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Discussion on whether the notion of complex numbers do exist

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Abstract: It is known that complex numbers, complex geometry, complex algebra etc. have become common mathematical tools for many scientists, physicists, and mathematicians for long time. Nonetheless, among serious mathematicians, still there are people who question if such entities do exist in nature, or are they only in the mind? For these authors, we admit that complex numbers are convenient tools to work out physics problems, but we shall be cautious to come up with complicated new mathematical structures, even if we hear such "generalization" method can be useful for finding the new physics (cf. Dirac). On the contrary, it is often the case that going simpler and simpler will lead you to new physics; for instance, instead of come up with monster group etc., perhaps one can begin to consider low dimensional geometry (cf. William Thurston, Three-dimensional geometry and topology, Princeton Univ. Press, NJ, 1997). Perhaps, the real space of crystalline corresponding to complex 3D geometry can be found useful in the near future.

Key words: complex numbers, generalization method, Dirac's advise, findings new physics through new mathematics

1. Introduction

To speak frankly, these authors wrote a number of papers in the past based on the notion of quaternion algebras and quaternion numbers. A senior physics professor once asked one of authors (VC): "Do you believe in such a universe of quaternion numbers?" At the time, we didn't think that much, it seems like the simplest approach to find 6D-version of Maxwell equations. At the time, we believe much in the following statement by J. Hadamard, somethihg like that: "a shortest way to reach infinity is through complex plane." But, more recently, there are a number of people who raise these issues, whether quaternion numbers do exist? There is also another quote: "Quaternions came from Hamilton after his really good work had been done; and, though beautifully ingenious, have been an unmixed evil to those who have touched them in any way, including Clerk Maxwell." - William Thomson, first baron Kelvin, 1892.

We also notice that there is recent paper by our former professor, who argues in favor of biquaternion algebra in relation to twistor etc. According to him, which can be paraphrased as follows: "The polynomial math of complicated quaternions, biquaternions, B is the most fitting contender for the job of a space-time variable based math (the idea presented first and foremost by D. Hestenes."[1] With all our respect, allow us to put these algebraic structures into a deeper look: whether such complex numbers do exist, let alone their extensions such as quaternion, biquaternion, Clifford algebra etc.? Nonetheless, there is also possibility that the 6D geometry, or more exactly (3D, symmetric) can be found in nature, especially in quasi-crystalline structure.

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Therefore all we can say is that may be this is still useful, although these 6D geometry may be seen more like a mathematical artefact. We will discuss shortly in subsequent section, an alternative to complex algebra and Clifford algebra, etc.

2. Short history of complex numbers

As Dempsey summarizes, which can be paraphrased as follows: “Complex numbers, albeit befuddling on occasion, are one of the most fascinating subjects with regards to math to have surfaced over the most recent five centuries. Despite the fact that it took time for them to get on as authentic numerical apparatuses, they have substantiated themselves once more what’s more again to be valuable in a wide assortment of math and designing points.”[6] According to Peters, which can be paraphrased as follows: “The most punctual hints of nonexistent numbers end up in Italy, settled inside a cubic condition. During the mid 1500s, the splitting line between college math also casual science contracted, and there was a quick improvement in variable based math. In the mid 1530s, numerical virtuoso Nicolo Tartaglia showed up at the scene ... and then Cardano etc.”[5] See also Dempsey [6].

3. Problem of Dirac’s recipe toward new physics

As Anderson & Joshi wrote, which can be para-phrased as follows: “One of the significant ways improvement happens in math is through a course of generalization. A portrayal given by Kitcher of one of the significant ways math advances, which he distinguishes as one of ”generalization.” This refines an idea communicated by Dirac in 1931 on the manner by which certain progressions in science can play a huge heuristic occupation in material science.”[7] See also how Maxwell and Heavyside worked out their way. [8, 9] Actually, Dirac advises as follows: “. . . .that a “powerful new method” for the physicist comprises of picking a branch of arithmetic and afterward continuing ”to foster it along appropriate lines, simultaneously searching for that manner by which it seems to loan itself normally to actual translation.”[10]

While initially such an advise sounds clear and worth to follow, but from the last few decades, there is a quite unhealthy trend, a kind of obsession to find new and the largest group ever, and then physicists try to find if there is signature of Nature’s approval of their wild adventure. Such a gloomy situation has been reported in Hossenfelder’s Lost in Math, which attracts responses from various luminaries such as Wilczek etc. As far as we can consider, these situations are caused because physicists tend to be absorbed more on mathematical structures, symmetry, beauty -so to speak. While they often forget to ask Nature what it actually says - through experiments. Such a simple problem. Even, there are rumours that Michelson-Morley experiment was designed and ordered as such to prove that “ether” the all-filling-primary fluid does not exist. Therefore, many more precise experiments which came later, such as Miller etc., are systematically discarded. They say: ether is not required – by definition, they would prefer “mathematical beauty” over reality itself (these are the attitude of many mathematicians and physicists alike, even if some of them do not agree with special relativity). Such and such is the case, until we found the arrogance of string theorists, who insist that string and superstring etc. should be the only game in town. Thanks to Peter Woit etc., we know that supertring theories are far from being the correct theory we sought for.[17] Part of the problem, as we can think, is that most physicists forget the latter part of Dirac’s advise above : “at the same time looking for that way in which it appears to lend itself naturally to physical interpretation.” [10] Therefore, what Dirac actually wrote is to find a balance between mathematical structures but we shall keep our feet on the grounds. See our paper in Journal J, where we argue that it is actually Kolmogorov’s ”theorem of contradiction” that shows the possibility of complicated mathematical theories to end up with so many paradoxes and problems, and it also was proved later on by Godel (1931). Therefore allow us to argue a few guides, including a simple one-to-one correspondence between mathematical variables and physical observables, as well as keeping our postulates to a minimum. [12] In a more general parlance, provided we can accept that actually all of us are crazy, especially we mathematical physicists in general, only with varying degree of madness; then what we argue is to keep principle of parsimony. This may be called, in a pun, as "Principle of minimal madness."

4. Remark by a senior mathematics professor

AK: "It is clear that the complex number was introduced to make quadratic equations always have solutions. Considering the nature of real number, only a fool will expect that the simplest such a case a quadratic of x=-1 will have a solution. The proposed solution i and -i is a joke. It has no meaning in the world of quadratic equations where everything is real numbers. Using complex plane and geometry, if we try to represent the situation of solving quadratic equations geometrically, we get nowhere. It is because the complex plane is not Euclidean plane. I am more and more convinced that the so called algebraists disserved mathematics. As far as I can see, the concept of algebraic equations is the source of all problems we have. Solving equations is an engineering problem which has little to do with mathematics. For me mathematics is to study structure and equations have little to do with structures. It is a mindless game which was popular among the mindless "merchant mathematicians" in the renaissance period. What do business people know about mathematics. There are more important things to do in life. Did Euler know that Aristotle already knew that a point is not a part of the plane? The problem is that people like Euler was too busy counting the money."

5. Discussion: An alternative route beyond complex algebra, Clifford algebra, etc.

In the aforementioned section, we argue that complex geometry, complex algebra Clifford algebra, including Hestenes's spacetime algebra etc., are not quite tenable. At this point, some readers may ask: "So, is there alternative route?" In this context, one of us (FS) presented new theories in recent papers, i.e. from Classical Algebraic Structures to NeutroAlgebraic Structures and AntiAlgebraic Structures. Here is a summary: In 2019 Smarandache [18] generalized the classical Algebraic Structures [that are too abstract for our real world] to NeutroAlgebraic Structures (or NeutroAlgebras) whose activities and maxims are to some degree valid, somewhat uncertain, and to some extent false as expansions of Partial Algebra, and to AntiAlgebraic Structures (or AntiAlgebras) whose tasks and sayings are absolutely false and on 2020 he kept on creating them [19, 20]. The NeutroAlgebras and AntiAlgebras are one more field of assessment, which is impelled from our real world. In old style arithmetical designs, all activities are 100 percent obvious, and all adages are 100 percent valid, yet, all things considered, by and large these limitations are excessively cruel, since in our reality we have things that. Definitions of Operation, NeutroOperation, AntiOperation: i.e. Whenever we characterize a procedure on a given set, it doesn't naturally imply that the activity is obvious. There are three prospects: 1) The activity is distinct (additionally called inward characterized) for every one of set's components [degree of truth T = 1] (as in old style arithmetical designs; this is a traditional Operation). Neutrosophically we compose: Operation(1,0,0). 2) The activity if distinct for certain components [degree of truth T], uncertain for different components [degree of indeterminacy I], and outer-defined for the other elements [degree of falsehood F], where (T,I,F) is different from (1,0,0) and from (0,0,1) (this is a NeutroOperation). Neutrosophically we write: NeutroOperation(T,I,F). 3) The operation is outer-defined for all set's elements [degree of falsehood F = 1] (this is an AntiOperation). Neutrosophically we write: AntiOperation(0,0,1). Meaning of Axiom, NeutroAxiom, AntiAxiom: for example

Comparably for an adage, characterized on a given set, blessed with some operation(s). At the point when we characterize an aphorism on a given set, it doesn't consequently imply that the maxim is valid for every one of set's components. We have three prospects once more: 1) The saying is valid for every one of set's components (absolutely obvious) [degree of truth T = 1] (as in traditional arithmetical designs; this is an old style Axiom). Neutrosophically we compose: Axiom(1,0,0). 2) The aphorism if valid for certain components [degree of truth T], uncertain for different components [degree of indeterminacy I], and misleading for different components [degree of AntiAxiom(0,0,1). He established the NeutroAlgebras in light of the fact that the regulations don't similarly apply to all residents, so they are NeutroLaws. A few regulations apply somewhat to a classification of residents, and to an alternate degree to another classification. Quite often there are exemptions for the law! In that capacity, there is an American folkloric joke: All individuals are conceived equivalent, yet certain individuals are more equivalent than others. A couple of models: - There are influential individuals that are over the regulations, and others that advantage of insusceptibility regarding the regulations. - For instance, in the official courtroom, special individuals benefit from preferred protection legal counselors over the lower classes, so they might get a lighter sentence. - The Double Standard are generally spread: some guideline applies to certain individuals, however not to other people. To summarize: The point of NeutroAlgebra is that this is realistic (i.e. in our every day life the laws (axioms, regulations) do NOT apply in the same degree to all elements (people) but in different degrees. So, the classical algebraic structures are idealistic (not real) since the axioms (regulations) apply to all elements in the same degree to each element.

6. Almost a conclusion

As we are not yet specialists in studying this philosophy of complex numbers and complex geometry. . . let us just make a remark that this problem shall be discussed more openly among mathematicians and also physicists. These problems shall include whether the notion of Minkowski metric do exist in nature, or is it just fantasy? We mean:

$$ds^2 = dx^2 + dy^2 + dz^2 + (ic.dt)^2 \tag{1}$$

From philosophic perspective, it can be shown that there is no way that time can be merged with space, and also including imaginary number only makes it worse. According to Kenneth Taylor, which can be paraphrased as follows: "Obviously, relativity is frequently and properly said to dismiss both outright space and outright time. In any case, dismissing outright reality actually doesn't exactly get us to the space-time continuum. To get to that objective, we want to discuss light. Light is, obviously, the quickest thing known to mankind." So, the essence of departure from Newtonian space concept, is that velocity of light is assumed to be the largest velocity in nature, then on the ground, Minkowskian metric obtains its justification. Moreover, physical properties of space were denied, and in turn, they tried to introduce certain properties into the notion of spacetime. But if we think rather deeply, we will realize that there is no physical property whatsoever that can be assigned to spacetime metric. With all our respect to those theoretical physics luminaries who coined term such as "geometro-dynamics," let us argue as follows: Are we sure that geometro-dynamics is a valid term? Let us, for the sake of argument, draw a triangle. It is a geometric shape, right? Now, watch and observe the triangle for let say, 4 days, will it move an inch? Definitely not, because a triangle is a drawing, not a physical object. Therefore, we can conclude: "The notion of geometro-dynamics is merely contradictio in terminus." Therefore, it is understandable that the late John Wheeler himself, who coined term geometro-dynamics, later on abandoned many features of that approach. See J. Stachel's article: The rise and fall of Geometrodynamics, and also W. Misner. [13–15] We can recall too, that there is senior professor in Germany who was brought to justice a few years ago, and then he admitted that there is no way to measure "spacetime curvature." This case file can be found in internet, related to LIGO project. Actually, there are several criticisms on that observation project [16]. Of course, by mentioning all of these, it does not mean that we are right all along, but let us face the truth as it is. Physics is more related to solid experimental evidences, not just made of tower of sand. As we wrote in introduction, with all our respect to all our former professors (at IGC, and also around many countries) who are so kind to teach many things (especially to one of us, VC), allow us to put these algebraic structures into a deeper look: whether such complex numbers do exist, let alone their extensions such as quaternion, biquaternion, Clifford algebra etc. Nonetheless, there is also possibility that the 6D geometry, or more exactly (3D, symmetric) can be found in nature, especially in quasicrystalline structure. Therefore all we can say is that may be this is still useful, although these 6D geometry may be seen more like a mathematical artefact. Zlabys et al wrote their abstract as follows: "Here we show that time and space crystalline structures can be combined together and even six-dimensional time-space lattices can be realized. As an example, we demonstrate that such time-space crystalline structures can reveal the six-dimensional quantum Hall effect quantified by the third Chern number."[3] All in all, if there is any of you who can measure precisely "spacetime curvature", you can contact us by email: victorchristianto@gmail.com and smarand@unm.edu.

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