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On N_m-semi-open sets in neutrosophic minimal structure spaces

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Abstract: The focuses of this article, we study the notions of N_m-semi-open sets, N_m-semi-interior, N_m-semi-closure, N_m-semi-continuous maps in neutrosophic minimal structures & some basic concepts.

Key words: Neutrosophic minimal structure spaces (in short, nms), N_m-semi-closed, N_m-semi-open and N_m-semi-continuous

1. Introduction

L. A. Zadeh's [16] Fuzzy set concepts laid the foundation of many theories such as neutrosophic sets, soft sets, etc. K. T. Atanassov's [4] intuitionistic fuzzy set theory in many areas such as topology, computer science and so on. F. Smarandache [14, 15] found that some objects have indeterminacy or neutral other than membership and non-membership. A. A. Salama & S. A. Alblowi[13], introduced and studied some fundamendal properties of neutrosophic set (in short., ns) & neutrosophic topological spaces (in short., nt). V. Popa & T. Noiri [12] introduced the notions of of minimal structure spaces. M. Karthika et al [11] introduced and studied nms. (ie., N_m-closed, N_m-open, N_m-closure, N_m-interior, union property, intersection property , N_m- maps and so on,...). We analysis of N_m-semi-closed sets, N_m-semi-open sets, N_m-semi-closure and N_m-semi-interior operators in nms. Finally, we introduce N_m-semi-continuous map and investigate some properties of such concepts.

2. Preliminaries

Definition 2.1. [13] A nt in Salama's sense on a nonempty set X is a family τ of ns in X satisfying three axioms:

1. Empty set $(0_\sim)$ and universal set$(1_\sim)$ are members of τ.
2. $K_1 \cap K_2 \in \tau$ where $K_1, K_2 \in \tau$.

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3. $\cup K_\delta \in \tau$ for every $\{K_\delta : \delta \in \Delta\} \leq \tau$.

Definition 2.2. [11] Let nms over a universal set Ω be denoted by N_m. N_m is said to be nms over Ω if it satisfies following the axiom: $0_\sim, 1_\sim \in N_m$. A family of nms is denoted by $(\Omega, N_{m\Omega})$.

3. N_m-semi-open

Definition 3.1. Let $(\Omega, N_{m\Omega})$ be a nms. A subset W of Ω is said to be N_m-semi-open set (in short, N_m-sos) if $W \leq N_m cl(N_m int(W))$. The complement of an N_m-sos is called an N_m-scs.

Remark 3.1. *Let $(\Omega, \mathcal{T})$ be a nt & $W \leq \Omega$. W is called an N_m-semi-open set(in short, N_m-sos) [10] if W $\leq \mathcal{N}cl(\mathcal{N}int(W))$. If the nms $N_{m\Omega}$ is a topology, clearly an N_m-sos is N_m-sos .*

Lemma 3.1. *Let $(\Omega, N_{m\Omega})$ be a nms. Then*

1. *Every N_m os is N_m-sos.*
2. *W is an N_m-sos iff $W \leq N_m cl(N_m int(W))$.*
3. *Every N_m-cs is N_m-scs.*
4. *W is an N_m-scs iff $N_m int(N_m cl(W)) \leq W$.*

Theorem 3.1. *Let $(\Omega, N_{m\Omega})$ be a nms. The arbitrary union of N_m-sos is a N_m-sos.*

Proof. Let W_δ be an N_m-sos for $\delta \in \Delta$. From Definition 3.1 and Proposition 3.8 (vi) [11], it follows $W_\delta \leq N_m cl(N_m int(W_\delta)) \leq N_m cl(N_m int(\bigcup W_\delta))$. This implies $\bigcup W_\delta \leq N_m clo(N_m inte(\bigcup W_\delta))$. Hence $\bigcup W_\delta$ is an N_m-sos. □

Remark 3.2. *Let $(\Omega, N_{m\Omega})$ be a nms. The intersection of any two N_m-sos may not be N_m-sos.*

Example 3.1. *Let $\Omega = \{\omega\}$ with $N_m = \{0_\sim, P_1, Q_1, R_1, S_1, 1_\sim\}$ and $N_m^C = \{1_\sim, I_1, J_1, K_1, L_1, 0_\sim\}$ where*
$P_1 = \prec (1, 0.5, 0.6)\succ$; $Q_1 = \prec (0, 0.9, 0.2)\succ$; $R_1 = \prec (1, 0.9, 0.2)\succ$; $S_1 = \prec (0.8, 0.5, 0.6)\succ$
$I_1 = \prec (0.6, 0.5, 1)\succ$; $J_1 = \prec (0.2, 0.1, 0)\succ$; $K_1 = \prec (0.2, 0.1, 1)\succ$; $L_1 = \prec (0.6, 0.5, 0)\succ$
Now we define the two N_m-sos as follows:
$A = \prec (1, 0.5, 0.6)\succ$; $B = \prec (0.8, 0.5, 0.2)\succ$
Here $N_m int(A) = P_1$, $N_m cl(N_m int(A)) = N_m cl(P_1) = 0_\sim^C$ and
$N_m int(B) = S_1$, $N_m cl(N_m int(B)) = N_m cl(S_1) = 0_\sim^C$. But $A \wedge B = \prec (0.8, 0.5, 0.6)\succ$ is not a N_m-sos in Ω.

Definition 3.2. Let $(\Omega, N_{m\Omega})$ be a nms. For a subset W of Ω. Then,

1. N_m-semi-closure of W = min $\{I : I \text{ is } N_m\text{-scs and } I \geq W\}$, it is denoted by N_m-scl(W).
2. N_m-semi-interior of W = max $\{G : G \text{ is } N_m\text{-sos and } G \leq W\}$, it is denoted by N_m-sint(W).

Theorem 3.2. *Let $(\Omega, N_{m\Omega})$ be a nms and $W \leq \Omega$. Then*

1. *N_m-sint(W) $\leq$ W.*
2. *If W $\leq$ Z, then N_m-sint(W) $\leq$ N_m-sint(Z).*
3. *W is N_m-sos iff N_m-sint(W) = W.*
4. *N_m-sint(N_m-sint(W)) = N_m-sint(W).*
5. *N_m-scl(Ω − W) = $\Omega - N_m$-sint(W) and N_m-sint($\Omega -$ W) = Ω − N_m-scl(W).*

Proof. (1), (2) Obvious.
(3) by theorem 3.1.
(4) by (3).
(5) $W \leq \Omega, \Omega - N_m - \sin t(W) = \Omega - \max\{U : U \leq W, U$ is $N_m - sos\} = \min\{\Omega - U : U \leq W, U$ is $N_m - sos\} = \min\{\Omega - U : \Omega - W \leq \Omega - U, U$ is $N_m - sos\} = N_m - scl(\Omega - W)$.
Similarly, we have N_m-sint(Ω − W) = Ω − N_m-scl(W). □

Theorem 3.3. *Let (Ω, $N_{m\Omega}$) be a nms and W $\leq$ Ω. Then*

1. *W $\leq$ N_m-scl(W).*
2. *If W $\leq$ Z, then N_m-scl(W) $\leq$ N_m-scl(Z).*
3. *F is N_m-scs iff N_m-scl(F) = F.*
4. *N_m-scl(N_m-scl(W)) = N_m-scl(W).*

Proof. Similar to by theorem 3.2. □

Theorem 3.4. *Let (Ω, $N_{m\Omega}$) be a nms & W $\leq$ Ω. Then*

1. *ω $\in$ N_m-scl(W) iff if W $\cap$ M $\neq \emptyset$ for every N_m-sos M containing ω.*
2. *ω $\in$ N_m-sint(W) iff there exists an N_m-sos K such that K $\leq$ W.*

Proof. (1) $\exists$ there is an N_m-sos M containing ω such that W $\cap$ M = $\emptyset$. Ω − M is an N_m-scs such that W $\leq$ Ω − M, $\omega \notin \Omega$ − M. This implies $\omega \notin$ N_m-scl(W).
The reverse relation is true.
(2) Obvious. □

Lemma 3.2. *(Ω, $N_{m\Omega}$) be a nms & W $\leq$ Ω.*

1. *$N_m$int($N_m$cl(W)) $\leq$ $N_m$int($N_m$cl(N_m-scl(W))) $\leq$ N_m-scl(W).*
2. *N_m-sint(W) $\leq$ $N_m$cl($N_m$int(N_m-sint(W))) $\leq$ $N_m$cl($N_m$int(W)).*

Proof. (1) For W $\leq$ Ω, by Theorem 3.3, N_m-scl(W) is an N_m-scs. Hence from Lemma 3.1, we have $N_m$int($N_m$cl(W)) $\leq$ $N_m$int($N_m$cl(N_m-scl(W))) $\leq$ N_m-scl(W).
(2) similar by the proof of (1). □

Definition 3.3. Let l : $(\Omega,\ N_{m\Omega}) \to (\Lambda,\ N_{m\Lambda})$ is called N_m-semi-continuous map (in short, N_m-sc) iff $l^{-1}(V) \in N_m$-sos whenever $V \in N_{m\Lambda}$.

Theorem 3.5. *Every neutrosophic minimal continuous is N_m-sc but not conversely.*

Proof. By Lemma 3.1 (1). □

Theorem 3.6. *Let $l : \Omega \to \Lambda$ be a map on two nms $(\Omega,\ N_{m\Omega})$ and $(\Lambda,\ N_{m\Lambda})$.*

1. *l is N_m-sc.*
2. *$l^{-1}(M)$ is an N_m-sos for each N_m os M in Λ.*
3. *$l^{-1}(Z)$ is an N_m-scs for each N_m-cs Z in Λ.*
4. *$l(N_m\text{-}scl(W)) \leq N_m cl(l(W))$ for $W \leq \Omega$.*
5. *$N_m\text{-}scl(l^{-1}(Z)) \leq l^{-1}(N_m cl(Z))$ for $Z \leq \Lambda$.*
6. *$l^{-1}(N_m int(Z)) \leq N_m\text{-}sint(l^{-1}(Z))$ for $Z \leq \Lambda$.*

Proof. (1) ⇒ (2) Let M be an N_m os in Λ & $\omega \in l^{-1}(M)$. By hypothesis, there exists an N_m-sos U_ω containing ω such that $l(U) \leq M$. This implies $\omega \in U_\omega \leq l^{-1}(M)$ for all $\omega \in l^{-1}(M)$. Hence by Theorem 3.1, $l^{-1}(M)$ is N_m-sos.
(2) ⇒ (3) Obvious.
(3) ⇒ (4) For $W \leq \Omega$, $l^{-1}(N_m cl(l(W))) = l^{-1}(\min\{S \leq \Lambda : l(W) \leq S$ and S is N_m-closed$\}) = \min\{l^{-1}(S) \leq \Omega : W \leq l^{-1}(S)$ and S is N_m-scs$\} \geq \min\{R \leq \Omega : W \leq R$ and R is N_m-scs$\} = N_m$-scl(W). Hence $l(N_m\text{-scl}(W)) \leq N_m cl(l(W))$.
(4) ⇒ (5) For $W \leq \Omega$, from (4), it follows $l(N_m\text{-scl}(l^{-1}(W))) \leq N_m cl(l(l^{-1}(W))) \leq N_m cl(W)$. Hence we get (5).
(5) ⇒ (6) For $Z \leq \Lambda$, fromm $N_m int(Z) = \Lambda - N_m cl(\Lambda - Z)$ and (5), it follows: $l^{-1}(N_m int(Z)) = l^{-1}(\Lambda - N_m cl(\Lambda - Z)) = \Omega - l^{-1}(N_m cl(\Lambda - Z)) \leq \Omega - N_m\text{-scl}(l^{-1}(\Lambda - Z)) = N_m\text{-sint}(l^{-1}(Z))$. Hence (6) is obtained.
(6) ⇒ (1) Let $\omega \in \Omega$ and M an N_m os containing $l(\omega)$. From (6) & Proposition 3.8 [11], it follows $\omega \in l^{-1}(M) = l^{-1}(N_m int(M)) \leq N_m\text{-sint}(l^{-1}(M))$. Theorem 3.4, ∃ N_m-sos U containing ω such that $\omega \in U \leq l^{-1}(M)$. Hence l is N_m-sc. □ □

Theorem 3.7. *$l : \Omega \to \Lambda$ be a map on two nms $(\Omega,\ N_{m\Omega})$ and $(\Lambda,\ N_{m\Lambda})$.*

1. *l is N_m-sc.*
2. *$l^{-1}(M) \leq N_m cl(N_m int(l^{-1}(M)))$ for each N_m os M in Λ.*
3. *$N_m int(N_m cl(l^{-1}(R))) \leq l^{-1}(R)$ for each N_m-cs R in Λ.*
4. *$l(N_m int(N_m cl(W))) \leq N_m cl(l(A))$ for $W \leq \Omega$.*
5. *$N_m int(N_m cl(l^{-1}(Z))) \leq l^{-1}(N_m cl(Z))$ for $Z \leq \Lambda$.*
6. *$l^{-1}(N_m int(Z)) \leq N_m cl(N_m int(l^{-1}(Z)))$ for $Z \leq \Lambda$.*

Proof. (1) $\Leftrightarrow$ (2) By theorem 3.6 and definition of N_m-sos.
(1) $\Leftrightarrow$ (3) By theorem 3.6 and lemma 3.1.
(3) $\Rightarrow$ (4) Let $\mathrm{W} \leq \Omega$. Then from Theorem 3.6(4) and Lemma 3.2, it follows $\mathrm{N}_m\mathrm{int}(\mathrm{N}_m\mathrm{cl}(\mathrm{W})) \leq \mathrm{N}_m\text{-scl}(\mathrm{W}))$ $\leq \mathrm{l}^{-1}(\mathrm{N}_m\mathrm{cl}(\mathrm{l}(\mathrm{W})))$. Hence $\mathrm{l}(\mathrm{N}_m\mathrm{int}(\mathrm{N}_m\mathrm{cl}(\mathrm{W}))) \leq \mathrm{N}_m\mathrm{cl}(\mathrm{l}(\mathrm{W}))$.
(4) $\Rightarrow$ (5) Obvious.
(5) $\Rightarrow$ (6) From (5) and Proposition 3.8 [11], it follows: $\mathrm{l}^{-1}(\mathrm{N}_m\mathrm{int}(\mathrm{Z})) = \mathrm{l}^{-1}(\Lambda - \mathrm{N}_m\mathrm{cl}(\Lambda - \mathrm{Z})) = \Omega$ $-\mathrm{l}^{-1}(\mathrm{N}_m\mathrm{cl}(\Lambda - \mathrm{Z})) \leq \Omega - \mathrm{N}_m\mathrm{int}(\mathrm{N}_m\mathrm{cl}(\mathrm{l}^{-1}(\Lambda - \mathrm{Z})))$
$= \mathrm{N}_m\mathrm{cl}(\mathrm{N}_m\mathrm{int}(\mathrm{l}^{-1}(\mathrm{Z})))$. Hence, (6) is obtained.
(6) $\Rightarrow$ (1) Let M be an $\mathrm{N}_m$os in Λ. Then by (6) and Proposition 3.8 [11], we have $\mathrm{l}^{-1}(\mathrm{M}) = \mathrm{l}^{-1}(\mathrm{N}_m\mathrm{int}(\mathrm{M}))$ $\leq \mathrm{N}_m\mathrm{cl}(\mathrm{N}_m\mathrm{int}(\mathrm{l}^{-1}(\mathrm{M})))$. This implies $\mathrm{l}^{-1}(\mathrm{M})$ is an N_m-sos. Hence by (2), l is N_m-semi-continuous. □

Conclusion

We presented several new notions and related properties by utilizing the concept of N_m-sos in nms.

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