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Exterior set in Neutrosophic biminimal structure spaces

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Abstract: We start with studying the concept of some fundamental properties of exterior set in neutrosophic biminimal structure space.

Key words: minimal structure spaces, neutrosophic biminimal structure spaces, exterior set in neutrosophic biminimal structure space

1. Introduction

The contribution of mathematics to the present-day technology in reaching to a fast trend cannot be ignored. The theories presented differently from classical methods in studies such as fuzzy set [24], intuitionstic fuzzy sets[4], intuitionistic set [6], neutrosophic set [23], etc., have great importance in this contribution of mathematics in recent years. Many works have been done on these sets by mathematicians in many areas of mathematics [1–3, 5, 7–16, 18]. The idea of minimal structure (in short, m-structure) was introduced by V. Popa and T. Noiri [19] in 2000. The notion of neutrosophic biminimal structure space (in short, nbiss) was introduced by S. Ganesan and C. Alexander [17] in 2021. Also they introduced and studied $N^1_{mX}N^2_{mX}$-closed sets and $N^1_{mX}N^2_{mX}$-open sets in nbiss and, also the application of index number(Statistical theory) is inspired from the concept of nbiss in real world. In this work, we introduced the concept of exterior set in nbiss and studied some of their basic properties.

2. Preliminaries

Definition 2.1. [17] Let H be a nonempty set & N^1_{mH}, N^2_{mH} be nms on H. A triple (H N^1_{mH}, N^2_{mH}) is said to be nbiss.

Definition 2.2. [17] Let (H, N^1_{mH}, N^2_{mh}) be a nbiss and E be any neutrosophic set. Then

1. Every E $\in$ N^j_{mH} is open & its complement is closed, respectively, for j = 1, 2.
2. N$_m cl_j$-closure of E = minimum {U : U is N^j_{mH}-closed set and U $\geq$ E}, respectively, for j = 1, 2 and it is denoted by N$_m cl_j$(E).

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3. $N_m int_j$-interior of E = maximum {W : W is N^j_{mH}-open set and W $\leq$ E}, respectively, for j = 1, 2 and it is denoted by $N_m int_j(E)$.

Definition 2.3. [17] A subset E of a nbiss (H, N^1_{mH}, N^2_{mH}) is said to be $N^1_{mH}N^2_{mH}$-closed if $N_m cl_1(N_m cl_2(E))$ = E.

Definition 2.4. [17] Let (H, N^1_{mH}, N^2_{mH}) be a nbiss and E be a subset of H. Then E is $N^1_{mH}N^2_{mH}$-closed iff $N_m cl_1(E) = E$ and $N_m cl_2(E) = E$.

Proposition 2.1. *[17] Let (H, N^1_{mH}, N^2_{mH}) be a nbiss. If E and F are $N^1_{mH}N^2_{mH}$-closed subsets of (H, N^1_{mH}, N^2_{mH}), then E $\wedge$ F is $N^1_{mH}N^2_{mH}$-closed.*

Proposition 2.2. *[17] Let (H, N^1_{mH}, N^2_{mH}) be a nbims. Then E is a $N^1_{mH}N^2_{mH}$-open subset of (H, N^1_{mH}, N^2_{mH}) if and only if E = $N_m int_1(N_m int_2(E))$.*

3. $N^i_{mH}N^j_{mH}$-EXTERIOR

Definition 3.1. Let (H, N^1_{mH}, N^2_{mH}) be a nbiss, E a subset of H and h $\in$ H. We called h to be the $N^i_{mH}N^j_{mH}$-exterior point of E if h $\in N_m int_i(N_m int_j(H \setminus E))$. We denote the set of all $N^i_{mH}N^j_{mH}$-exterior points of E by $N_{mH}\mathrm{Ext}_{ij}(E)$ where i, j = 1, 2 and i $\neq$ j.
From definition we have $N_{mH}\mathrm{Ext}_{ij}(E) = H \setminus N_m cl_i(N_m cl_j(E))$.

Example 3.1. *Let $H = \{h\}$ with $N^1_{mH} = \{0_\sim, A, 1_\sim\}$; $(N^1_{mH})^C = \{1_\sim, B, 0_\sim\}$ and*
$N^2_{mH} = \{0_\sim, U, 1_\sim\}$; $(N^2_{mH})^C = \{1_\sim, V, 0_\sim\}$ where
$A = \prec (0.9, 0.3, 0.8)\succ$: $B = \prec (0.8, 0.7, 0.9)\succ$
$U = \prec (0.5, 0.5, 0.7)\succ$: $V = \prec (0.7, 0.5, 0.5)\succ$
WKT $0_\sim = \{\prec h, 0, 0, 1 \succ : h \in H\}$, $1_\sim = \{\prec h, 1, 1, 0 \succ : h \in H\}$ and $0^C_\sim = \{\prec h, 1, 1, 0 \succ : h \in H\}$, $1^C_\sim = \{\prec h, 0, 0, 1 \succ : h \in H\}$.
Now we define $G = \prec (0.3, 0.4, 0.5)\succ$.
Then $N_{mH}Ext_{ij}(\prec (0.3, 0.4, 0.5)\succ) = H \setminus N_m cl_i(N_m cl_j(\prec (0.3, 0.4, 0.5)\succ))$. Hence $N_{mH}Ext_{12}(\prec (0.3, 0.4, 0.5)\succ) = H \setminus N_m cl_1(N_m cl_2(\prec (0.3, 0.4, 0.5)\succ)) = 0_\sim$.

Lemma 3.1. *Let (H, N^1_{mH}, N^2_{mH}) be a nbiss and E be a subset of H. Then for each i, j = 1, 2 & i $\neq$ j, we have;*

1. *$N_{mH}Ext_{ij}(E) \cap E = 0_\sim$.*
2. *$N_{mH}Ext_{ij}(0_\sim) = 1_\sim$.*
3. *$N_{mH}Ext_{ij}(1_\sim) = 0_\sim$.*

Proof. (1) Since $N_{mH}\mathrm{Ext}_{ij}(E) = H \setminus N_m cl_i(N_m cl_j(E))$ and $E \subset N_m cl_i(N_m cl_j(E))$, $(H \setminus N_m cl_i(N_m cl_j(E))) \cap E \subseteq (H \setminus E) \cap E = 0_\sim$. Therefore $(H \setminus N_m cl_i(N_m cl_j(E))) \cap E = 0_\sim$. Hence $N_{mH}\mathrm{Ext}_{ij}(E) \cap E = 0_\sim$.
(2) $N_{mH}\mathrm{Ext}_{ij}(0_\sim) = 1_\sim \setminus N_m cl_i(N_m cl_j(0_\sim)) = 1_\sim \setminus 0_\sim = 1_\sim$.
(3) $N_{mH}\mathrm{Ext}_{ij}(H) = 1_\sim \setminus N_m cl_i(N_m cl_j(1_\sim)) = 1_\sim \setminus 1_\sim = 0_\sim$. $\square$

Theorem 3.1. *Let (H, N^1_{mH}, N^2_{mH}) be a nbiss and E, F be a subset of H. If E $\subseteq$ F, then $N_{mH}\,Ext_{ij}$ (F) $\subseteq$ $N_{mH}\,Ext_{ij}$ (E) where i, j = 1, 2 & i $\neq$ j.*

Proof. Assume that (H, N^1_{mH}, N^2_{mH}) is a nbiss, E, F are subset of H and E $\subseteq$ F. Thus $N_m cl_i(N_m cl_j(E)) \subseteq N_m cl_i(N_m cl_j(F))$ and H $\setminus$ $N_m cl_i(N_m cl_j(F)) \subseteq$ H $\setminus$ $N_m cl_i(N_m cl_j(E))$. Hence $N_{mX}\,\mathrm{Ext}_{ij}$ (F) $\subseteq$ $N_{mH}\,\mathrm{Ext}_{ij}$ (E) for each i, j = 1, 2 and i $\neq$ j. □

Theorem 3.2. *Let (H, N^1_{mH}, N^2_{mH}) be a nbiss and E a subset of H. Then for each i, j = 1, 2 and i $\neq$ j, E is $N^i_{mH}N^j_{mH}$-closed if and only if $N_{mH}\,Ext_{ij}$ (E) = H $\setminus$ E.*

Proof. (1) $\Rightarrow$ (2) Let E be a subset of H. Assume that E is $N^i_{mH}N^j_{mH}$-closed. Thus $E = mcl_i(N_m cl_j(E))$. Therefore $N_{mH}\,\mathrm{Ext}_{ij}$ (E) = H $\setminus mcl_i(N_m cl_j(E)) = H\setminus$ E.
(2) $\Rightarrow$ (1) Assume that $N_{mH}\,\mathrm{Ext}_{ij}$ (E) = H $\setminus$ E. Thus H $\setminus mcl_i(N_m cl_j(E)) = H\setminus$ E. Consequently $mcl_i(N_m cl_j(E)) = E$ and E is $N^i_{mH}N^j_{mH}$-closed. □

Corollary 3.1. *Let (H, N^1_{mH}, N^2_{mH}) be a nbiss and E a subset of H. Then for each i, j = 1, 2 & i $\neq$ j, E is $N^i_{mH}N^j_{mH}$-open if and only if $N_{mH}\,Ext_{ij}$ (H $\setminus$ E) = E.*

Proof. (1) $\Rightarrow$ (2) Let E be a subset of H. Assume that E is $N^i_{mH}N^j_{mH}$-open. Thus H $\setminus$ E is $N^i_{mH}N^j_{mH}$-closed. Therefore $N_{mH}\,\mathrm{Ext}_{ij}$ (H $\setminus$ E) = H $\setminus$ (H $\setminus$ E) = E.
(2) $\Rightarrow$ (1) Assume that $N_{mX}\,\mathrm{Ext}_{ij}$ (H $\setminus$ E) = E. Thus E = $N_{mH}\,\mathrm{Ext}_{ij}$ (H $\setminus$ E) = H $\setminus mcl_i(N_m cl_j(H\setminus$ E)) = $N_m int_i(N_m int_j(E))$. Hence E is $N^i_{mH}N^j_{mH}$-open. □

Theorem 3.3. *Let (H, N^1_{mH}, N^2_{mH}) be a nbiss and E be a subset of H. If E is$N^i_{mH}N^j_{mH}$-closed, then $N_{mH}\,Ext_{ij}$ (H $\setminus$ $N_{mH}\,Ext_{ij}$ (E)) = $N_{mH}\,Ext_{ij}$ (E), where i, j = 1, 2 & i $\neq$ j.*

Proof. Assume that E is $N^i_{mH}N^j_{mH}$-closed. Thus $N_{mH}\,\mathrm{Ext}_{ij}$ (E) = H $\setminus$ E. Hence $N_{mH}\,\mathrm{Ext}_{ij}$ (H $\setminus$ $N_{mH}\,\mathrm{Ext}_{ij}$ (E)) = $N_{mH}\,\mathrm{Ext}_{ij}$ (H $\setminus$ (H $\setminus$ E)) = $N_{mH}\,\mathrm{Ext}_{ij}$ (E). □

Theorem 3.4. *Let (H, N^1_{mH}, N^2_{mH}) be a nbiss and E, F be subsets of H. Then for each i, j = 1, 2 & i $\neq$ j, we have;*

1. *$N_{mH}\,Ext_{ij}$ (E) $\cup$ $N_{mH}\,Ext_{ij}$ (F) $\subseteq$ $N_{mH}\,Ext_{ij}$ (E $\cap$ F).*
2. *If E and F are $N^i_{mH}N^j_{mH}$-closed, then $N_{mH}\,Ext_{ij}$ (E) $\cup$ $N_{mH}\,Ext_{ij}$ (F) = $N_{mH}\,Ext_{ij}$ (E $\cap$ F).*

Proof. Assume that (H, N^1_{mH}, N^2_{mH}) is a nbiss, E and F are subsets of H. (1). Since E $\cap$ F $\subseteq$ E and E $\cap$ F $\subseteq$ F, we have $N_{mH}\,\mathrm{Ext}_{ij}$ (E) $\subseteq$ $N_{mH}\,\mathrm{Ext}_{ij}$ (E $\cap$ F) and $N_{mH}\,\mathrm{Ext}_{ij}$ (F) $\subseteq$ $N_{mH}\,\mathrm{Ext}_{ij}$ (E $\cap$ F). It follows that $N_{mH}\,\mathrm{Ext}_{ij}$ (E) $\cup$ $N_{mH}\,\mathrm{Ext}_{ij}$ (F) $\subseteq$ $N_{mH}\,\mathrm{Ext}_{ij}$ (E $\cap$ F).
(2). Assume that E and F are $N^i_{mH}N^j_{mH}$-closed. Then E $\cap$ F is $N^i_{mH}N^j_{mH}$-closed. Thus $N_{mH}\,\mathrm{Ext}_{ij}$ (E $\cap$ F) = H $\setminus$ (E $\cap$ F) = (H $\setminus$ E) $\cup$ (H $\setminus$ F) = $N_{mH}\,\mathrm{Ext}_{ij}$ (E) $\cup$ $N_{mH}\,\mathrm{Ext}_{ij}$ (F). □

Theorem 3.5. *Let (H, N^1_{mH}, N^2_{mH}) be a nbiss and E, F be subsets of H. Then for each i, j = 1, 2 & i $\neq$ j, we have;*

1. *$N_{mH}\,Ext_{ij}$ (E ∪ F) ⊆ $N_{mH}\,Ext_{ij}$ (E) ∩ $N_{mH}\,Ext_{ij}$ (F).*

2. *If E and F are $N^i_{mH}N^j_{mH}$-open, then $N_{mH}\,Ext_{ij}$ (E ∪ F) = $N_{mH}\,Ext_{ij}$ (E) ∩ $N_{mH}\,Ext_{ij}$ (F).*

Proof. Assume that (H, N^1_{mH}, N^2_{mH}) is a nbiss, E and F are subsets of H. (1). Since E ⊆ E ∪ F and F ⊆ E ∪ F, we have $N_{mH}\,\mathrm{Ext}_{ij}$ (E ∪ F) ⊆ $N_{mH}\,\mathrm{Ext}_{ij}$ (E) and $N_{mH}\,\mathrm{Ext}_{ij}$ (E ∪ F) ⊆ $N_{mH}\,\mathrm{Ext}_{ij}$ (F). It follows that $N_{mH}\,\mathrm{Ext}_{ij}$ (E ∪ F) ⊆ $N_{mH}\,\mathrm{Ext}_{ij}$ (E) ∩ $N_{mH}\,\mathrm{Ext}_{ij}$ (F).

(2). Assume that E and F are $N^i_{mH}N^j_{mH}$-open. Then E ∪ F is $N^i_{mH}N^j_{mH}$-open. It follows that H \ E, H \ F and H \ (E ∪ F) are $N^i_{mH}N^j_{mH}$-closed. Thus by Theorem 3.4(2), we have $N_{mH}\,\mathrm{Ext}_{ij}$ (H \ E) ∪ $N_{mH}\,\mathrm{Ext}_{ij}$ (H \ F) = $N_{mH}\,\mathrm{Ext}_{ij}$ ((H \ E) ∩ (H \ F)) = $N_{mH}\,\mathrm{Ext}_{ij}$ (H \ (E ∪ F)) = E ∪ F. □

Conclusion

We presented several new notions and related properties by utilizing the concept of exterior set in nbiss.

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