, but not conversely.*

*Proof.* Let $\phi : (\mathcal{E}, \mathcal{T}) \to (\mathcal{F}, \Sigma)$ is $gb^*$-closed map and $\vartheta_0$ be an closed set in $(\mathcal{E}_0, \mathcal{T}_0)$. So, it exists is a closed set $\vartheta$ in $(\mathcal{E}, \mathcal{T})$, such that $\vartheta_0 = \vartheta \cap \mathcal{E}_0$. As $\mathcal{E}_0$ is closed in $(\mathcal{E}, \mathcal{T})$. Hence $\phi\left(\vartheta_0\right)$ is $gb^*$-closed in $(\mathcal{F}, \Sigma)$. Then $\phi : (\mathcal{E}_0, \mathcal{T}_0) \to (\mathcal{F}, \Sigma)$ is $gb^*$-closed. □

**Example 2.4.** *Let* $\phi : (\mathcal{E}, \mathcal{T}) \to (\mathcal{E}, \Sigma)$ *be a map, such that* $\phi\left(x\right) = x$, $\phi\left(y\right) = x$, $\phi\left(z\right) = y$, $\phi\left(t\right) = z$, *where* $\mathcal{E} = \{x, y, z, t\}, \mathcal{T} = \{\mathcal{E}, \emptyset, \{x\}, \{y, z, t\}\}$, $\Sigma = \{\mathcal{E}, \emptyset, \{y\}, \{x, y, z\}\}$ *and* $\mathcal{E}_0 = \{x\}$. *Then* $\phi : (\mathcal{E}_0, \mathcal{T}_0) \to (\mathcal{E}, \Sigma)$ *is* $gb^*$*-closed map, but* $\phi : (\mathcal{E}, \mathcal{T}) \to (\mathcal{E}, \Sigma)$ *is not* $gb^*$*-closed map.*

**Lemma 2.1.** *Let* $\Omega_1$ *and* $\Omega_2$ *are* $gb^*$*-closed in* $(\mathcal{E}, \mathcal{T})$, *then* $\Omega_1 \cup \Omega_2$ *is* $gb^*$*-closed in* $(\mathcal{E}, \mathcal{T})$.

*Proof.* Let $\Omega_1$ and $\Omega_2$ are $gb^*$-closed in $(\mathcal{E}, \mathcal{T})$, if $bcl\left(\Omega_1 \cup \Omega_2\right) \subseteq \theta$, with $\theta$ is $g^*$open in $(\mathcal{E}, \mathcal{T})$, we have $bcl\left(\Omega_1\right) \subset bcl\left(\Omega_1 \cup \Omega_2\right) \subseteq \theta$, then $\Omega_1 \subseteq \theta$, the same way we find $\Omega_2 \subseteq \theta$. Hence $\Omega_1 \cup \Omega_2 \subseteq \theta$. □

**Theorem 2.7.** *Let* $\mathcal{E}_1$ *and* $\mathcal{E}_2$ *sets in* $\mathcal{E}$ *and let* $\phi : (\mathcal{E}, \mathcal{T}) \to (\mathcal{F}, \Sigma)$ *is map, such as* $\mathcal{E}_1 \cup \mathcal{E}_2 = \mathcal{E}$. *If* $\phi_{\mathcal{E}_1} : (\mathcal{E}_1, \mathcal{T}_1) \to (\mathcal{F}, \Sigma)$ *and* $\phi_{\mathcal{E}_2} : (\mathcal{E}_2, \mathcal{T}_2) \to (\mathcal{F}, \Sigma)$ *are* $gb^*$*-closed map, then* $\phi : (\mathcal{E}, \mathcal{T}) \to (\mathcal{F}, \Sigma)$ *is* $gb^*$*-closed map.*

*Proof.* Let $\phi_{\mathcal{E}_1} : (\mathcal{E}_1, \mathcal{T}_1) \to (\mathcal{F}, \Sigma)$, $\phi_{\mathcal{E}_2} : (\mathcal{E}_2, \mathcal{T}_2) \to (\mathcal{F}, \Sigma)$ are $gb^*$-closed map and $\vartheta$ be an closed set in $\mathcal{E}$, then

$$\begin{aligned} \phi(\vartheta) &= \phi(\vartheta \cap \mathcal{E}) = \phi(\vartheta \cap (\mathcal{E}_1 \cup \mathcal{E}_2)) = \phi((\vartheta \cap \mathcal{E}_1) \cup (\vartheta \cap \mathcal{E}_2)) \\ &= \phi(\vartheta \cap \mathcal{E}_1) \cup \phi(\vartheta \cap \mathcal{E}_2). \end{aligned}$$

We have $\vartheta \cap \mathcal{E}_1$ and $\vartheta \cap \mathcal{E}_2$ are a closed in $(\mathcal{E}_1, \mathcal{T}_1)$ and $(\mathcal{E}_2, \mathcal{T}_2)$, respectively, then $\phi(\vartheta \cap \mathcal{E}_1)$ and $\phi(\vartheta \cap \mathcal{E}_2)$ are $gb^*$-closed in $(\mathcal{F}, \Sigma)$. By lemma 2.1, we get that $\phi(\vartheta \cap \mathcal{E}_1)$ and $\phi(\vartheta \cap \mathcal{E}_2)$ are $gb^*$-closed in $(\mathcal{F}, \Sigma)$. Hence $\phi(\vartheta)$ is $gb^*$-closed in $(\mathcal{F}, \Sigma)$. Then $\phi : (\mathcal{E}, \mathcal{T}) \to (\mathcal{F}, \Sigma)$ is $gb^*$-closed. □

The following corollary is a new generalization of the Theorem 2.7.

**Corollary 2.4.** *Let $\mathcal{E}_1, \mathcal{E}_2, ..., \mathcal{E}_n$ be sets, such as $\mathcal{E} = \bigcup_{i=1}^{i=n} \mathcal{E}_i$, if $\phi_{\mathcal{E}_i} : (\mathcal{E}_i, \mathcal{T}_i) \to (\mathcal{F}, \Sigma)$ is $gb^*$-closed map, for $i \in [1, n] \cap \mathbb{N}$, then $\phi : (\mathcal{E}, \mathcal{T}) \to (\mathcal{F}, \Sigma)$ is $gb^*$-closed map.*

The following result is some properties of $gb^*$-open map.

**Theorem 2.8.** *Let $\mathcal{E}_0$ be a open set in $(\mathcal{E}, \mathcal{T})$ and let $\phi : (\mathcal{E}, \mathcal{T}) \to (\mathcal{F}, \Sigma)$ is $gb^*$-open map, then $\phi : (\mathcal{E}_0, \mathcal{T}_0) \to (\mathcal{F}, \Sigma)$ is $gb^*$-open map, but not conversely.*

*Proof.* Let $\phi : (\mathcal{E}, \mathcal{T}) \to (\mathcal{F}, \Sigma)$ is $gb^*$-closed map and $\theta_0$ be an open set in $(\mathcal{E}_0, \mathcal{T}_0)$. So, it exists is a open set $\theta$ in $(\mathcal{E}, \mathcal{T})$, such that $\theta_0 = \theta \cap \mathcal{E}_0$. As $\mathcal{E}_0$ is open in $(\mathcal{E}, \mathcal{T})$. Hence $\phi(\theta_0)$ is $gb^*$-open in $(\mathcal{F}, \Sigma)$. Then $\phi : (\mathcal{E}_0, \mathcal{T}_0) \to (\mathcal{F}, \Sigma)$ is $gb^*$-open. □

**Example 2.5.** *Let $\phi : (\mathcal{E}, \mathcal{T}) \to (\mathcal{E}, \Sigma)$ be a map such that $\phi(x) = x$, $\phi(y) = x$, $\phi(c) = y$, $\phi(d) = c$, where $\mathcal{E} = \{x, y, c, d\}$, $\mathcal{T} = \{\mathcal{E}, \emptyset, \{x\}, \{y, c, d\}\}$, $\Sigma = \{\mathcal{E}, \emptyset, \{y\}, \{x, y, c\}\}$ and $\mathcal{E}_0 = \{x\}$. Then $\phi : (\mathcal{E}_0, \mathcal{T}) \to (\mathcal{E}, \Sigma)$ is $gb^*$-open map, but $\phi : (\mathcal{E}_0, \mathcal{T}) \to (\mathcal{E}, \Sigma)$ is not $gb^*$-open map.*

**Lemma 2.2.** *Let $\Omega_1$ and $\Omega_2$ are $gb^*$-open in $(\mathcal{E}, \mathcal{T})$, such as $\Omega_1 \cap \Omega_2 \neq \emptyset$, then $\Omega_1 \cup \Omega_2$ is $gb^*$-open in $(\mathcal{E}, \mathcal{T})$.*

*Proof.* Let $\Omega_1$ and $\Omega_2$ are $gb^*$-open in $(\mathcal{E}, \mathcal{T})$, such as $\Omega_1 \cap \Omega_2 \neq \emptyset$, we have $\emptyset$ is $gb^*$-closed, then $(\Omega_1 \cup \Omega_2)^c$ is $gb^*$-closed in $(\mathcal{E}, \mathcal{T})$, thus $\Omega_1 \cup \Omega_2$ is $gb^*$-open in $(\mathcal{E}, \mathcal{T})$. □

**Theorem 2.9.** *Let $\mathcal{E}_1, \mathcal{E}_2$ be sets disjoint, such as $\mathcal{E}_1 \cup \mathcal{E}_2 = \mathcal{E}$. If $\phi_{\mathcal{E}_1} : (\mathcal{E}_1, \mathcal{T}_1) \to (\mathcal{F}, \Sigma)$ and $\phi_{\mathcal{E}_2} : (\mathcal{E}_2, \mathcal{T}_2) \to (\mathcal{F}, \Sigma)$ are $gb^*$-open map, $\phi : \mathcal{E} \to \mathcal{F}$ is an injective map, then $\phi : (\mathcal{E}, \mathcal{T}) \to (\mathcal{F}, \Sigma)$ is $gb^*$-open map.*

*Proof.* Let $\phi_{\mathcal{E}_1} : (\mathcal{E}_1, \mathcal{T}_1) \to (\mathcal{F}, \Sigma)$ and $\phi_{\mathcal{E}_2} : (\mathcal{E}_2, \mathcal{T}_2) \to (\mathcal{F}, \Sigma)$ are $gb^*$-open map, $\theta$ be an open set in $(\mathcal{E}, \mathcal{T})$, then

$$\phi(\theta) = \phi(\theta \cap \mathcal{E}) = \phi(\theta \cap \mathcal{E}_1) \cup \phi(\theta \cap \mathcal{E}_2).$$

We have $\theta \cap \mathcal{E}_1$ and $\theta \cap \mathcal{E}_2$ are a open in $(\mathcal{E}_1, \mathcal{T}_1)$ and $(\mathcal{E}_2, \mathcal{T}_2)$, respectively, then $\phi(\theta \cap \mathcal{E}_1)$ and $\phi(\theta \cap \mathcal{E}_2)$ are $gb^*$-open in $(\mathcal{F}, \Sigma)$, now we show that $\phi(\theta \cap \mathcal{E}_1) \cap \phi(\theta \cap \mathcal{E}_2) = \emptyset$. Suppose that $\phi(\theta \cap \mathcal{E}_1) \cap \phi(\theta \cap \mathcal{E}_2) \neq \emptyset$, let $y \in \phi(\theta \cap \mathcal{E}_1) \cap \phi(\theta \cap \mathcal{E}_2)$, there exists $x_1 \in \mathcal{E}_1$ and $x_2 \in \mathcal{E}_2$, such that, $x_1 \neq x_2$ and $y = \phi(x_1) = \phi(x_2)$, which contradicts, by $\phi$ is injective map. By lemma 2.2, we get $\phi(\theta)$ is $gb^*$-open in $(\mathcal{F}, \Sigma)$. Then $\phi : (\mathcal{E}, \mathcal{T}) \to (\mathcal{F}, \Sigma)$ is $gb^*$-open. □

The following results are consequences of Theorem 2.9.

**Corollary 2.5.** *Let $\mathcal{E}_1, \mathcal{E}_2, ..., \mathcal{E}_n$ be sets, such as $\mathcal{E} = \bigcup_{i=1}^{i=n} \mathcal{E}_i$ and for $i \neq j$, $\mathcal{E}_1 \cap \mathcal{E}_2 = \emptyset$. If, for $i \in [1, n] \cap \mathbb{N}$, $\phi_{\mathcal{E}_i}$ is $gb^*$-open map, then $\phi : (\mathcal{E}, \mathcal{T}) \to (\mathcal{F}, \Sigma)$ is $gb^*$-open map.*

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