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Usabilitiy of Chebyshev Pseudospectral method with finite element method on circular discs

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Abstract: In this study, the thermal stress behavior of a disk modeled under a linear increasing temperature distribution is investigated. The temperature limit conditions are determined as 30^{o}C, 40^{o}C, 50^{o}C, 60^{o}C and 70^{o}C. The stresses were determined analytically using a computer program developed in the study, using the Finite element method (FEM) ANSYS (2022-R1) program, and using the Psedudospectral Chebyshev method. With the Chebyshev method, high-precision solutions can be obtained using a small number of braids. The main purpose of this study is to investigate the thermal stresses occurring in circular discs with different methods. The results obtained were compared with each stress analysis, linear temperature distribution, psedudospectral chebyshev method other and presented with graphs.

Key words: Stress analysis, Composite materials, Temperature-related- thermal effects

1. Introduction

Temperature is one of the basic characters of materials. Changing the temperature can cause unwanted problems in the materials. Today, disks are used in aviation, unmanned aerial vehicles, aircraft and jet industries and high-tech productions produced against corrosion [1]. At least two materials with different chemical and physical properties from each other and together have created new materials are called composite materials. Composite materials have the characteristics of high strength and low density. That is why they are preferred in aviation structures with the greatest need [1]. Composite materials are often preferred today due to their high strength and applicable properties. Composite materials are one of the developing branches of materials science. Due to the fact that it has its own unique production method, it is also widely used in the aerospace and defense industry, automotive, marine, aviation, industry. Reinforcement in composite material is a very important factor in ensuring mechanical strength [2]. The pseudospectral Chebyshev method is a method that transforms the differential equation obtained according to one or more independent variables into a system of linear or nonlinear equations [3, 4]. There are a lot of studies in the literature on the behavior of thermal stresses that occur with disks. In a study conducted; functionally, graded circular cylinders and disks with displacement and stresses under uniform internal and external pressure were studied [5]. In different studies conducted; the stresses occurring in disks modeled for different temperatures were investigated by analytical and finite element method. The results obtained were compared with other studies in the literature. The results obtained were found to be compatible with each other [6–9]. Composite materials are used everywhere in machine parts, as well as in the literature on studies of their sensitivity to radiation. In a study conducted, the shielding properties of BaTi4O9 ceramics were investigated. In another study, the effect of Fe 2 O3 composite material on radiation

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and the effect of boron waste on the radiation shielding properties of cement were investigated in radiation behavior [10–13]. Another study; the temperature distributions in a hollow sphere have been investigated analytically depending on the Cattanotte-Vernotte approach [14]. Analytical solutions occurring in hyperbolic rotating disks subjected to different boundary conditions have been investigated [15, 16]. It has been revealed that studies using the Pseudospectral Chebyshev Method have yielded accurate results [17, 18]. In this study, it was assumed that the temperature increases linearly from the inner surface of the disk to the outer surface. Composite stresses; the analytical solution using the computer program was calculated using the ANSI 2022 R program, which is a finite element method, and the Pseudospectral Chebyshev method.

2. Material And Method

The conditions under which the temperature increases linearly from the inner surface of the disk to its outer surface are referenced. It is assumed that the temperature distribution of the disk subjected to thermal stress changes in the radial direction. The disk seen in Figure 1 is fixed and its dimensions are modeled as a=40 mm, b=80 mm. Different temperature values of 30^{o}C, 40^{o}C, 50^{o}C, 60^{o}C and 70^{o}C were taken as reference.

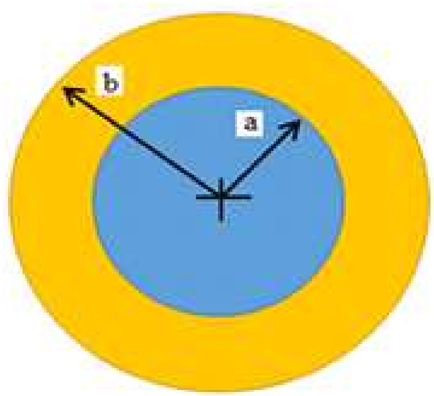

Figure 1. A composite disk modeled.

The mechanical properties of composite disk materials are given in Table 1.

Table 1. Mechanical properties of the disk material [19].

	$E_\theta(GPa)$	$E_r(GPa)$	k	$\alpha_r(1/^oC)$	$\alpha_\theta(1/^oC)$	$\upsilon_{\theta r}$
Composite disk	92	75	1.107	28.5×10^{-6}	16×10^{-6}	0.29

2.1. Equations and Formulas

α_r ve α_θ are the thermal expansion coefficients in the radial and tangential directions under conditions where the plane stress condition is valid for the composite thin disk. $a_{\theta\theta}$ a_{rr} $a_{r\theta}$ denotes the constants of the elasticity matrix. The expressions of the elasticity constants in terms of engineering constants are as follows [20]. Figure 2 shows the disk modeled under a linear increasing temperature distribution from the inside to the outside of the disk.

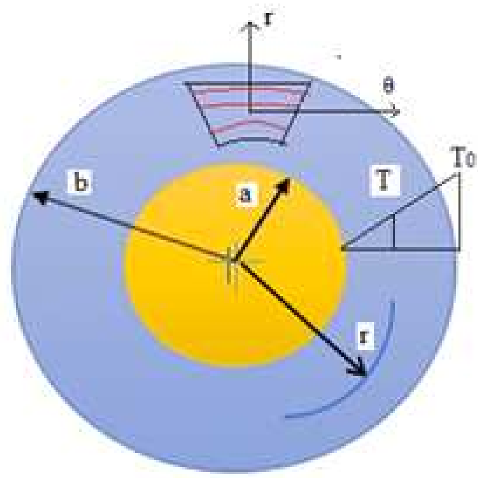

Figure 2. The linear temperature change occurring in the composite disk.

$\sigma_z = 0$ general equilibrium equation for a thin disk [20].

$$\frac{r\,(d\sigma_r)}{dr} + (\sigma_r) - (\sigma_\theta) = 0 \tag{1}$$

is given in the form. In equation (1), r is the radius of the disk at any point, σ_r is the radial stress, and σ_θ is the tangential stress. Here, the disc material is taken as i = 1.

$$\varepsilon_r = \frac{du}{dr} \tag{2}$$

$$\varepsilon_\theta = \frac{u}{r} \tag{3}$$

Here u is the displacement in the radial direction. ε_r denotes radial deformation, ε_θ denotes deformation in the tangential direction. Strain-stress relation;

$$\varepsilon_{ri} = \frac{1}{E}\left(\ \sigma_r - \upsilon\sigma_\theta\right) + \alpha_i T_r \tag{4}$$

$$\varepsilon_\theta = \frac{1}{E}\left(\ \sigma_\theta - \upsilon\sigma\right) + \alpha T_r \tag{5}$$

$$s_r = \frac{F}{r} \tag{6}$$

$$\sigma_\theta = \frac{dF}{dr} \tag{7}$$

It shaped. (6) and (7) are applied in equations (4) and (5);

$$\varepsilon_r = \frac{1}{E}\left(\ \frac{F}{r} - \upsilon\frac{dF}{dr}\right) + \alpha T_r \tag{8}$$

$$\varepsilon_{\theta i} = \frac{1}{E}\left(\ \frac{dF}{dr} - \upsilon\frac{F}{r}\right) + \alpha T_r \tag{9}$$

obtained. Eligibility equation for elongation;

$$r\frac{d\ \varepsilon}{dr}+\varepsilon_\theta-\varepsilon_r= 0 \tag{10}$$

as obtained. The general equation (11) is obtained by using the equilibrium equation (1-7) in which the stress function can be defined as F.

$$r^2\frac{d^2F}{dr^2}+r\frac{dF}{dr}-F = -r^2\alpha_i E_i T_r^{'} \tag{11}$$

From equation (11);

$$r^2F^{''}+rF^{'}-k^2F =\frac{(\alpha_r-\alpha_\theta)\,T}{a_{\theta\theta}}r-\frac{a_{\theta\theta}T^{'}}{a_{\theta\theta}}r^2 \tag{12}$$

Substituting T and $T^{'}$ yields Equation (13);

$$r^2F^{''}+rF^{'}-k^2F =\frac{(\alpha_r-\alpha_\theta)\,(r^2-ar)}{a_{\theta\theta}(b-a)}T_O-\frac{a_{\theta\theta}T_O}{a_{\theta\theta}(b-a)}r^2 \tag{13}$$

In order to obtain Chebyshev points that have more mesh points at the boundary points, the Chebyshev differential matrix D can be calculated using equation (14). In this way, high precision can be achieved: by multiplying the vector by a finite number of times in this method, the derivatives of the vector are obtained with high precision.

$$r_j = \cos\{j\pi|N\}, j = 0, 1, 2, 3 \ldots N \tag{14}$$

$$F'\,(r_j) = (DF)_J \tag{15}$$

$$F\text{"}(r_j) = (D^2F)_J \tag{16}$$

$$F = [F_0 \ldots F_n]^T,\ r_j \tag{17}$$

rj is numbered from right to left and can be defined in the december [-1,1]. The calculation of the derivative matrix can be done from the Matlab m file [21]. When using first- and second-order derivatives, the Chebyshev differential matrix, in equation (13);

$$\left\lceil\frac{dF}{dr}\,(r_n)\right\rceil\equiv D\,\lceil F\,(r_n)\rceil \tag{18}$$

$$\left\lceil\frac{dF^2}{dr}\,(r_n)\right\rceil\equiv D^2\,\lceil F\,(r_n)\rceil \tag{19}$$

If discretization is performed, such as equations (18) and (19); the differential equation modeling the system is given below. Where L_1 is the linear coefficient, and RHSF is the right-hand side equation.

$$L_1F = RHSF \tag{20}$$

When the boundary conditions $\sigma_r(a) = 0$ and $\sigma_r(b) = 0$ are applied in Equation 20, the non-obvious solution of RHSF is given below.

$$L_1 = r^2D^2 + rD - k^2r_j \tag{21}$$

$$RHSF = \frac{(\alpha_r - \alpha_\theta)(r^2 - ar)}{a_{\theta\theta}(b-a)}T_O - \frac{a_{\theta\theta}T_O}{a_{\theta\theta}(b-a)}r^2 \tag{22}$$

3. Discussions

In this study, the error margin of the Chebyshev method was investigated by determining the thermal stresses occurring in a composite disk using a computer program, the Finite Element method using the ANSYS program, and the Chebyshev method using the Chebyshev method. The ANSYS 2022 R1 program was used.

Table 2. Tangential and radial stresses occurring in the disk

Temperature ΔT(°C)	Surface	Analytical Solution		Finite Element Method		Pseudospectral Chebyshev Methods	
		σt (MPa)	σr (MPa)	σt (MPa)	σr (MPa)	σt (MPa)	σr (MPa)
30 °C	Inner	20.86	0	20.76	0	20.85	0
	Outer	-43.05	0	-42.93	0	-43.05	0
40°C	Inner	27.81	0	27.69	0	27.80	0
	Outer	-57.40	0	-57.26	0	-57.42	0
50°C	Inner	34.76	0	34.61	0	34.74	0
	Outer	-71.75	0	-71.57	0	-71.75	0
60°C	Inner	41.72	0	41.52	0	41.69	0
	Outer	-86.11	0	-85.89	0	-86.12	0
70°C	Inner	48.67	0	48.45	0	48.65	0
	Outer	-100.46	0	-100.17	0	-100.46	0

The solutions by finite element method (Ansys 2021-R1) are given in Figure 2 and Figure 3.

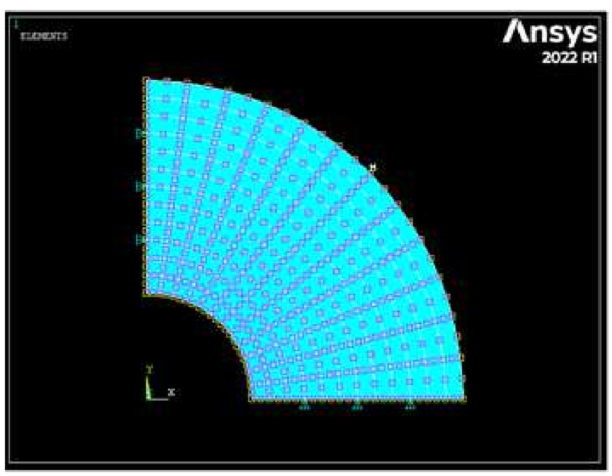

Figure 3. A composite disk modeled by the finite element method.

In Figure 4 the tangential determined on the disk obtained by the Finite Element method is given. Figure 5 shows the Radial stresses calculated by the Finite element method.

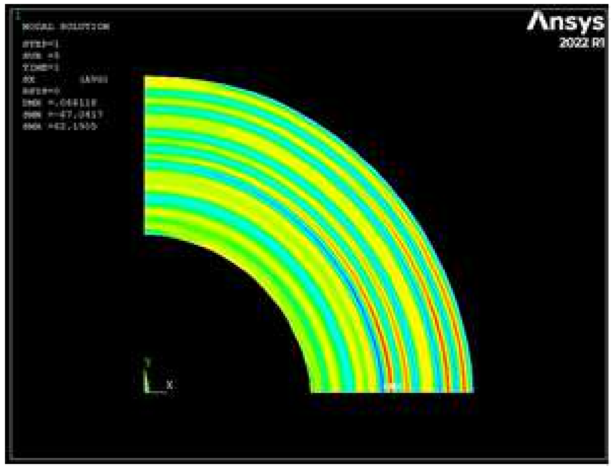

Figure 4. Distribution of tangential stresses by the finite element method.

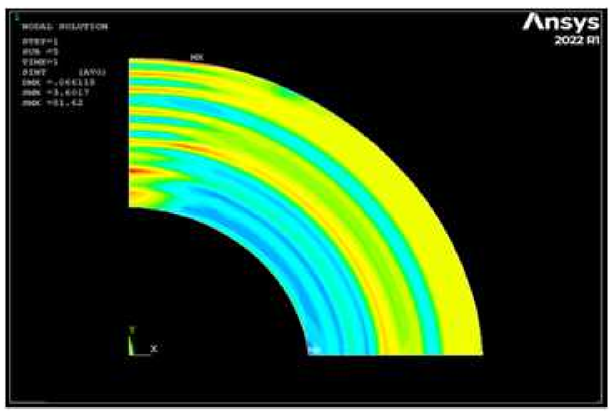

Figure 5. Distribution of radial stresses (SEY).

Figure 6 shows the radial stresses obtained as a result of different methods.

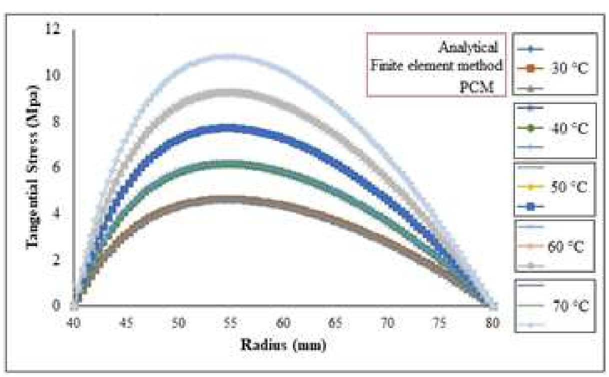

Figure 6. Radial stresses occurring in the disk.

Figure 7 shows the tangential stresses.

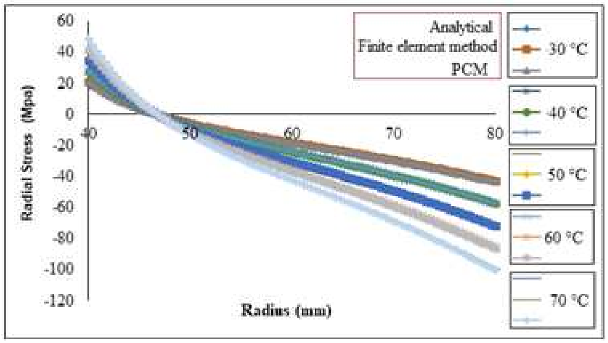

Figure 7. Tangential stresses occurring in the disk.

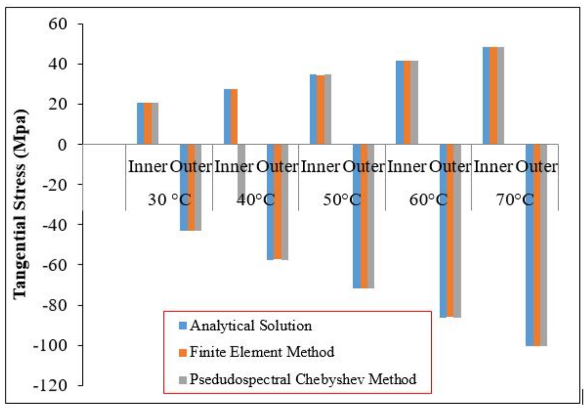

Figure 8. Stresses that occur in a certain area of the disk.

4. Conclusion

- As a result of this study, the following results were obtained:
- It has been observed that there is an increase in the stress values with increasing temperature in the stress values determined for three different methods
- It has been determined that the tangential stresses formed in the disk in three methods are of greater magnitude than the radial stresses.
- Radial stresses are formed in the form of tensile stress.
- Tangential stresses are in the form of tensile stresses from the innermost part of the disk to the middle region. It occurred in the form of a pressing stress from the middle part of the disk to the outer region.
- It has been determined that the number of nodes determined when using the finite element method (ANSYS-2022) is greater than the number of nodes used in the Chebsyhev method.
- It is concluded that the results obtained using the Chebyseh method are closer to the analytical result than the results of the finite element method, and the Chebsyhev method can give faster and more accurate results in solving thermal stresses

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