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# Continuum Hypothesis Revisited: From Cantor, to Godel, then to Discrete Cellular Space model

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**Abstract:** This article is a follow-up to our previous article (cf. Octogon Mathematical Magazine, 2018). As we know, Continuum Hypothesis is one unresolved problem in mathematics, and it is likely to affect physics theories too, once a reasonable solution has been achieved. The Continuum Hypothesis can be stated as follows: if you are given a line with an infinite set of points marked out on it, then just two things can happen: either the set is countable, or it has as many elements as the whole line. There is no third infinity between the two (cf. Juliette Kennedy's article, IAS, 2011, https://www.ias.edu/ideas/2011/kennedy-continuum-hypothesis). Here we review CH from Godel view etc, then we also review Smarandache's partially denying of axioms, and then we will review our proposed Discrete Cellular Space.

**Key words:** Continuum Hypothesis, Discrete Cellular Space, Godel's view on Continuum Hypothesis, quasicrystalline

## 1. Introduction

The word 'geometry' comes from the Greek words 'geo', which means the 'earth', and 'metrein', signifying 'to gauge'. Math seems to have started from issues that emerged for estimating land. This part of math was contemplated in different structures in each old civilisation, be it in Egypt, Babylonia, China, India, Greece, the Incas, and so forth. Individuals of those ancient civilisations dealt with a few useful issues which required the improvement of math in different ways. Euclidean Geometry is the investigation of Geometry dependent on definitions, vague terms (such as points, line and plane) and the presumptions of the mathematician Euclid (330 BC). As historians of mathematics told us, it was in 1900 at the Paris conference that Hilbert presented his list of unsolved mathematical problems; as number one on that list, which was entitled Mathematische Probleme, stands the continuum problem, already conjectured by Cantor.[1] The famous Poincaré's remark to be mentioned in Russell's classic book, The Principles of Mathematics ( Russell 1937, p.347), and it can be paraphrased as follows: "The continuum consequently considered is only an assortment of people organized in a specific request, endless in number, it is valid, yet outside to one another. This isn't the customary idea, where there should be, between the components of the continuum, a kind of personal bond which makes an entire of them, in which the point isn't preceding the line, yet the line direct."[3] In Godel's article, he once wrote, that "Cantor's continuum problem is simply the question: How many points are there in a straight line in Euclidean space?"[2] According to Koellner (2011): "The Continuum Hypothesis is one of the most central open problems in set theory, one that is significant for both numerical and philosophical reasons. The issue

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really emerged with the introduction of set hypothesis; for sure, in many regards it animated the introduction of set hypothesis. In 1874 Cantor had shown that there is a balanced correspondence between the regular numbers and the mathematical numbers. All the more shockingly, he showed that there is nobody to-one correspondence between the regular numbers and the genuine numbers. Taking the presence of a coordinated correspondence as a measure for when two sets have a similar size (something he unquestionably did by 1878), this outcome shows that there is more than one degree of limitlessness and subsequently brought forth the higher endless in science."[6] As we see that this is unsolved problem in mathematics, and even according to Hamkins (2015), a dream solution to this problem is unattainable. See also Cohen (1953), Feferman (2011). Why does this problem matter to physics sciences? Because as we know, theoretical physics have become so abstract but yet many more problems are unresolved, as Sabine Hossenfelder wrote in her book, Lost in Math (2019). As an education expert puts it: "it is such that we are as confused as ever, only at a much higher level." Therefore one of things needed to review, is to check if there is problem in the very corner stone of geometry itself, i.e. Euclid axioms. In the following section, let us discuss a possible path to find out if this CH problem is still within our reach. First of all, let us review Euclid's five axioms and other related definitions.

## 2. Euclid's five axioms and several definitions

The followings are the axioms of standard Euclidean Geometry. They show up toward the beginning of Book I of The Elements by Euclid. Propose 1: A straight line fragment can be drawn joining any two focuses. Hypothesize 2: Any straight line fragment can be stretched out endlessly to shape a straight line. Propose 3: Given any line portion, a circle can be drawn involving the fragment as the range with one endpoint as the middle. Hypothesize 4: All right points are consistent. Propose 5: If a straight line falling on two straight lines make the inside points on a similar side under two right points, the two straight lines, whenever delivered endlessly, meet on that side on which are the points not exactly the two right points. Then, let us audit a portion of Euclid's Book 1 Definitions. (1). A point is what has no part. This can be perceived to imply that a point is something that can't be partitioned into anything more modest. (2). A line is breadthless length. A line is a develop that has no thickness. It very well may be considered as a constant progression of focuses. (3). A straight line is a line which lies equally with the focuses on itself. (4). A plane point is the tendency to each other of two lines in a plane which meet each other and don't lie in an orderly fashion.

## 3. Motivation of this study

Quite some time ago, these writers began a small book dedicated to discretization and quantization in astrophysics. It becomes clear, that discretization of space requires deeper understanding. Later on, we put forth some ideas which appeared later in Octogon Mathematical Magazine (2018). Around a year later, we got involved in another book project, which several contributing physicist fellows. Among some of us, and also with other contributors, discussion arises on several unsolved problems in mathematics, including continuum problem, and whether theoretical physicists have concern on that issue [44]. It is clear, that once this problem in underpinning of mathematics has been solved, then the implications will be profound to many diverse area of physics fields. One hint to find the solution to this Continuum Hypothesis problem is: The basic idea is NOT to get number correct, BUT to get the ideas correct. Mathematics is not solving equations! It is understanding.

## 4. Previous efforts to solve Continuum Hypothesis

In this section, let us review five existing attempts to solve CH problem. For an introduction, see for example [47], Let us discuss one by one, as follows: (a) Cantor's set theory. In 1874, Georg Cantor, then, at that point, a youthful teacher at Halle University, distributed a four-page note in Crelle's Journal, showing that the arrangement of mathematical numbers is countable, and the arrangement of genuine numbers uncountable. Cantor, in his attempt to build Cardinal chain reached the first continuum Cardinal. The first continuum Cardinal was at the limit of the chain of finite Cardinals. As he kept going up constructing Cardinal chain, he reached the largest Cardinal C. Then a shock wave hit him. He discovered that C+1 is a Cardinal such that:

$$C \leq C + 1. \tag{1}$$

When Cantor announced this shocking result which proved that his set theory is inconsistent, Russell who badly needed Cantor's set theory for his Principia Mathematica was in shock. His life time project was demolished. In his desperate attempt to save Cantor's set theory, Russell ended up with a simplest proof for the inconsistency of Cantor's set theory, aka Russell's paradox. As a desperate patch up solution to this shocking problem, ZF and von Neumann developed an axiomatic set theory. Though we still have not found an inconsistency proof for this new set theory, and most of us have already given up on this, all attempts to show that this formal set theory is consistent failed. To make the matter even more discouraging, Paul Cohen showed in his PhD, that the Continuum hypothesis which is the formal version of Cantor's first continuum set is independent upon the ZF. Disappointed by his own discovery, Cohen left set theory and moved to mathematical analysis. Under this discouraging situation, the only bright light was the result which showed that ZF minus (what ever) one axiom is always consistent. Practically minded mathematicians say this is enough. Conclusion: Cantor set theory is inconsistent. Cantor himself proved it. He was the first. Then Russel presented the simple most proof which goes as follows: Let R=x:x not in x. If R in R, then R not in R. If R not in R then R in R. Cantor's original proof used an infinite chain of cardinals. Russell's proof is simple most. But they say the same thing: Set theory of Cantor is inconsistent. So, on mathematics side, things are not in good shape and we are seriously concerned about the implication of this to theoretical physics. See also Rede [41]. (b) Dedekind cut theorem. From 1872 onwards, Cantor compared with Richard Dedekind (1831- 1916), who was 14 years his senior and had quite recently advanced the meaning of Dedekind cuts of genuine numbers. Before that, Dedekind proposed an answer, called Dedekind cut. A Dedekind cut is a fragment of the plan of normal numbers into two subsets An and B, so much that any part of An isn't precisely any part of B; Dedekind displayed that the plan of cuts acts unequivocally as one would guess that the arrangement of genuine numbers ought to act, with the cut (A,B) tending to the novel veritable number between A moreover B. Dedekind slice would in this manner have the option to be used to foster authentic numbers.[37] Dedekind looked to draw motivation from specific properties of the line, when we mean to place in correspondence and arrange the arrangement of genuine numbers on it. The property of coherence of the straight line introduces itself as a mathematical, perceptual and subjective person that Dedekind tried to foster a conventional treatment.[40] Crosby concludes Dedekind method with remark, which can be paraphrased as follows: "On Richard Dedekind's independent endeavor to exhibit the progression of the genuine numbers utilizing just arithmetical thinking. It is worth taking note of that his meaning of progression didn't go unchallenged by other numerical personalities of his time. The most noticeable elective perspective cases that a continuum can't be compositional in nature. That is, as Dedekind's genuine numbers are made out of discrete components, they can't be persistent. Paul du Bois-Reymond, a German mathematician who was alive at the point when "Progression and Irrational Numbers" was distributed, called the decrease of a continuum to discrete components "a program whose philosophical cogency, and surprisingly legitimate consistency, had been tested many occasions throughout the long term" [39]. Such a remark can be viewed as early indication that the solution of CH problem shall be found in redefining what is "discreteness." (c) Godel's restatement of the problem. Godel gave a restatement of CH problem, which can be paraphrased as follows: "This shortage of results, even with regards to the most key inquiries in this field, might be because of some degree to simply numerical troubles. ... This negative mentality towards Cantor's set hypothesis, nonetheless, is in no way, shape or form a fundamental result of a nearer assessment of its establishments, yet at the same just the after effect of specific philosophical originations of the idea of math, which concede numerical items just to the degree wherein they are interpretable as acts and developments of our own brain."[38] (d) Woodin's recent result. A review of Woodin's result has been given by Dehornoy, including his conjecture: "Conjecture 1 (Woodin, 1999). Every set theory that is compatible with the existence of large cardinals and makes the properties of sets with hereditary cardinality ....under forcing implies that the Continuum Hypothesis be false."[42] (e) Lakoff & Nunez's cognitive function approach. In our interpretation, Lakoff & Nunez's approach is the closest to Godel's remark: "wherein they are (or alternately are accepted to be) interpretable as acts and developments of our own brain." Therefore, let us see where the problem began. Firstly, let us quote from Robinson, which can be paraphrased as follows: "Concerning the establishment of science, my position (assessment) depends on the accompanying two fundamental standards: (1) No matter what an importance of words is utilized, endless sets don't exist. (They do not exist by and by or in principle). All the more explicitly, any assertion on endless sets is just inane. (2) However, we should in any case direct numerical works and exercises not surprisingly. In other words, when we work, we should in any case regard endless sets as though they really existed."[43] Such a conclusion shall be startling, indicating that the very notion of infinite sets etc. do not exist at all, except in the human mind. Lakoff & Nunez make it more clear in preface in their book, which can be paraphrased as follows: "How would we see such fundamental ideas as endlessness, zero, lines, focuses, and sets utilizing our ordinary theoretical device? How are we to figure out numerical thoughts that, to the fledgling, are paradoxical—thoughts like space-filling bends, little numbers, the point at in-limit, and non-very much established sets (i.e., sets that "contain themselves" as individuals)?" [23]. It becomes more clear that such an alteration from daily experience with "line segment" began with Descartes. Lakoff & Nunez wrote in chapter 12 in their book: "Euclid characterized a surface as "that which has length and broadness just," a line as "breadthless length," and a point as "that which has no part." Euclid utilized the customary idea of a come up short on: A surface needs thickness, a line needs expansiveness also thickness, and a point (which is comprised of no skillet) comes up short on these." [23]p. 265]. Moreover, they conclude: "Space has been conceptualized in two altogether different ways in the historical backdrop of math. Before the mid-nineteenth century, space was conceptualized as the vast majority ordinarily consider it-in particular, as normally consistent. Here is how we as a whole contemplate space in daily existence. ... Descartes' creation of logical calculation changed science for eternity. His focal similitude, Numbers Are Points on a Line (see Case Study 1), permitted one to conceptualize number juggling and polynomial math in mathematical terms and to envision capacities and logarithmic conditions in spatial terms. The reasonable mix of the source and target areas of this illustration allows us to move to and fro conceptually among numbers and focuses on a line." [23] [p. 260] So we know, why for most mathematicians, they assign real number line to define finite line segment, which actually do not exist in reality. The aforementioned discussions can be found helpful in order to see where we get lost.

## 5. On theorem of partially denying of axioms and known attempts to solve Continuum Hypothesis

Quite some time ago, Smarandache when he was young, introduced partial denial of axiom, especially considering Euclid's fifth axiom. Smarandache (b. 1954) partially negated Euclid's V postulate: "There exist straight lines and exterior points to them such that from those exterior points one can construct to the given straight lines: 1. only one parallel – in a certain zone of the geometric space [therefore, here functions the Euclidean geometry]; 2. more parallels, but in a finite number – in another space zone; 3. an infinite number of parallels, but numerable. [7] We can make the following remark on partial denying of axioms, which can be paraphrased as follows: "While the Non-Euclidean Geometries came about because of the complete invalidation of only one explicit maxim (Euclid's Fifth Postulate), the AntiGeometry results from the all out nullification of any adage and even of additional sayings from any mathematical proverbial framework (Euclid's, Hilbert's, and so on), and the NeutroAxiom results from the halfway refutation of at least one aphorisms [and no all out nullification of no axiom] from any mathematical aphoristic framework."[46] Now, we can say that among existing attempts to solve Continuum Hypothesis include: Dedekind cuts, algorithm approach etc., of which none has achieved to solve CH problem. In our view, the deep root cause why this problem has not been solved until today is: to remind you the aforementioned remark by Godel, i.e. the simplest formulation of the problem can be restated as follows: "How many points are there in a straight line in Euclidean space?" That is, according to principle of parsimony, we don't have to complicate the arguments beyond what is necessary, such as what Cantor did (while surely we appreciate his inventive transfinite sets etc.). In other words, we shall consider continuum hypothesis as it is from a more realistic perspective, not to do with real numbers or infinite numbers. To assert a finite length of line segment with real numbers only lead us to complicated arguments.

## 6. Discrete Cellular Space and its implications

In this section, allow us to review in a more accessible way, our arguments as we presented in Octogon Mathematical Magazine (2018). From previous section, we can recall that Definition of a point is as follows: (1). A point is that which has no part. In other words, that can cause serious contradiction. Let us start that we assume that a line segment is composed of infinite number of points, but a point is defined as "circle with zero diameter." By definition, a circle is the arrangement of all places in a plane that are equidistant from a given point called the focal point of the circle. We utilize the image to address a circle. The line section from the focal point of the circle to any point on the circle is a sweep of the circle. Furthermore, by meaning of a circle, all radii have a similar length. We likewise utilize the term span to mean the length of a sweep of the circle. That would imply:

$$\text{"}o + o + o + \text{... ad infinitum = finite length of line segment."} \tag{2}$$

Of course, that is confusing and contradictory, because by definition, a series of infinite number of zeroes never become a finite length. In our perspective, that is nothing to do with real numbers, but it is required to revisit our definition of what a point is. A more realistic definition can be given as follows: a point is defined as "a circle with small but non-zero diameter cell, let say we call it, z." From that starting point, we can arrive to a more palatable argument,i.e.:

$$\text{"}z + z + z + \text{... ad infinitum = finite length of line segment."} \tag{3}$$

Therefore we arrive to a plausible solution to continuum hypothesis, that space is composed of Discrete Cells; that is why we call it Discrete Cellular Space hypothesis. While for some mathematician readers, that proposed solution may be found too pragmatics, we suppose that for many physics sciences, astrophysics etc., that solution shall be sufficient "for all practical purposes" – provided we are allowed to use that popular phrase for physicists. QED. With regards to Smarandache's aforementioned theorem, the proposed DCS model do not really make use of such a partial denial of axiom, except just a redefinition and clarification of the first axiom of Euclid. (Postscript note: In Sm. Hybrid Geometries, an axiom may be denied 100 percent but in different ways (see a book-chapter by E. Gonzalez), for example: the 5-th postulate of Euclid is denied as: a) there are lines and exterior points such that there is no parallel to the lines; b) there are lines and points such that there are many parallels to the points. For further reading on Smarandache geometries, see: http://fs.unm.edu/Geometries.htm). Nonetheless, his theorem of partially deny an axiom can be viewed as a guide, i.e. we can find out what happens if we relax Euclidean axioms one by one. Implications may be found in cosmology model, as we know there is Lindquist-Wheeler model, or Conrad Ranzan's cellular universe [10], and also foam-like model of the Universe [11]. What's more interesting here is that recently one of these authors communicated with Prof. E. Panarella, from Physics Essays journal, where we discuss a paper on discretization [12]. We suggest that it may be possible to extend further our DCS model to be more linked to on-going research in quantum gravity. See for instance Friedel-Livine, who stated which can be paraphrased as follows: "... Refining our depiction of the 3d calculation, we supporter to consider each 3d cells as air pockets, implying that we will portray the 3d math of every cells as the condition of the 2d calculation of its limit. Then, at that point, the 3d cells are stuck along shared limit surfaces and consistency conditions transform into matching imperatives between the two portrayals of the math of the limit according to the viewpoint of the two 3d cells sharing it. This image prompts 3d calculation as an organizations of air pockets."[13] This seems very interesting as well as workable approach, to find connection between DCS hypothesis and such network bubble, related to 3D cellular structure. Moreover, such a model of network bubble can be connected also to Wheeler's foam gravity model, which is composed of crystals, as David T. Crouse remarks: "John Wheeler's quantum foam such that the foam becomes a gravity crystal permeating all space and producing measurable inertial anomalies of astronomical bodies." [13] Nonetheless, allow us to remind fellow physicists to keep working in the above 3D cells, as they lead to direct connection with 3D crystalline structure as we will discuss as follows.

## 7. Discussion A: Remark on philosophical aspects

In this section, we will discuss several aspects which may be asked by readers. First of all, it is common to assert a finite length of line segment with real numbers only lead us to complicated arguments; it is called "real number line." (see for instance Scott [27]). But as we all know, that is only perceived by human cognition, in particular in a mathematician's cognitive process. As Lakoff and Nunez put it in preface of their book: "We discovered that it did: What is called the real-number line is not a line as most people understand it. What is called the continuum is not continuous in the ordinary sense of the term."[23] Nunez also wrote on definition of point, which can be paraphrased as follows: "... a point, which is the most straightforward substance in Euclidean math can't be really seen. A point, as characterized by Euclid is a dimensionless element, a substance that has just area however no expansion. No super-magnifying instrument can at any point permit us to see a point in fact. A point, with its accuracy and clear character, is a romanticized conceptual element."[24, 25] Therefore, the aforementioned arguments are essentially to alter the notion of "real number line" into a natural meaning of a line segment, composed of circles with small but finite cells. Now, let us discuss on method, we use a direct method by redefining the meaning of point, because that approach has nearest correspondence with our daily experience, as we argue in [28]. That particular direct approach may be favorable to physicists. While we do not wish to compare with others, we can mention difference between Landau and de Gennes, which can be paraphrased as follows: " The style of Landau was to go to the core of the issue, make not many yet significant suppositions, and infer apparently by wizardry a few vigorous outcomes. de Gennes' methodology bore in excess of a passing likeness to that of Landau." [29] And one more note, we can mention that there are other quite similar hypothesis with ours, for instance by Y. Breek, who argue for several postulates: "Postulate 1. (Discreteness): Space is discrete and composed of the underlying elementary units. The resulting discrete structure can be geometrically represented as a graph, network, or lattice (see Figure 1). The graph does not exist in space; rather, the graph itself is space." See Breek [30]. While it seems to correspond with our DCS, Breek does not consider the cellular structure of 3D space itself.

## 8. Discussion B: Remark on a few mathematical aspects

A German mathematician/Physicist Riemann in the mid 19th century managed to articulate the concept of limit using what is now called "- method" and manage to articulate the concept of $\lim_{x\mathrm{B}a} f(x)$ as :

$$\lim_{x\mathrm{Ba}} f(x) = b(>0)(>\ddot{0})([|x-a| < |f(x)-b| <]. \tag{4}$$

With this limit concept Riemann obtained the instantaneous change of rate of a function f(x) as follows:

$$f(x) = \lim_{x\ \mathrm{B}a}\left((f\ddot{f}+h) - f(x))/h\right). \tag{5}$$

In physics, we represent a motion as a function f(t) from time t to locations. Then clearly by $f'(t)$ we mean the instantaneous speed at t. We can define the acceleration of f(t) at t as $f"(t)$. This well understood definition of the speed of f(x) at time x as $f'(x)$ has been rather thoughtlessly accepted. Mathematics and physics are very different disciplines and we can not just adopt mathematics to describe physical reality. This is to say there is a little more going on in physics than in mathematics which is philosophically obvious because mathematics is a generalization of physics. Lack of philosophical understanding everywhere in physics has been slowing down the development of physics in many places. We know that "time" never reverses, it is an autonomous process which advances "without" any interferes, we must have very different mathematics for physics. Mathematics can not handle physics. Mathematics and physics are based upon totally different philosophies despite apparent similarity. Few physicists understand this difference. This is how mathematics was wrongly used in physics creating serious problems. Many mathematicians rightly think that as mathematics is more general than physics, we must be more careful when we use mathematics to describe physics. It is unfortunate that the pride of the King of Science is so high that there is little hope in communicating with physicists on these serious issues. Let us be more specific on this issue. the $\epsilon$-$\delta$ definition of $f'(x)$ has nothing to do with physics. This definition came from Cauchy-Rieman. Neither of them are philosophers nor physicists. They are just bloody articulate mathematicians who did not understand real world. For physicists,

$$f(x) = \lim_{x\mathrm{B}a}((f(\ddot{x}+h) - f(x))/h) \tag{6}$$

does not mean

$$\lim_{x\mathrm{B}a} f(x) = b(>0)(>\ddot{0})[|x-a| < |f(x)-b| <] \tag{7}$$

at all. For physicists limx→a is a physical process of x approaching a on a real line where x and a are geometric points. It was the grandeur complex of grand mathematicians like Cauchy and Riemann that they "improved" (or "articulate") the limit concept of limx→af(x). Certainly it was "improved" conceptually up in the air and we still have no idea how this abstract concept of limx→a has anything to do with physics. All of this conflict created a troubled dichotomy of pure mathematics and mathematics for real world. Certainly this abstract mathematics produced many fancy "deep results" most of which were never ever used in real life. This grandeur-complex driven abstract mathematics in the end reached the grandest set theory of Cantor, which Cantor himself proved inconsistent. Recently a book was published entitled "Lost in Mathematics"[45]. This book was the complain on the role pure mathematics played in the modern development of physics. Starting with Maxwell's EM field theory, Einstein's relativity theory and Heisenberg- Schrödinger's quantum mechanics which produced the grandest fallacy of the "Ultimate Physics, most empirically verified theory in, namely Quantum Mechanics". It is unfortunate that the author (i.e. S. Hossenfelder) was not aware of the fatal errors of these legendary ultimate theories of physics which dominated physics world for almost 1.5 centuries. She was just complaining about the oppressive usage of these "advanced mathematics" in theoretical physics. As we said just above the situation is much worse. Basically none of these figures of fame of the last 1.5 century of theoretical physics understood the mathematics they used. All of it turned out to be just embarrassing mathematical jokes. Contrary and almost amusingly, none of these pure mathematicians who promoted this irrelevant mathematization of physics understood physics at all. So the comedy and the tragedy is that physicists did not understand mathematics they use and mathematicians who promote their highly questionable mathematical theories did not understand physics at all. Going back to mathematics and science: The problem we mentioned just above about, the concept of limit, mathematical v.s. physical, has a lot to do with what happened to pure mathematics at the turn of the 20th century where Cantorian set theory destroying the grand hope by many prominent mathematicians such as Russell, Frege etc. motivated some determined mathematicians to abandon the over generality of set theory and pure mathematics in general which is based upon the fake empire of set theory. In one end, determined constructivism mathematicians such as Kronecker and Barwise concluded that only under the restriction on mathematical constructions to comply with the Principle of Constructivity, rejecting any fancy abstract metaphysical constructions, we can build trustable consistent mathematical theories. This idea was purified in the form of what we mathematicians call Recursion Theory and its generalization in the form of "Numeration Theory" which was started and developed by Soviet Mathematicians such as Malcev and Ershov at The Academy of Science Novosibirsk. In the limited context of theoretical computer science, this theory was discussed among constructivism mathematicians. It is our current view that the only mathematics which is solidly grounded and trust worthy is this theory. The rest are rather up in the air fantasy dreaming world. We are proposing that a restriction similar to this is badly needed in theoretical physics too. The situation this field was in up until recently was as seriously confused as the pure mathematics of the late 19th century, if not more. The only difference is that thanks to the academic honesty of mathematicians, in mathematics these issues were openly discussed without any suppression.

## 9. Application 1. What is space? A new hypothesis of super-crystalline vacua

In this section, allow us to extend ideas in the aforementioned section on possibility that the space consists of discrete cells, to become cells composed of superconductor quasi-crystalline. We discuss some features of this model. It is known that continuum problem is a fundamental question in pure mathematics and also theoretical physics field: whether the space is discrete or continuous. As we argued before how plausible it is discrete cellular space (DCS) to solve CH problem. Now, we can extend further by assuming that the 3D space is composed of network of dense-packed cells. That way the space system looks both as graph network as well as discrete cellular pattern. In this section, allow us to put forth a new hypothesis that the discrete cellular structure of space consists of cells of superconductor quasi-cristallyne. That proposition can be viewed as an alternative to Finkelstein's old hypothesis of hypercrystalline vacua [15]. There are furthermore (cautiously aperiodic) quasiperiodic jewels for which a depiction in regards to a change of a central design or a course of action of no less than two establishments is either inappropriate or unfathomable. We fight that one should insinuate all such valuable stones as quasicrystals, paying little brain to their point-pack equity. The most notable model for such jewels is a quasiperiodic tiling, for instance, the prestigious Penrose tiling. One consumes space with "unit cells" or "tiles" such that keeps up long-range demand without periodicity, and produces an essentially discrete diffraction diagram. Unquestionably, quasiperiodic diamonds having balances that are unlawful for discontinuous jewels, for instance, the watched icosahedral, octagonal, decagonal, and dodecagonal valuable stones—can't be outlined by changing a secret periodic design with a comparable equilibrium, and are along these lines all quasicrystals. Quasiperiodic pearls with no-no equilibriums can be formed as a difference in a discontinuous design, yet that need not be what is going on.[16] Ongoing discovery proposes that semi translucent has superconductive stage in exceptionally low temperature.[17] Consider the possibility that the quasicrystalline model isn't in semiconductor solid...but a superconductor quasicrystalline. Maybe, we might refer to it as: "super-glasslike vacua speculation." Quasi-translucent strong is additionally great since it gets multiple aspects, which might be exceptionally important. This likewise would bring into an amicable view among Finkelstein and Penrose and some of Frank Tony Smith's examinations. The following thing to consider is a super-semi crytalline strong (SQC). In view of its "fractal properties," we can expect that the Superconductor Quasi-Crystalline (SQC) can stretch out down to the construction of room, like what Finkelstein visualized. The semi precious stone design of room might be made out of strong matter or delicate matter, of which its overall elements has been illustrated by Fan et al. [18] See also several discussions on new findings of natural quasicrystals in Nature [19–22]. It is worth to remark here, that the proposed super-crystalline model of 3D space resembles the dense-packed spheron model of the late Prof. Linus Pauling. Such a dense-packed spheres have been discussed by many mathematicians. Australian mathematician, Mahler, also once wrote on dense-packed sphere. The difference here is, the dense-packed crystalline model now to be hypothesised to form the 3D space itself.

It is also possible to consider 'crystalline symmetry' within those real-space; see for instance Song et al. [31-32]. While this extended hypothesis of super-quasi-crystal structure of 3D space seems rather weird, we are sure that this is one of the most plausible direction toward description of what the space is made of. More research is of course recommended.

## 10. Application 2: Direct Discrete Formulation of Field Laws

There are other applications on such discrete cell model, for instance Finite Cell Method has been suggested by Parvizian et al. [33]. But it seems the most promising application is the so-called "The Cell method for Direct Discrete Formulation of Field Laws" [34, 35]. See also a recent publication describing application of the Cell method in finite formulation of parallel computation. As Tonti wrote, which can be para-phrased as follows: "The quintessence of the technique is to straightforwardly give a discrete formulation of field laws, without utilizing and requiring a differential detailing. It is demonstrated that, for direct interjection, the firmness grid so got coincides with the one of the Finite Element Method" [34]. Moreover, Tonti describes: "On managing differential definitions, it is very regular to utilize coordinate frameworks. Despite what might be expected, a

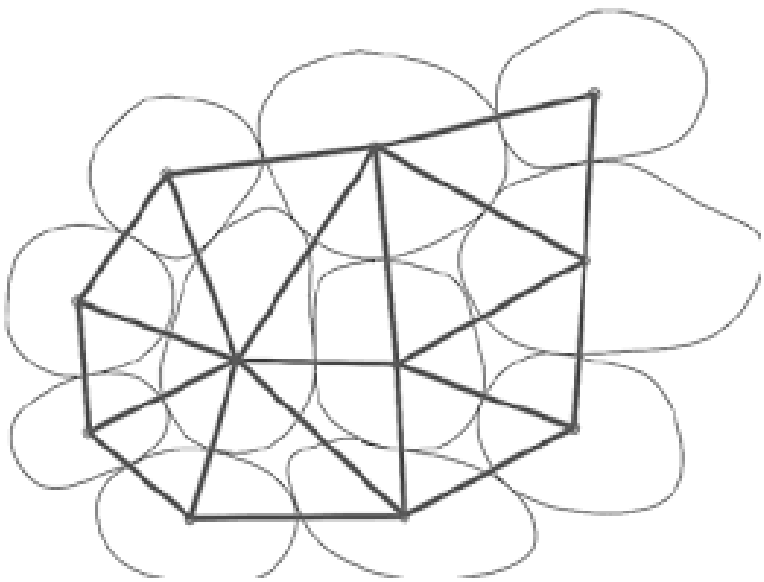

**Figure 1**. Discrete cellular model of 3D space

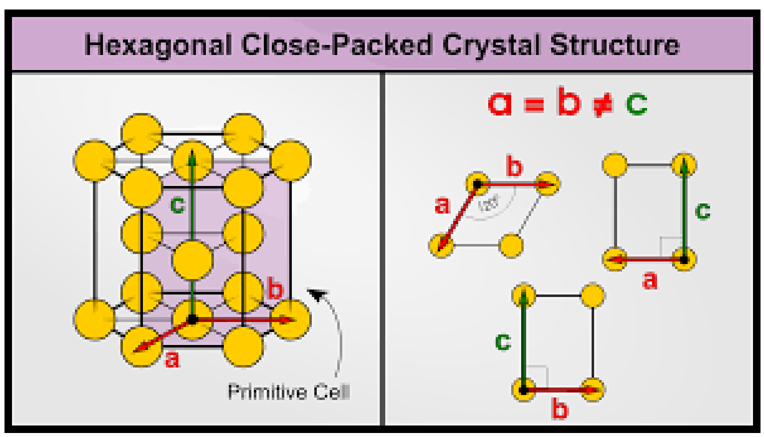


**Figure 2**. Hexagonal cellular close-packed crystal structure.

direct discrete definition manages worldwide factors, that are normally associated with limited sizes of spaces, and limited time spans, for example volumes, surfaces, lines, time spans just as focuses what's more moments. We will signify them as spatial and transient elements. Following the act of arithmetical geography, a branch of geography that utilizes cell buildings, the vertices, edges, faces what's more cells are considered as "cells" of aspect zero, one, two and three individually. In short they are signified as 0-cells, 1-cells, 2-cells and 3-cells. Likewise a cell complex isn't imagined as a bunch of little volumes yet as an assortment of cells of different aspects."[33] It can be expected that the Cell Method can be applied to various problems in sciences, given more availability of fast computers.

### 11. Concluding remarks

In this review article, we revisit our previous hypothesis called Discrete Cellular Space (Octogon, 2018), in order to answer the known unsolved problem, called Continuum Hypothesis. We believe that a tenable solution to this problem shall be found in close connection with empirical science, i,e. the notion of discrete cellular space, which can be attributed more to discrete mathematics, such as cellular automata modeling. In the last section, we gave an outline of extension of DCS hypothesis toward super-quasi-crystalline model of 3D space. While this extended hypothesis of super-quasi-crystal structure of 3D space seems rather weird, we are sure that this is one of the most plausible direction toward description of what the space is made of. All in all, allow us to close this article with a quote from Karl Popper: "For me, both philosophy and science lose all their attraction when they give up that pursuit [of knowledge and understanding of the world] – when they become specialisms and cease to see, and to wonder at, the riddles of our world. Specialization maybe a great temptation for the scientist. For the philosopher it is the mortal sin." (cf. Karl Popper, "The World of Parmenides.")

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