On $N_m$-semi-open sets in neutrosophic minimal structure spaces

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Abstract: The focuses of this article, we study the notions of $N_m$-semi-open sets, $N_m$-semi-interior, $N_m$-semi-closure, $N_m$-semi-continuous maps in neutrosophic minimal structures & some basic concepts.

Key words: Neutrosophic minimal structure spaces (in short, nms), $N_m$-semi-closed, $N_m$-semi-open and $N_m$-semi-continuous

1. Introduction

L. A. Zadeh’s [16] Fuzzy set concepts laid the foundation of many theories such as neutrosophic sets, soft sets, etc. K. T. Atanassov’s [4] intuitionistic fuzzy set theory in many areas such as topology, computer science and so on. F. Smarandache [14, 15] found that some objects have indeterminacy or neutral other than membership and non-membership. A. A. Salama & S. A. Albloowi [13], introduced and studied some fundamental properties of neutrosophic set (in short., ns) & neutrosophic topological spaces (in short., nt). V. Popa & T. Noiri [12] introduced the notions of of minimal structure spaces. M. Karthika et al [11] introduced and studied nms. (ie., $N_m$-closed, $N_m$-open, $N_m$-closure, $N_m$-interior, union property, intersection property, $N_m$- maps and so on,...). We analysis of $N_m$-semi-closed sets, $N_m$-semi-open sets, $N_m$-semi-closure and $N_m$-semi-interior operators in nms. Finally, we introduce $N_m$-semi-continuous map and investigate some properties of such concepts.

2. Preliminaries

Definition 2.1. [13] A nt in Salama’s sense on a nonempty set X is a family $\tau$ of ns in X satisfying three axioms:

1. Empty set $(0_\sim)$ and universal set $(1_\sim)$ are members of $\tau$.

2. $K_1 \cap K_2 \in \tau$ where $K_1, K_2 \in \tau$.

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3. $\cup K_\delta \in \tau$ for every $\{K_\delta : \delta \in \Delta\} \leq \tau$.

**Definition 2.2.** [11] Let nms over a universal set $\Omega$ be denoted by $N_m$. $N_m$ is said to be nms over $\Omega$ if it satisfies following the axiom: $0 \sim, 1 \sim \in N_m$. A family of nms is denoted by $(\Omega, N_m)$. 

3. $N_m$-semi-open

**Definition 3.1.** Let $(\Omega, N_m)$ be a nms. A subset $W$ of $\Omega$ is said to be $N_m$-semi-open set (in short, $N_m$-sos) if $W \leq N_m \text{cl}(N_m \text{int}(W))$. The complement of an $N_m$-sos is called an $N_m$-scs.

**Remark 3.1.** Let $(\Omega, T)$ be a nt & $W \leq \Omega$. $W$ is called an $N_m$-semi-open set (in short, $N_m$-sos) [10] if $W \leq N_m \text{cl}(N_m \text{int}(W))$. If the nms $N_m$ is a topology, clearly an $N_m$-sos is $N_m$-sos.

**Lemma 3.1.** Let $(\Omega, N_m)$ be a nms. Then

1. Every $N_m$-os is $N_m$-sos.
2. $W$ is an $N_m$-sos iff $W \leq N_m \text{cl}(N_m \text{int}(W))$.
3. Every $N_m$-cs is $N_m$-scs.
4. $W$ is an $N_m$-scs iff $N_m \text{int}(N_m \text{cl}(W)) \leq W$.

**Theorem 3.1.** Let $(\Omega, N_m)$ be a nms. The arbitrary union of $N_m$-sos is a $N_m$-sos.

**Proof.** Let $W_\delta$ be an $N_m$-sos for $\delta \in \Delta$. From Definition 3.1 and Proposition 3.8 (vi) [11], it follows $W_\delta \leq N_m \text{cl}(N_m \text{int}(W_\delta)) \leq N_m \text{cl}(N_m \text{int}(\bigcup W_\delta))$. This implies $\bigcup W_\delta \leq N_m \text{cl}(N_m \text{int}(\bigcup W_\delta))$. Hence $\bigcup W_\delta$ is an $N_m$-sos. 

**Remark 3.2.** Let $(\Omega, N_m)$ be a nms. The intersection of any two $N_m$-sos may not be $N_m$-sos.

**Example 3.1.** Let $\Omega = \{\omega\}$ with $N_m = \{0_\sim, P_1, Q_1, R_1, S_1, 1_\sim\}$ and $N_m^C = \{1_\sim, I_1, J_1, K_1, L_1, 0_\sim\}$ where $P_1 = \prec (1, 0.5, 0.6)\succ; Q_1 = \prec (0, 0.9, 0.2)\succ; R_1 = \prec (1, 0.9, 0.2)\succ; S_1 = \prec (0.8, 0.5, 0.6)\succ; I_1 = \prec (0.6, 0.5, 1)\succ; J_1 = \prec (0.2, 0.1, 0)\succ; K_1 = \prec (0.2, 0.1, 1)\succ; L_1 = \prec (0.6, 0.5, 0)\succ$.

Now we define the two $N_m$-sos as follows:

$A = \prec (1, 0.5, 0.6)\succ; B = \prec (0.8, 0.5, 0.2)\succ$.

Here $N_m \text{int}(A) = P_1$, $N_m \text{cl}(N_m \text{int}(A)) = N_m \text{cl}(P_1) = 0_\sim^C$ and $N_m \text{int}(B) = S_1$, $N_m \text{cl}(N_m \text{int}(B)) = N_m \text{cl}(S_1) = 0_\sim^C$. But $A \cap B = \prec (0.8, 0.5, 0.6)\succ$ is not a $N_m$-sos in $\Omega$.

**Definition 3.2.** Let $(\Omega, N_m)$ be a nms. For a subset $W$ of $\Omega$. Then,

1. $N_m$-semi-closure of $W = \text{min}\{I : I \text{ is } N_m \text{-scs and } I \geq W\}$, it is denoted by $N_m - \text{sc}(W)$.
2. $N_m$-semi-interior of $W = \text{max}\{G : G \text{ is } N_m \text{-sos and } G \leq W\}$, it is denoted by $N_m - \text{sint}(W)$.

**Theorem 3.2.** Let $(\Omega, N_m)$ be a nms and $W \leq \Omega$. Then
1. \( N_m \)-sint(W) \( \leq \) W.

2. If \( W \leq Z \), then \( N_m \)-sint(W) \( \leq \) \( N_m \)-sint(Z).

3. \( W \) is \( N_m \)-sos iff \( N_m \)-sint(W) = W.

4. \( N_m \)-sint\( (N_m \)-sint(W)) = \( N_m \)-sint(W).

5. \( N_m \)-scl(\( \Omega - W \)) = \( \Omega - N_m \)-sint(W) and \( N_m \)-sint(\( \Omega - W \)) = \( \Omega - N_m \)-scl(W).

\textbf{Proof.} (1), (2) Obvious.

(3) by theorem 3.1.

(4) by (3).

(5) \( W \leq \Omega, \Omega - N_m - \text{sint}(W) = \Omega - \max\{U : U \leq W, U \text{ is } N_m - \text{sos}\} = \min\{\Omega - U : U \leq W; U \text{ is } N_m - \text{sos}\} = N_m - \text{scl}(\Omega - W). \)

Similarly, we have \( N_m \)-sint(\( \Omega - W \)) = \( \Omega - N_m \)-scl(W). \( \square \)

\textbf{Theorem 3.3.} Let \( (\Omega, N_{m\Omega}) \) be a nms and \( W \leq \Omega \). Then

1. \( W \leq N_m \)-scl(W).

2. If \( W \leq Z \), then \( N_m \)-scl(W) \( \leq \) \( N_m \)-scl(Z).

3. \( F \) is \( N_m \)-scs iff \( N_m \)-scl(F) = F.

4. \( N_m \)-scl\( (N_m \)-scl(W)) = \( N_m \)-scl(W).

\textbf{Proof.} Similar to by theorem 3.2. \( \square \)

\textbf{Theorem 3.4.} Let \( (\Omega, N_{m\Omega}) \) be a nms \& \( W \leq \Omega \). Then

1. \( \omega \in N_m \)-scl(W) iff if \( W \cap M \neq \emptyset \) for every \( N_m \)-sos M containing \( \omega \).

2. \( \omega \in N_m \)-sint(W) iff there exists an \( N_m \)-sos K such that \( K \leq W \).

\textbf{Proof.} (1) \( \exists \) there is an \( N_m \)-sos M containing \( \omega \) such that \( W \cap M = \emptyset \). \( \Omega - M \) is an \( N_m \)-scs such that \( W \leq \Omega - M, \omega \notin \Omega - M. \) This implies \( \omega \notin N_m \)-scl(W).

The reverse relation is true.

(2) Obvious. \( \square \)

\textbf{Lemma 3.2.} \( (\Omega, N_{m\Omega}) \) be a nms \& \( W \leq \Omega \).

1. \( N_m \)-int\( (N_m \)-cl(W)) \( \leq \) \( N_m \)-int\( (N_m \)-cl(N_m \)-scl(W))) \( \leq \) \( N_m \)-scl(W).

2. \( N_m \)-sint(W) \( \leq \) \( N_m \)-cl\( (N_m \)-int\( (N_m \)-sint(W))) \( \leq \) \( N_m \)-cl\( (N_m \)-int(W)).

\textbf{Proof.} (1) For \( W \leq \Omega \), by Theorem 3.3, \( N_m \)-scl(W) is an \( N_m \)-scs. Hence from Lemma 3.1, we have \( N_m \)-int\( (N_m \)-cl(W)) \( \leq \) \( N_m \)-int\( (N_m \)-cl(N_m \)-scl(W))) \( \leq \) \( N_m \)-scl(W).

(2) similar by the proof of (1). \( \square \)
Definition 3.3. Let \( l : (\Omega, N_m \Omega) \rightarrow (\Lambda, N_m \Lambda) \) is called \( N_m \)-semi-continuous map (in short, \( N_m \)-sc) iff \( l^{-1}(V) \subseteq N_m \)-sos whenever \( V \subseteq N_m \Lambda \).

Theorem 3.5. Every neutrosophic minimal continuous is \( N_m \)-sc but not conversely.

Proof. By Lemma 3.1 (1).

Theorem 3.6. Let \( l : \Omega \rightarrow \Lambda \) be a map on two nms \((\Omega, N_m \Omega)\) and \((\Lambda, N_m \Lambda)\).

1. \( l \) is \( N_m \)-sc.
2. \( l^{-1}(M) \) is an \( N_m \)-sos for each \( N_m \)-os \( M \) in \( \Lambda \).
3. \( l^{-1}(Z) \) is an \( N_m \)-scs for each \( N_m \)-cs \( Z \) in \( \Lambda \).
4. \( l(N_m \text{-scl}(W)) \leq l(N_m \text{-cl}(W)) \) for \( W \subseteq \Omega \).
5. \( N_m \text{-scl}(l^{-1}(Z)) \leq l^{-1}(N_m \text{-cl}(Z)) \) for \( Z \subseteq \Lambda \).
6. \( l^{-1}(N_m \text{-int}(Z)) \leq N_m \text{-sint}(l^{-1}(Z)) \) for \( Z \subseteq \Lambda \).

Proof. (1) \( \Rightarrow \) (2) Let \( M \) be an \( N_m \)-os in \( \Lambda \) \& \( \omega \in l^{-1}(M) \). By hypothesis, there exists an \( N_m \)-sos \( U_{\omega} \), containing \( \omega \) such that \( l(U) \subseteq M \). This implies \( \omega \in U_{\omega} \subseteq l^{-1}(M) \) for all \( \omega \in l^{-1}(M) \). Hence by Theorem 3.1, \( l^{-1}(M) \) is \( N_m \)-sos.

(2) \( \Rightarrow \) (3) Obvious.

(3) \( \Rightarrow \) (4) For \( W \subseteq \Omega \), \( l(N_m \text{-cl}(l(W))) = l^{-1}(\min\{S \subseteq \Lambda : l(W) \subseteq S \text{ and } S \text{ is } N_m \text{-closed}\}) = \min\{l^{-1}(S) \subseteq \Omega : S \subseteq \Lambda \text{ and } S \text{ is } N_m \text{-sos}\} \geq \min\{R \subseteq \Omega : W \subseteq R \text{ and } R \text{ is } N_m \text{-cs}\} = N_m \text{-scl}(W) \). Hence \( l(N_m \text{-scl}(W)) \leq l(N_m \text{-cl}(W)) \).

(4) \( \Rightarrow \) (5) For \( W \subseteq \Omega \), from (4), it follows \( l(N_m \text{-scl}(l^{-1}(W))) \leq l(N_m \text{-cl}(l^{-1}(W))) \leq l(N_m \text{-cl}(W)) \). Hence we get (5).

(5) \( \Rightarrow \) (6) For \( Z \subseteq \Lambda \), from \( N_m \text{-int}(Z) = \Lambda - N_m \text{-cl}(\Lambda - Z) \) and (5), it follows: \( l^{-1}(N_m \text{-int}(Z)) = l^{-1}(\Lambda - N_m \text{-cl}(\Lambda - Z)) = \Omega - l^{-1}(N_m \text{-cl}(\Lambda - Z)) = \Omega - N_m \text{-scl}(l^{-1}(\Lambda - Z)) = N_m \text{-sint}(l^{-1}(Z)) \). Hence (6) is obtained.

(6) \( \Rightarrow \) (1) Let \( \omega \in \Omega \) and \( M \) an \( N_m \)-os containing \( l(\omega) \). From (6) \& Proposition 3.8 [11], it follows \( Z \in l^{-1}(M) = l^{-1}(N_m \text{-int}(M)) \leq N_m \text{-sint}(l^{-1}(M)) \). Theorem 3.4, \( \exists N_m \)-sos \( U \) containing \( \omega \) such that \( \omega \in U \subseteq l^{-1}(M) \). Hence \( l \) is \( N_m \)-sc. \( \square \)

Theorem 3.7. \( l : \Omega \rightarrow \Lambda \) be a map on two nms \((\Omega, N_m \Omega)\) and \((\Lambda, N_m \Lambda)\).

1. \( l \) is \( N_m \)-sc.
2. \( l^{-1}(M) \subseteq N_m \text{-cl}(N_m \text{-int}(l^{-1}(M))) \) for each \( N_m \)-os \( M \) in \( \Lambda \).
3. \( N_m \text{-int}(N_m \text{-cl}(l^{-1}(R))) \subseteq l^{-1}(R) \) for each \( N_m \)-cs \( R \) in \( \Lambda \).
4. \( l(N_m \text{-int}(N_m \text{-cl}(W))) \subseteq N_m \text{-cl}(l(A)) \) for \( W \subseteq \Omega \).
5. \( N_m \text{-int}(N_m \text{-cl}(l^{-1}(Z))) \subseteq l^{-1}(N_m \text{-cl}(Z)) \) for \( Z \subseteq \Lambda \).
6. \( l^{-1}(N_m \text{-int}(Z)) \subseteq N_m \text{-cl}(N_m \text{-int}(l^{-1}(Z))) \) for \( Z \subseteq \Lambda \).
Proof. (1) ⇔ (2) By theorem 3.6 and definition of $N_m$-sos.
(1) ⇔ (3) By theorem 3.6 and lemma 3.1.
(3) ⇒ (4) Let $W \leq \Omega$. Then from Theorem 3.6(4) and Lemma 3.2, it follows $N_m \text{int}(N_m \text{cl}(W)) \leq N_m \text{scl}(W)$, hence $l(N_m \text{int}(N_m \text{cl}(W))) \leq N_m \text{cl}(l(W))$.
(4) ⇒ (5) Obvious.
(5) ⇒ (6) From (5) and Proposition 3.8 [11], it follows: $1^{-1}(N_m \text{int}(Z)) = 1^{-1}(\Lambda - N_m \text{cl}(\Lambda - Z)) = \Omega$
$-1^{-1}(N_m \text{cl}(\Lambda - Z)) \leq \Omega - N_m \text{int}(N_m \text{cl}(1^{-1}(\Lambda - Z)))$
$= N_m \text{cl}(N_m \text{int}(1^{-1}(Z)))$. Hence, (6) is obtained.
(6) ⇒ (1) Let M be an $N_m$ os in $\Lambda$. Then by (6) and Proposition 3.8 [11], we have $1^{-1}(M) = 1^{-1}(N_m \text{int}(M)) \leq N_m \text{cl}(N_m \text{int}(1^{-1}(M)))$. This implies $1^{-1}(M)$ is an $N_m$-sos. Hence by (2), $l$ is $N_m$-semi-continuous.

Conclusion
We presented several new notions and related properties by utilizing the concept of $N_m$-sos in nms.

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References


