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On Micro b-open Sets

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Abstract: In this paper, a new kind of set called a Micro b-open set in micro topological space is introduced. Basic properties of Micro b-open sets are analyzed and also this set used to introduce and study the new types of functions called Micro b-irresolute, Micro b-continuous and Micro Wb-continuous functions.

Key words: Nano topology; Micro topology; Micro open set; Micro b-open set

1. Introduction

The motivating insight behind Topology is that some geometric problems depend not on the exact shape of the objects involved, but rather on the way they are put together. One of the first papers in Topology was the demonstration by Leonhard Euler, on The Seven Bridges of Konigsberg, which is believed to be one of the first academic treatises in Modern Topology. Topology is a major area of Mathematics concerned with properties that are preserved under continuous deformations of objects, such as deformations that involve stretching, but no tearing or gluing. The term 'Topology' was introduced in Germany in 1847 by Johann Benedict Listing. Modern topology depends strongly on the ideas of set theory developed by George Cantor in the later part of the 19th century. Henri Poincare introduced the concepts of homotopy and homology which are now considered as part of algebraic topology. Maurice Frechet, Volterra, Arzela, Hadamard, Ascoli and others introduced the metric space in 1906 which is now regarded as a special case of general topological space. In 1914, Felix Hausdorff coined the term 'topological space' and gave the definition of Hausdorff space. The fact that topological spaces show up naturally in almost every branch of Mathematics has made it one of the great unifying ideas of Mathematics. Different type of topological spaces are introduced by many topologist. The concept of nano topology was introduced by Thivagar et al [7] which is defined in terms of the lower and upper approximations and the boundary region of a subset of an universe. The notion of approximations and boundary region of a set was originally proposed by Pawlak [12] in order to introduce the concept of rough set theory. In 2019, Chandrasekar [1] introduced the concept of micro topology which is a simple extension of nano topology and he also studied the concepts of Micro pre-open and Micro semi-open sets. Chandrasekar and Swathi [2] introduced Micro α-open in micro topological space. Thivagar and Richard [9] established the weak forms of nano open set and also they introduced nano-regular open set in a nano topological space. Parimala et al [11] introduced the notion nano b-open set. More relevant research can be referred to [6, 13, 13]. In the past few years, different forms of b-open sets have been studied [3–5].

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2. Preliminaries

Definition 2.1. [10] Let U be a nonempty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U,R) is said to be the approximation space. Let $X \subseteq U$.

1. The lower approximation of U with respect to R is the set of all objects, which can be for certain classified as X with respect to R and its is denoted by $L_R(X)$. That is, $L_R(X) = \bigcup_{x\in U}\{R(x) : R(x) \subseteq X\}$, where $R(x)$ denotes the equivalence class determined by x.
2. The upper approximation of U with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is, $U_R(X) = \bigcup_{x\in U}\{R(x) : R(x) \cap X \neq \phi\}$.
3. The boundary region of U with respect to R is the set of all objects, which can be classified neither as X nor as not-X with respect to R and it is denoted by $B_R(X)$. That is, $B_R(X) = U_R(X) - L_R(X)$.

Definition 2.2. [7, 8] Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$, where $X \subseteq U$. Then, $\tau_R(X)$ satisfies the following axioms:

1. U and $\phi \in \tau_R(X)$.
2. The union of the elements of any subcollection of $\tau_R(X)$ is in $\tau_R(X)$.
3. The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

That is, $\tau_R(X)$ is a topology on U called the nano topology on U with respect to X. We call $(U, \tau_R(X))$ as the nano topological space. The elements of $\tau_R(X)$ are called as nano open sets. A subset F of U is nano closed if its complement is nano open.

Definition 2.3. [7] Let $(U, \tau_R(X))$ be a nano topological space with respect to X where $X \subseteq U$ and $K \subseteq U$, then the nano interior of K is defined as the union of all nano open subsets of K and it is denoted by $NInt(K)$. The nano closure of K is defined as the intersection of all nano closed sets containing K and it is denoted by $NCl(K)$.

Definition 2.4. A subset K of a nano topological space $(U, \tau_R(X))$ is called:

1. nano-regular open if $NInt(NCl(K)) = K$ [9].
2. nano α-open if $K \subseteq NInt(NCl(NInt(K)))$ [9].
3. nano pre-open if $K \subseteq NInt(NCl(K))$ [9].
4. nano semi-open if $K \subseteq NCl(NInt(K))$ [9].
5. nano b-open if $K \subseteq NCl(NInt(K)) \cup NInt(NCl(K))$ [11].

Theorem 2.1. *[9]Any nano-regular open set is nano open.*

Definition 2.5. [1] Let $(U, \tau_R(X))$ be a nano topological space. Then, $\mu_R(X) = \{N \cup (N' \cap \mu) : N, N' \in \tau_R(X)$ and $\mu \notin \tau_R(X)\}$ is called the micro topology on U with respect to X. The triplet $(U, \tau_R(X), \mu_R(X))$ is called micro topological space and the elements of $\mu_R(X)$ are called Micro open sets and the complement of a Micro open set is called a Micro closed set. Every nano topology is micro topology.

Definition 2.6. [1] The Micro closure of a set A is denoted by $Mic\text{-}cl(A)$ and is defined as $Mic\text{-}cl(A) = \cap\{B : B$ is Micro closed and $A \subseteq B\}$. The Micro interior of a set A is denoted by $Mic\text{-}int(A)$ and is defined as $Mic\text{-}int(A) = \cup\{B : B$ is Micro open and $A \supseteq B\}$.

Definition 2.7. Let $(U, \tau_R(X), \mu_R(X))$ be a micro topological space and $A \subseteq U$. Then,

1. A is called Micro α-open if $A \subseteq Mic\text{-}int(Mic\text{-}cl(Mic\text{-}int(A)))$ [2].
2. A is called Micro pre-open if $A \subseteq Mic\text{-}int(Mic\text{-}cl(A))$ [1].
3. A is called Micro semi-open if $A \subseteq Mic\text{-}cl(Mic\text{-}int(A))$ [1].
4. A is called Micro pre-closed if $Mic\text{-}cl(Mic\text{-}int(A)) \subseteq A$ [1].
5. A is called Micro semi-closed if $Mic\text{-}int(Mic\text{-}cl(A)) \subseteq A$ [1].

Theorem 2.2. *Let $(U, \tau_R(X), \mu_R(X))$ be a micro topological space. Then:*

1. *Every Micro open set is Micro pre-open [1].*
2. *Every Micro open set is Micro semi-open [1].*
3. *Every Micro open set is Micro α-open [2].*
4. *Every Micro α-open set is Micro semi-open [2].*
5. *Every Micro α-open set is Micro pre-open [2].*

Theorem 2.3. *[1] Let $(U, \tau_R(X), \mu_R(X))$ be a micro topological space. Let A and B be any two subsets of U. Then:*

1. *A is a Micro open set if and only if $Mic\text{-}int(A) = A$.*
2. *A is a Micro closed set if and only if $Mic\text{-}cl(A) = A$.*
3. *$Mic\text{-}cl(A \cup B) \supseteq Mic\text{-}cl(A) \cup Mic\text{-}cl(B)$.*
4. *$Mic\text{-}int(A \cup B) \supseteq Mic\text{-}int(A) \cup Mic\text{-}int(B)$.*
5. *$Mic\text{-}cl(A \cap B) \subseteq Mic\text{-}cl(A) \cap Mic\text{-}cl(B)$.*
6. *$Mic\text{-}int(A \cap B) \subseteq Mic\text{-}int(A) \cap Mic\text{-}int(B)$.*
7. *$Mic\text{-}int(U \setminus A) = U \setminus Mic\text{-}cl(A)$.*
8. *$Mic\text{-}cl(U \setminus A) = U \setminus Mic\text{-}int(A)$.*

Definition 2.8. Let $(U, \tau_R(X), \mu_R(X))$ and $(V, \tau_R(Y), \mu_R(Y))$ be two micro topological spaces. Then, A function $f : U \to V$ is said to be:

1. Micro-continuous if $f^{-1}(K)$ is Micro open in U, for every Micro open set K in V [1].
2. Micro-α continuous if $f^{-1}(K)$ is Micro α-open in U, for every Micro open set K in V [2].
3. Micro-pre continuous if $f^{-1}(K)$ is Micro pre-closed in U, for every Micro closd set K in V [1].
4. Micro-semi continuous if $f^{-1}(K)$ is Micro semi-closed in U, for every Micro closed set K in V [1].

Theorem 2.4. [2] Every Micro-continuous is Micro-α continuous.

3. Micro b-open Sets

Definition 3.1. Let $(U, \tau_R(X), \mu_R(X))$ be a micro topological space and $A \subseteq U$. Then, A is called a Micro b-open set if $A \subseteq Mic\text{-}int(Mic\text{-}cl(A)) \cup Mic\text{-}cl(Mic\text{-}int(A))$. $Mic\text{-}bO(U, X)$ denotes the collection of all Micro b-open sets. A subset B of U is called Micro b-closed if and only if its complement is Micro b-open. Moreover, $Mic\text{-}bC(U, X)$ denotes the collection of all Micro b-closed sets.

Remark 3.1. *It can be shown that a subset B of U is Micro b-closed if and only if $Mic\text{-}cl(Mic\text{-}int(B)) \cap Mic\text{-}int(Mic\text{-}cl(B)) \subseteq B$.*

Theorem 3.1. *Let $(U, \tau_R(X), \mu_R(X))$ be a micro topological space. Then:*

1. *Every Micro pre-open is Micro b-open.*
2. *Every Micro semi-open is Micro b-open.*

Proof. 1. If A is Micro pre-open, then $A \subseteq$Mic-int(Mic-$cl(A)) \subseteq$Mic-int(Mic-$cl(A)) \cup$Mic-cl(Mic-$int(A))$. Thus, A is Micro b-open.

2. If A is Micro semi-open, then $A \subseteq$Mic-cl(Mic-$int(A)) \subseteq$Mic-int(Mic-$cl(A)) \cup$Mic-cl(Mic-$int(A))$. Thus, A is Micro b-open.

□

The converse of the above theorem need not be true in general as it is shown below.

Example 3.1. *Consider $U = \{r, m, o\}$ with $U/R = \{\{r\}, \{m, o\}\}$ and $X = \{r\}$. Then, $\tau_R(X) = \{U, \phi, \{r\}\}$.*

1. *If $\mu = \{o\}$, then $\mu_R(X) = \{U, \phi, \{r\}, \{o\}, \{r, o\}\}$ and $\{r, m\}$ is Micro b-open but not Micro pre-open.*
2. *If $\mu = \{m, o\}$, then $\mu_R(X) = \{U, \phi, \{r\}, \{m, o\}\}$ and $\{o\}$ is Micro b-open but not Micro semi-open.*

Remark 3.2. 1. *The concepts of Micro b-open and nano b-open sets are independent.*

2. *The concepts of Micro α-open and nano α-open sets are independent.*
3. *The concepts of Micro pre-open and nano pre-open sets are independent.*
4. *The concepts of Micro semi-open and nano semi-open sets are independent.*
5. *The concepts of Micro pre-open and Micro semi-open sets are independent.*

Example 3.2. 1. *From Example 3.1 (1), we have:*

   (a) *$\{o\}$ is Micro b-open, Micro α-open, Micro pre-open and Micro semi-open but $\{o\}$ is not nano b-open, nano α-open, nano pre-open and nano semi-open.*

   (b) *$\{r, m\}$ is both nano α-open and nano pre-open but neither Micro α-open nor Micro pre-open.*

2. *From Example 3.1 (2), $\{r, m\}$ is nano semi-open but not Micro semi-open.*
3. *From Example 3.1 (1), $\{r, m\}$ is Micro semi-open but not Micro pre-open and from (2), $\{o\}$ is Micro pre-open but not Micro semi-open.*

Example 3.3. *Consider $U = \{r, m, o\}$ with $U/R = \{\{r, m, o\}\}$ and $X = \{m, o\}$. Then, $\tau_R(X) = \{U, \phi\}$. If $\mu = \{r\}$, then $\mu_R(X) = \{U, \phi, \{r\}\}$ and $\{m\}$ is nano b-open but not Micro b-open.*

Corollary 3.1. Let $(U, \tau_R(X), \mu_R(X))$ be a micro topological space. Then,

1. Every Micro open is Micro b-open.
2. Every nano open is Micro b-open.
3. Every nano-regular open is Micro b-open.
4. Every Micro α-open is Micro b-open.

Proof. Let A be Micro open, then $A = A \cup A = Mic\text{-}int(A) \cup Mic\text{-}int(A) \subseteq Mic\text{-}int(Mic\text{-}cl(A)) \cup Mic\text{-}cl(Mic\text{-}int(A))$.

2. Follows from Definition 2.5 and (1).
3. Follows from Theorem 2.1 and (2).
4. Follows from Theorems 2.2 (4) and 3.1 (2).

□

The converse of the above corollary need not be true in general as it is shown below.

Example 3.4. *From Example 3.1 (1), $\{r, m\}$ is Micro b-open but not nano-regular open, nano open, Micro open and Micro α-open.*

Remark 3.3. From Theorems 2.1, 2.2, 3.1 and Definition 2.5, we obtain the following diagram of implications:

$$\begin{tikzcd}
\text{nano open} \arrow[d] & \text{nano-regular open} \arrow[l] & \text{Micro pre-open} \arrow[d] \\
\text{Micro open} \arrow[r] & \text{Micro } \alpha\text{-open} \arrow[ur] \arrow[dr] & \text{Micro } b\text{-open} \\
 & & \text{Micro semi-open} \arrow[u]
\end{tikzcd}$$

Theorem 3.2. *An arbitrary union of Micro b-open sets is Micro b-open.*

Proof. Let $\{A_i : i \in I\}$ be a family of Micro b-open sets. Then for each i, $A_i \subseteq Mic\text{-}int(Mic\text{-}cl(A_i)) \cup Mic\text{-}cl(Mic\text{-}int(A_i))$ and so

$$\cup_{i\in I} A_i \subseteq \cup_{i\in I}[Mic\text{-}int(Mic\text{-}cl(A_i)) \cup Mic\text{-}cl(Mic\text{-}int(A_i))]$$

$$\subseteq [\cup_{i\in I}(Mic\text{-}int(Mic\text{-}cl(A_i)))] \cup [\cup_{i\in I}(Mic\text{-}cl(Mic\text{-}int(A_i)))]$$

$$\subseteq [\cup_{i\in I} Mic\text{-}int(Mic\text{-}cl(A_i))] \cup [\cup_{i\in I} Mic\text{-}cl(Mic\text{-}int(A_i))]$$

$$\subseteq [Mic\text{-}int(\cup_{i\in I} Mic\text{-}cl(A_i))] \cup [Mic\text{-}cl(\cup_{i\in I} Mic\text{-}int(A_i))]$$

$$\subseteq [Mic\text{-}int(Mic\text{-}cl(\cup_{i\in I} A_i))] \cup [Mic\text{-}cl(Mic\text{-}int(\cup_{i\in I} A_i))].$$

Thus, $\cup_{i\in I} A_i$ is a Micro b-open set. □

Remark 3.4. *An arbitrary intersection of Micro b-closed sets is Micro b-closed.*

2. *The intersection of two Micro b-open sets may not be Micro b-open.*

Example 3.5. *From Example 3.1 (1), $A = \{r, m\}$ and $B = \{m, o\}$ are Micro-b-open, but $A \cap B = \{m\}$ which is not Micro-b-open.*

Remark 3.5. *Let $(U, \tau_R(X), \mu_R(X))$ be a micro topological space. Let A and B be any two subsets of U. Then:*

1. *Mic-$cl(A)$ is Micro closed.*
2. *$A \subseteq Mic$-$cl(A)$.*
3. *$U \setminus Mic$-$cl(U \setminus A) = Mic$-$int(A)$.*
4. *$U \setminus Mic$-$int(U \setminus A) = Mic$-$cl(A)$.*
5. *Mic-$cl(A \cup B) \subseteq Mic$-$cl(A) \cup Mic$-$cl(B)$.*
6. *Mic-$int(A \cap B) \supseteq Mic$-$int(A) \cap Mic$-$int(B)$.*
7. *$x \in Mic$-$cl(A)$ if and only if for every Micro open subset L of U containing x, $A \cap L \neq \phi$.*

Theorem 3.3. *Let $(U, \tau_R(X), \mu_R(X))$ be a micro topological space and $A \subseteq U$. Then:*

1. *For every Micro open set G, we have Mic-$cl(A) \cap G \subseteq Mic$-$cl(A \cap G)$.*
2. *For every Micro closed set F, we have Mic-$int(A \cup F) \subseteq Mic$-$int(A) \cup F$.*

Proof. 1. Let $x \in Mic$-$cl(A) \cap G$, then $x \in Mic$-$cl(A)$ and $x \in G$. Let L be a Micro open set containing x, then $L \cap G$ is also a Micro open set containing x. By Remark 3.5 (7), we have $(L \cap G) \cap A \neq \phi$. This implies $L \cap (A \cap G) \neq \phi$. This is true for every L containing x, hence by Remark 3.5 (7), $x \in Mic$-$cl(A \cap G)$. Thus, Mic-$cl(A) \cap G \subseteq Mic$-$cl(A \cap G)$.

2. It is obvious.

□

Theorem 3.4. *The intersection of a Micro b-open set and a Micro open set is Micro b-open.*

Proof. Let A be Micro b-open and G be Micro open, then $A \subseteq Mic\text{-}int(Mic\text{-}cl(A)) \cup Mic\text{-}cl(Mic\text{-}int(A))$ and $G = Mic$-$int(G)$. Then, we have

$$G \cap A \subseteq G \cap [Mic\text{-}int(Mic\text{-}cl(A)) \cup Mic\text{-}cl(Mic\text{-}int(A))]$$

$$= [G \cap Mic\text{-}int(Mic\text{-}cl(A))] \cup [G \cap Mic\text{-}cl(Mic\text{-}int(A))]$$

$$= [Mic\text{-}int(G) \cap Mic\text{-}int(Mic\text{-}cl(A))] \cup [G \cap Mic\text{-}cl(Mic\text{-}int(A))]$$

$$\subseteq [Mic\text{-}int(G \cap Mic\text{-}cl(A))] \cup [Mic\text{-}cl(G \cap Mic\text{-}int(A))]$$

$$\subseteq [Mic\text{-}int(Mic\text{-}cl(G \cap A))] \cup [Mic\text{-}cl(Mic\text{-}int(G \cap A))].$$

This shows that $G \cap A$ is Micro b-open. □

Definition 3.2. Let $(U, \tau_R(X), \mu_R(X))$ be a micro topological space. Then:

1. The union of all Micro b-open sets contained in A is called the Micro b-interior of A and denoted by Mic-$bint(A)$.
2. The intersection of all Micro b-closed sets containing A is called the Micro b-closure of A and denoted by Mic-$bcl(A)$.

Now, we state the following theorem without proof.

Theorem 3.5. *Let $(U, \tau_R(X), \mu_R(X))$ be a micro topological space. For any subsets A, B of U, we have the following:*

1. *A is Micro b-open if and only if $A = Mic$-$bint(A)$.*
2. *A is Micro b-closed if and only if $A = Mic$-$bcl(A)$.*
3. *If $A \subseteq B$, then Mic-$bint(A) \subseteq Mic$-$bint(B)$ and Mic-$bcl(A) \subseteq Mic$-$bcl(B)$.*
4. *Mic-$bint(A) \cup Mic$-$bint(B) \subseteq Mic$-$bint(A \cup B)$.*
5. *Mic-$bint(A \cap B) \subseteq Mic$-$bint(A) \cap Mic$-$bint(B)$.*
6. *Mic-$bcl(A) \cup Mic$-$bcl(B) \subseteq Mic$-$bcl(A \cup B)$.*
7. *Mic-$bcl(A \cap B) \subseteq Mic$-$bcl(A) \cap Mic$-$bcl(B)$.*
8. *Mic-$bint(U \setminus A) = U \setminus Mic$-$bcl(A)$.*
9. *Mic-$bcl(U \setminus A) = U \setminus Mic$-$bint(A)$.*
10. *$U \setminus Mic$-$bcl(U \setminus A) = Mic$-$bint(A)$.*
11. *$U \setminus Mic$-$bint(U \setminus A) = Mic$-$bcl(A)$.*
12. *$x \in Mic$-$bint(A)$ if and only if there exists a Micro b-open set L such that $x \in L \subseteq A$.*

In general the equalities of (5) and (6) of the above theorem do not hold, as it is shown in the following examples.

Example 3.6. *From Example 3.1 (1),*

1. *if we let $A = \{m, o\}$ and $B = \{r, m\}$, then Mic-$bint(A) = \{m, o\}$, Mic-$bint(B) = \{r, m\}$ and Mic-$bint(A \cap B) = \phi$, where $A \cap B = \{m\}$ this implies that Mic-$bint(A \cap B) = \phi \neq \{m\} = Mic$-$bint(A) \cap Mic$-$bint(B)$.*
2. *if we let $A = \{r\}$ and $B = \{o\}$, then Mic-$bcl(A) = \{r\}$, Mic-$bcl(B) = \{o\}$ and Mic-$bcl(A \cup B) = U$, where $A \cup B = \{r, o\}$ this implies that Mic-$bcl(A \cup B) = U \neq \{r, o\} = Mic$-$bcl(A) \cup Mic$-$bcl(B)$.*

Theorem 3.6. *Let A be a subset of a micro topological space $(U, \tau_R(X), \mu_R(X))$. Then, $x \in Mic$-$bcl(A)$ if and only if for every Micro b-open subset L of U containing x, $A \cap L \neq \phi$.*

Proof. Let $x \in Mic$-$bcl(A)$ and suppose that $L \cap A = \phi$ for some Micro b-open set L which contains x. Then, $(U \setminus L)$ is Micro b-closed and $A \subset (U \setminus L)$, thus Mic-$bcl(A) \subset (U \setminus L)$. But this implies that $x \in (U \setminus L)$, a contradiction. Thus, $L \cap A \neq \phi$.
Conversely, let $A \subseteq U$ and $x \in U$ such that for each Micro b-open set L_1 which contains x, $L_1 \cap A \neq \phi$. If $x \notin Mic$-$bcl(A)$, there is a Micro b-closed set F such that $A \subseteq F$ and $x \notin F$. Then, $(U \setminus F)$ is a Micro b-open set with $x \in (U \setminus F)$, and thus $(U \setminus F) \cap A \neq \phi$, which is a contradiction. $\square$

Theorem 3.7. Let $(U, \tau_R(X), \mu_R(X))$ be a micro topological space and $L \subseteq U$. Then, L is Micro b-open if and only if for each $s \in L$, there exists a Micro b-open set D such that $s \in D \subseteq L$.

Proof. It is obvious. □

Definition 3.3. A function $f : U \to V$ is said to be Micro b-irresolute if $f^{-1}(K)$ is a Micro b-open set in U, for every Micro b-open set K in V.

Theorem 3.8. *Let $f : U \to V$ be a function, then the following statements are equivalent:*

1. *f is Micro b-irresolute.*
2. *$f(Mic\text{-}bcl(A)) \subseteq Mic\text{-}bcl(f(A))$ holds for every subset A of U.*
3. *$f^{-1}(B)$ is Micro b-closed in U, for every Micro b-closed subset B of V.*

Proof. $(2) \Rightarrow (3)$: Let B be a Micro b-closed set in V, then $Mic\text{-}bcl(B) = B$. By using (2), we have $f(Mic\text{-}bcl(f^{-1}(B)) \subseteq Mic\text{-}bcl(B) = B$. Thus, $Mic\text{-}bcl(f^{-1}(B)) \subseteq f^{-1}(B)$ and hence $f^{-1}(B)$ is Micro b-closed in U.
$(3) \Rightarrow (2)$: If $A \subseteq U$, then $Mic\text{-}bcl(f(A))$ is Micro b-closed in V and by (3) $f^{-1}(Mic\text{-}bcl(f(A)))$ is Micro b-closed in U. Furthermore, $A \subseteq f^{-1}(f(A)) \subseteq f^{-1}(Mic\text{-}bcl(f(A)))$. Thus, $Mic\text{-}bcl(A) \subseteq f^{-1}(Mic\text{-}bcl(f(A)))$, consequently, $f(Mic\text{-}bcl(A)) \subseteq f(f^{-1}(Mic\text{-}bcl(f(A)))) \subseteq Mic\text{-}bcl(f(A))$.
$(3) \Leftrightarrow (1)$: Obvious. □

Definition 3.4. A function $f : U \to V$ is said to be:

1. Micro b-continuous at a point $x \in U$ if for each Micro open subset K of V containing $f(x)$, there exists a Micro b-open subset L of U containing x such that $f(L) \subseteq K$. The function f is said to be Micro b-continuous if it has this property at each $x \in U$.
2. Micro Wb-continuous at a point $x \in U$ if for each Micro open subset K of V containing $f(x)$, there exists a Micro b-open subset L of U containing x such that $f(L) \subseteq Mic\text{-}cl(K)$. The function f is said to be Micro Wb-continuous if it has this property at each $x \in U$.

Remark 3.6. *It is obvious from the above definition that Micro b-continuous implies Micro Wb-continuous. However, the converse is not true in general as it is shown in the following example.*

Example 3.7. *Consider $U = \{a, b, c\}$ with $U/R = \{\{a\}, \{b, c\}\}$ and $X = \{a\}$. Then, $\tau_R(X) = \{U, \phi, \{a\}\}$. If $\mu = \{b\}$, then $\mu_R(X) = \{U, \phi, \{a\}, \{b\}, \{a, b\}\}$ and $Mic\text{-}bO(U, X) = \{U, \phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}\}$. Consider $V = \{a, b, c, d\}$ with $V/R = \{\{a\}, \{c\}, \{b, d\}\}$ and $Y = \{b, d\}$. Then, $\tau_R(Y) = \{V, \phi, \{b, d\}\}$. If $\mu = \{a\}$, then $\mu_R(Y) = \{V, \phi, \{a\}, \{b, d\}, \{a, b, d\}\}$. Define a function $f : U \to V$ as follows:*

$$f(x) = \begin{cases} c & \text{if } x = a \\ a & \text{if } x = b \\ d & \text{if } x = c \end{cases}$$

Then, f is Micro Wb-continuous, but not Micro b-continuous, because $\{b, d\}$ is a Micro open set in V containing $f(c) = d$, but there exist no Micro b-open set L in U containing c such that $f(L) \subseteq \{b, d\}$.

Theorem 3.9. *A function $f : U \to V$ is Micro b-continuous if and only if the inverse image of every Micro open set in V is Micro b-open in U.*

Proof. Let f be Micro b-continuous and K be any Micro open set in V. If $f^{-1}(K) = \phi$, then $f^{-1}(K)$ is a Micro b-open set in U but if $f^{-1}(K) \neq \phi$, then there exists $x \in f^{-1}(K)$ which implies $f(x) \in K$. Since f is Micro b-continuous, then there exists a Micro b-open set L in U containing x such that $f(L) \subseteq K$. This implies that $x \in L \subseteq f^{-1}(K)$ and hence $f^{-1}(K)$ is Micro b-open.
Conversely, let K be any Micro open set in V containing $f(x)$, then $x \in f^{-1}(K)$ and by hypothesis $f^{-1}(K)$ is a Micro b-open set in U containing x, so $f(f^{-1}(K)) \subseteq K$. Thus, f is Micro b-continuous. □

Remark 3.7. *Let $f : U \to V$ be a function, then*

1. *f is Micro-pre continuous if $f^{-1}(K)$ is Micro pre-open in U, for every Micro open set K in V.*
2. *f is Micro-semi continuous if $f^{-1}(K)$ is Micro semi-open in U, for every Micro open set K in V.*

Remark 3.8. *Every Micro b-irresolute is Micro b-continuous.*

2. *Every Micro-continuous is Micro b-continuous.*
3. *Every Micro-α continuous is Micro b-continuous.*
4. *Every Micro-pre continuous is Micro b-continuous.*
5. *Every Micro-semi continuous is Micro b-continuous.*
6. *Every Micro-α continuous is Micro-semi continuous.*
7. *Every Micro-α continuous is Micro-pre continuous.*

The converse of the above remark need not be true in general as it is shown below.

Example 3.8. *Consider $U = \{a, b, c\}$ with $U/R = \{\{a\}, \{b, c\}\}$ and $X = \{a\}$. Then, $\tau_R(X) = \{U, \phi, \{a\}\}$. If $\mu = \{b\}$, then $\mu_R(X) = \{U, \phi, \{a\}, \{b\}, \{a, b\}\}$ and $Mic\text{-}bO(U, X) = \{U, \phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}\}$. Consider $V = \{r, m, n\}$ with $V/R = \{\{n\}, \{r, m\}\}$ and $Y = \{m\}$. Then, $\tau_R(Y) = \{V, \phi, \{r, m\}\}$. If $\mu = \{r, n\}$, then $\mu_R(Y) = \{V, \phi, \{r\}, \{r, m\}, \{r, n\}\}$. Define a function $f : U \to V$ as follows:*

$$f(x) = \begin{cases} r & \text{if } x = a \\ m & \text{if } x = b \\ n & \text{if } x = c \end{cases}$$

Then, f is both Micro b-continuous and Micro-semi continuous but not Micro-continuous, Micro-α continuous and Micro-pre continuous.

Example 3.9. *Consider $U = \{a, b, c\}$ with $U/R = \{\{a\}, \{b, c\}\}$ and $X = \{a\}$. Then, $\tau_R(X) = \{U, \phi, \{a\}\}$. If $\mu = \{b\}$, then $\mu_R(X) = \{U, \phi, \{a\}, \{b\}, \{a, b\}\}$ and $Mic\text{-}bO(U, X) = \{U, \phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}\}$. Consider $V = \{r, m, n\}$ with $V/R = \{\{n\}, \{r, m\}\}$ and $Y = \{n\}$. Then, $\tau_R(Y) = \{V, \phi, \{n\}\}$. If $\mu = \{r, m\}$, then $\mu_R(Y) = \{V, \phi, \{n\}, \{r, m\}\}$ and $Mic\text{-}bO(V, Y) = \{V, \phi, \{r\}, \{m\}, \{n\}, \{r, m\}, \{r, n\}, \{m, n\}\}$. Define a function $f : U \to V$ as follows:*

$$f(x) = \begin{cases} n & \text{if } x = a \\ m & \text{if } x = b \\ r & \text{if } x = c \end{cases}$$

Then, f is Micro b-continuous but not Micro b-irresolute.

Example 3.10. *Consider $V = \{r, m, n\}$ with $V/R = \{\{n\}, \{r, m\}\}$ and $Y = \{n\}$. Then, $\tau_R(Y) = \{V, \phi, \{n\}\}$. If $\mu = \{r, m\}$, then $\mu_R(Y) = \{V, \phi, \{n\}, \{r, m\}\}$ and $Mic\text{-}bO(V, Y) = \{V, \phi, \{r\}, \{m\}, \{n\}, \{r, m\}, \{r, n\}, \{m, n\}\}$. Define a function $f : V \to V$ as follows:*

$$f(x) = \begin{cases} n & \text{if } x = m \\ m & \text{if } x = r \\ r & \text{if } x = n \end{cases}$$

Then, f is both Micro b-continuous and Micro-pre continuous but neither Micro-α continuous nor Micro-semi continuous.

Remark 3.9. *From Theorem 2.4, Remark 3.6 and Remark 3.8, we obtain the following diagram of implications:*

$$\begin{tikzcd}
\text{Micro } b\text{-irresolute} \arrow[d] & \text{Micro-pre continuous} \arrow[dl] & \\
\text{Micro } b\text{-continuous} \arrow[d] & \text{Micro-}\alpha\text{ continuous} \arrow[u] \arrow[d] & \text{Micro-continuous} \arrow[l] \\
\text{Micro } Wb\text{-continuous} & \text{Micro-semi continuous} \arrow[ul] &
\end{tikzcd}$$

Theorem 3.10. *For a function $f : U \to V$, the following statements are equivalent:*

1. *f is Micro b-continuous.*
2. *$f^{-1}(K)$ is a Micro b-open set in U, for each Micro open subset K of V.*
3. *$f^{-1}(F)$ is a Micro b-closed set in U, for each Micro closed subset F of V.*
4. *$f(Mic\text{-}bcl(A)) \subseteq Mic\text{-}cl(f(A))$, for each subset A of U.*
5. *$Mic\text{-}bcl(f^{-1}(B)) \subseteq f^{-1}(Mic\text{-}cl(B))$, for each subset B of V.*
6. *$f^{-1}(Mic\text{-}int(B)) \subseteq Mic\text{-}bint(f^{-1}(B))$, for each subset B of V.*

Proof. (1) $\Rightarrow$ (2): Directly from Theorem 3.9.
(2) $\Rightarrow$ (3): Let F be any Micro closed subset of V. Then, $V \setminus F$ is a Micro open subset of V. By (2), $f^{-1}(V \setminus F) = U \setminus f^{-1}(F)$ is a Micro b-open set in U and hence $f^{-1}(F)$ is a Micro b-closed set in U.
(3) $\Rightarrow$ (4): Let A be any subset of U. Then, $f(A) \subseteq Mic\text{-}cl(f(A))$ and $Mic\text{-}cl(f(A))$ is a Micro closed set in V. Hence, $A \subseteq f^{-1}(Mic\text{-}cl(f(A)))$. By (3), we have $f^{-1}(Mic\text{-}cl(f(A)))$ is a Micro b-closed set in U. Therefore, $Mic\text{-}bcl(A) \subseteq f^{-1}(Mic\text{-}cl(f(A)))$. Hence, $f(Mic\text{-}bcl(A)) \subseteq Mic\text{-}cl(f(A))$.
(4) $\Rightarrow$ (5): Let B be any subset of V. Then, $f^{-1}(B)$ is a subset of U. By (4), we have $f(Mic\text{-}bcl(f^{-1}(B))) \subseteq Mic\text{-}cl(f(f^{-1}(B))) \subseteq Mic\text{-}cl(B)$. Hence, $Mic\text{-}bcl(f^{-1}(B)) \subseteq f^{-1}(Mic\text{-}cl(B))$.
(5) $\Leftrightarrow$ (6): Let B be any subset of V. Then, apply (5) to $V \setminus B$ we obtain $Mic\text{-}bcl(f^{-1}(V \setminus B)) \subseteq f^{-1}(Mic\text{-}cl(V \setminus B)) \Leftrightarrow Mic\text{-}bcl(U \setminus f^{-1}(B)) \subseteq f^{-1}(V \setminus Mic\text{-}int(B)) \Leftrightarrow U \setminus Mic\text{-}bint(f^{-1}(B)) \subseteq U \setminus f^{-1}(Mic\text{-}int(B)) \Leftrightarrow f^{-1}(Mic\text{-}int(B)) \subseteq Mic\text{-}bint(f^{-1}(B))$. Thus, $f^{-1}(Mic\text{-}int(B)) \subseteq Mic\text{-}bint(f^{-1}(B))$.
(6) $\Rightarrow$ (1): Let $x \in U$ and K be any Micro open subset of V containing $f(x)$. By (6), we have $f^{-1}(Mic\text{-}int(K)) \subseteq Mic\text{-}bint(f^{-1}(K))$ implies that $f^{-1}(K) \subseteq Mic\text{-}bint(f^{-1}(K))$. Hence, $f^{-1}(K)$ is a Micro b-open set in U which contains x and clearly $f(f^{-1}(K)) \subseteq K$. Thus, f is Micro b-continuous. $\square$

Theorem 3.11. *For a function $f : U \to V$, the following statements are equivalent:*

1. *f is Micro Wb-continuous.*
2. *$f^{-1}(K) \subseteq Mic\text{-}bint(f^{-1}(Mic\text{-}cl(K)))$, for every Micro open subset K of V.*
3. *$Mic\text{-}bcl(f^{-1}(Mic\text{-}int(F))) \subseteq f^{-1}(F)$, for every Micro closed subset F of V.*
4. *$Mic\text{-}bcl(f^{-1}(Mic\text{-}int(Mic\text{-}cl(B)))) \subseteq f^{-1}(Mic\text{-}cl(B))$, for every subset B of V.*
5. *$f^{-1}(Mic\text{-}int(B)) \subseteq Mic\text{-}bint(f^{-1}(Mic\text{-}cl(Mic\text{-}int(B))))$, for every subset B of V.*
6. *$Mic\text{-}bcl(f^{-1}(K)) \subseteq f^{-1}(Mic\text{-}cl(K))$, for every Micro open subset K of V.*

Proof. (1) $\Rightarrow$ (2): Let K be a Micro open subset of V such that $x \in f^{-1}(K)$. Then, $f(x) \in K$ and there exists $L \in Mic\text{-}bO(U, X)$ containing x such that $f(L) \subseteq Mic\text{-}cl(K)$. Thus, we obtain $x \in L \subseteq f^{-1}(Mic\text{-}cl(K))$. This implies that $x \in Mic\text{-}bint(f^{-1}(Mic\text{-}cl(K)))$.

(2) $\Rightarrow$ (3): Let F be any Micro closed subset of V. Suppose that $x \notin f^{-1}(F)$, then $V \setminus F$ is Micro open in V and $x \in U \setminus f^{-1}(F) = f^{-1}(V \setminus F)$. By (2), we have $x \in Mic\text{-}bint(f^{-1}(Mic\text{-}cl(V \setminus F))) = Mic\text{-}bint(f^{-1}(V \setminus Mic\text{-}int(F))) = Mic\text{-}bint(U \setminus f^{-1}(Mic\text{-}int(F))) = U \setminus (Mic\text{-}bcl(f^{-1}(Mic\text{-}int(F))))$. Therefore, we obtain $x \notin Mic\text{-}bcl(f^{-1}(Mic\text{-}int(F)))$.

(3) $\Rightarrow$ (4): Let B be any subset of V. Then, $Mic\text{-}cl(B)$ is Micro closed in V and by (3), we have that if $x \in Mic\text{-}bcl(f^{-1}(Mic\text{-}int(Mic\text{-}cl(B))))$, then $x \in f^{-1}(Mic\text{-}cl(B))$.

(4) $\Rightarrow$ (5): Let B be any subset of V and $x \in f^{-1}(Mic\text{-}int(B))$. Then, we have $x \in f^{-1}(Mic\text{-}int(B)) = U \setminus f^{-1}(Mic\text{-}cl(V \setminus B))$. Then, $x \notin f^{-1}(Mic\text{-}cl(V \setminus B))$ and by (4), $x \in U \setminus Mic\text{-}bcl(f^{-1}(Mic\text{-}int(Mic\text{-}cl(V \setminus B)))) = Mic\text{-}bint(f^{-1}(Mic\text{-}cl(Mic\text{-}int(B))))$.

(5) $\Rightarrow$ (6): Let K be any Micro open subset of V. Suppose that $x \notin f^{-1}(Mic\text{-}cl(K))$. Then, $f(x) \notin Mic\text{-}cl(K)$ and there exists a Micro open set W containing $f(x)$ such that $W \cap K = \phi$; hence $Mic\text{-}cl(W) \cap K = \phi$. By (5), we have $x \in Mic\text{-}bint(f^{-1}(Mic\text{-}cl(W)))$ and hence there exists $L \in Mic\text{-}bO(U, X)$ such that $x \in L \subseteq f^{-1}(Mic\text{-}cl(W))$. Since $Mic\text{-}cl(W) \cap K = \phi$, then $L \cap f^{-1}(K) = \phi$ and by Theorem 3.6, $x \notin Mic\text{-}bcl(f^{-1}(K))$. Therefore, if $x \in Mic\text{-}bcl(f^{-1}(K))$, then $x \in f^{-1}(Mic\text{-}cl(K))$.

(6) $\Rightarrow$ (1): Let $x \in U$ and K any Micro open subset of V containing $f(x)$. Then, we have $x \in f^{-1}(K) \subseteq f^{-1}(Mic\text{-}int(Mic\text{-}cl(K))) = U \setminus f^{-1}(Mic\text{-}cl(V \setminus Mic\text{-}cl(K)))$. By (6), $x \notin Mic\text{-}bcl(f^{-1}(V \setminus Mic\text{-}cl(K)))$ and hence $x \in Mic\text{-}bint(f^{-1}(Mic\text{-}cl(K)))$. Therefore, there exists $L \in Mic\text{-}bO(U, X)$ such that $x \in L \subseteq f^{-1}(Mic\text{-}cl(K))$ and hence $f(L) \subseteq Mic\text{-}cl(K)$. This shows that f is Micro Wb-continuous. $\square$

Theorem 3.12. *For a function $f : U \to V$, the following statements are equivalent:*

1. *f is Micro Wb-continuous at $x \in U$.*
2. *$x \in Mic\text{-}bint(f^{-1}(Mic\text{-}cl(K)))$, for each Micro open set K containing $f(x)$.*

Proof. (1) $\Rightarrow$ (2): Let K be any Micro open set containing $f(x)$. Then, there exists a Micro b-open set L containing x such that $f(L) \subseteq Mic\text{-}cl(K)$ and so $L \subseteq f^{-1}(Mic\text{-}cl(K))$. Since L is Micro b-open, then $x \in L = Mic\text{-}bint(L) \subseteq Mic\text{-}bint(f^{-1}(Mic\text{-}cl(K)))$.

(2) $\Rightarrow$ (1): Let $x \in Mic\text{-}bint(f^{-1}(Mic\text{-}cl(K)))$, for each Micro open set K containing $f(x)$. Take $L = Mic\text{-}bint(f^{-1}(Mic\text{-}cl(K)))$. Then, $f(L) \subseteq Mic\text{-}cl(K)$. Moreover, L is Micro b-open. Thus, f is Micro Wb-continuous at $x \in U$. □

Theorem 3.13. *For a function $f : U \to V$, the following statements are equivalent:*

1. *f is Micro Wb-continuous.*
2. *$Mic\text{-}bcl(f^{-1}(K)) \subseteq f^{-1}(Mic\text{-}cl(K))$, for every Micro pre-open K in V.*
3. *$f^{-1}(K) \subseteq Mic\text{-}bint(f^{-1}(Mic\text{-}cl(K)))$, for every Micro pre-open K in V.*

Proof. (1) $\Rightarrow$ (2): Let K be any Micro pre-open subset of V such that $x \in Mic\text{-}bcl(f^{-1}(K))$. Suppose that $x \notin f^{-1}(Mic\text{-}cl(K))$. Then, there exists a Micro open set W containing $f(x)$ such that $W \cap K = \phi$. Thus, we have $W \cap Mic\text{-}cl(K) = \phi$ and hence $Mic\text{-}cl(W) \cap Mic\text{-}int(Mic\text{-}cl(K)) = \phi$. Since K is Micro pre-open, then $K \subseteq Mic\text{-}int(Mic\text{-}cl(K))$ and we have $K \cap Mic\text{-}cl(W) = \phi$. Since f is Micro Wb-continuous at $x \in U$ and W is a Micro open set containing $f(x)$, then there exists a Micro b-open subset L of U containing x such that $f(L) \subseteq Mic\text{-}cl(W)$. So, $f(L) \cap K = \phi$ and hence $L \cap f^{-1}(K) = \phi$. This shows that $x \notin Mic\text{-}bcl(f^{-1}(K))$. This is a contradiction. Therefore, $x \in f^{-1}(Mic\text{-}cl(K))$.
(2) $\Rightarrow$ (3): Let K be Micro pre-open in V and $x \in f^{-1}(K)$. Then, $f^{-1}(K) \subseteq f^{-1}(Mic\text{-}int(Mic\text{-}cl(K))) = U \setminus f^{-1}(Mic\text{-}cl(V \setminus Mic\text{-}cl(K)))$. Therefore, $x \notin f^{-1}(Mic\text{-}cl(V \setminus Mic\text{-}cl(K)))$. Then by (2), $x \notin Mic\text{-}bcl(f^{-1}(V \setminus Mic\text{-}cl(K)))$. Hence, $x \in U \setminus Mic\text{-}bcl(f^{-1}(V \setminus Mic\text{-}cl(K))) = Mic\text{-}bint(f^{-1}(Mic\text{-}cl(K)))$.
(3) $\Rightarrow$ (1): This follows from Theorem 3.11, since every Micro open set is Micro pre-open. □

4. Conclusions

In this work, the properties of a new types of Micro open sets are discussed. Also, we defined Micro b-irresolute, Micro b-continuous and Micro Wb-continuous functions, and obtained some of their properties. In the future, we will try to introduce the Micro b-separate axioms, and Micro b-connectedness in a micro topological space.

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