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On $\mathbb{S}_{p^\star}$-open sets in ideal nano topological spaces

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Abstract: The aim of the present paper is to introduce the class of $\mathbb{S}_{p^\star}$-nI-open (strongly nano pre*-I-open) sets which is strictly placed between the class of all pre-nI-open and the class of all pre*-nI-open subsets of U. Relationships with some other types of sets were given.

Key words: $\mathbb{S}_{p^\star}$-nI-open, $pre^\star$-nI-open and pre-nI-open

1. Introduction

An ideal I [16] on a space (X, τ) is a non-empty collection of subsets of X which satisfies the following conditions.

1. $A \in I$ and $B \subset A$ imply $B \in I$ and
2. $A \in I$ and $B \in I$ imply $A \cup B \in I$.

Given a space (X, τ) with an ideal I on X if $\wp(X)$ is the set of all subsets of X, a set operator $(.)^\star : \wp(X) \to \wp(X)$, called a local function of A with respect to τ and I is defined as follows: for $A \subset X$, $A^\star(I, \tau) = \{x \in X : U \cap A \notin I \text{ for every } U \in \tau(x)\}$ where $\tau(x) = \{U \in \tau : x \in U\}$ [1]. The closure operator defined by $cl^\star(A) = A \cup A^\star(I, \tau)$ [15] is a Kuratowski closure operator which generates a topology $\tau^\star(I, \tau)$ called the $\star$-topology which is finer then τ. We will simply write $A^\star$ for $A^\star(I, \tau)$ and $\tau^\star$ for $\tau^\star(I, \tau)$. If I is an ideal on X, then (X, τ, I) is called an ideal topological space or an ideal space.

Rajasekaran et.al [10] introduced pre-nI-open sets and α-nI-open sets in the concept of ideal nano topological spaces.

The aim of the present paper is to introduce the class of $\mathbb{S}_{p^\star}$-nI-open (strongly nano pre*-I-open) sets which is strictly placed between the class of all pre-nI-open and the class of all pre*-nI-open subsets of U. Relationships with some other types of sets were given.

2. Preliminaries

Definition 2.1. [8] Let U be a non-empty finite set of objects called the universe and R be an equivalence

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relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$.

1. The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X) = \bigcup_{x \in U}\{R(x) : R(x) \subseteq X\}$, where R(x) denotes the equivalence class determined by x.
2. The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is, $U_R(X) = \bigcup_{x \in U}\{R(x) : R(x) \cap X \neq \phi\}$.
3. The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not - X with respect to R and it is denoted by $B_R(X)$. That is, $B_R(X) = U_R(X) - L_R(X)$.

Definition 2.2. [2] Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then R(X) satisfies the following axioms:

1. U and $\phi \in \tau_R(X)$,
2. The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$,
3. The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

Thus $\tau_R(X)$ is a topology on U called the nano topology with respect to X and $(U, \tau_R(X))$ is called the nano topological space. The elements of $\tau_R(X)$ are called nano-open sets (briefly n-open sets). The complement of a n-open set is called n-closed.

In the rest of the paper, we denote a nano topological space by $(U, \mathcal{N})$, where $\mathcal{N} = \tau_R(X)$. The nano-interior and nano-closure of a subset O of U are denoted by $I_n(O)$ and $C_n(O)$, respectively.

A nano topological space $(U, \mathcal{N})$ with an ideal I on U is called [5] an ideal nano topological space and is denoted by $(U, \mathcal{N}, I)$. $G_n(x) = \{G_n \mid x \in G_n, G_n \in \mathcal{N}\}$, denotes [5] the family of nano open sets containing x.

In future an ideal nano topological spaces $(U, \mathcal{N}, I)$ is referred as a space.

Definition 2.3. [5] Let $(U, \mathcal{N}, I)$ be a space with an ideal I on U. Let $(.)^\star_n$ be a set operator from $\wp(U)$ to $\wp(U)$ ($\wp(U)$ is the set of all subsets of U).

For a subset $O \subseteq U$, $O^\star_n(I, \mathcal{N}) = \{x \in U : G_n \cap O \notin I$, for every $G_n \in G_n(x)\}$ is called the nano local function (briefly, n-local function) of A with respect to I and $\mathcal{N}$. We will simply write $O^\star_n$ for $O^\star_n(I, \mathcal{N})$.

Theorem 2.1. *[5] Let $(U, \mathcal{N}, I)$ be a space and O and B be subsets of U. Then*

1. $O \subseteq B \Rightarrow O^\star_n \subseteq B^\star_n$,
2. $O^\star_n = C_n(O^\star_n) \subseteq C_n(O)$ *($O^\star_n$ is a n-closed subset of $C_n(O)$)*,
3. $(O^\star_n)^\star_n \subseteq O^\star_n$,
4. $(O \cup B)^\star_n = O^\star_n \cup B^\star_n$,
5. $V \in \mathcal{N} \Rightarrow V \cap O^\star_n = V \cap (V \cap O)^\star_n \subseteq (V \cap O)^\star_n$,
6. $J \in I \Rightarrow (O \cup J)^\star_n = O^\star_n = (O - J)^\star_n$.

Theorem 2.2. *[5] Let $(U,\mathcal{N},I)$ be a space with an ideal I and $O \subseteq O_n^\star$, then $O_n^\star = C_n(O_n^\star) = C_n(O)$.*

Definition 2.4. [7] A subset A of a space $(U,\mathcal{N},I)$ is $n\star$-dense in itself (resp. $n\star$-perfect and $n\star$-closed) if $O \subseteq O_n^\star$ (resp. $O = O_n^\star$, $O_n^\star \subseteq O$).

The complement of a $n\star$-closed set is said to be $n\star$-open.

Definition 2.5. [3] A subset O of U in a nano topological space $(U,\mathcal{N})$ is called nano-codense (briefly n-codense) if $U - O$ is n-dense.

Theorem 2.3. *[5] Let $(U,\mathcal{N},I)$ be an ideal nano space. Then is $\mathcal{I}$ is n-codense $\iff$ $O \subseteq O^\star$ for every n-open set O.*

Definition 2.6. [5] Let $(U,\mathcal{N},I)$ be a space. The set operator $C_n^\star$ called a nano $\star$-closure is defined by $C_n^\star(O) = O \cup O_n^\star$ for $O \subseteq U$.

It can be easily observed that $C_n^\star(O) \subseteq C_n(O)$.

Theorem 2.4. *[6] In a space $(U,\mathcal{N},I)$, if O and B are subsets of U, then the following results are true for the set operator n-$cl^\star$.*

1. $O \subseteq C_n^\star(O)$,
2. $C_n^\star(\phi) = \phi$ *and* $C_n^\star(U) = U$,
3. $IfO \subset B$, *then* $C_n^\star(O) \subseteq C_n^\star(B)$,
4. $C_n^\star(O) \cup C_n^\star(B) = C_n^\star(O \cup B)$.
5. $C_n^\star(C_n^\star(O)) = C_n^\star(O)$.

Definition 2.7. [2] A subset O of a nano space $(U,\mathcal{N})$, is called a

1. nano pre-open (resp. np-open) set [2] if $O \subseteq I_n(C_n(O))$.
2. nano semi-open (resp. ns-open) set [2] if $O \subseteq C_n(I_n(O))$.
3. nano β-open (resp. $n\beta$-open) set [14] if $O \subseteq C_n(I_n(C_n(O)))$.

Definition 2.8. A subset O of an ideal nano space $(U,\mathcal{N},I)$, is called a

1. nano-I-open (resp. nI-open) [6] if $O \subseteq I_n(O_n^\star)$.
2. nano pre-I-open (resp. pre-nI-open) [10] if $O \subseteq I_n(C_n^\star(O))$.
3. nano α-I-open (resp. α-nI-open) [10] if $O \subseteq I_n(C_n^\star(I_n(O)))$.
4. nano b-I-open (resp. b-nI-open) if [10] is $O \subseteq I_n(C_n^\star(O) \cup C_n^\star(I_n(O))$.
5. nano pre*-I-open (resp. $pre^\star$-nI-open) [4] if $O \subseteq I_n^\star(C_n(O))$.
6. strongly nano β-I-open (resp. $\mathcal{S}\beta$-nI-open) [11] if $O \subseteq C_n^\star((I_n(C_n^\star(O)))$.
7. almost strong nano I-open (resp. almost $\mathcal{S}nI$-open) [11] if $O \subseteq C_n^\star(I_n(O_n^\star))$.
8. nano $t^\#$-I-set (resp. $t^\#$-nI-set) [12] if $I_n(O) = C_n^\star(I_n(O))$.
9. nano pre-nI-regular [12] if O is pre-nI-open and $t^\#$-nI-set.
10. weakly nano semi-I-open (resp. $\mathbb{W}_s$-nI-open) set [13] if $O \subseteq C_n^\star(I_n(C_n(O)))$.

3. On strongly pre*-open sets in ideal nano space

Definition 3.1. A subset O of an ideal nano space $(U, \mathcal{N}, I)$, is called a

1. strongly nano pre*-I-open (resp. $\mathbb{S}_{p^\star}$-nI-open) if $O \subseteq I_n^\star(C_n^\star(O))$.
2. strongly nano pre*-I-closed (resp. $\mathbb{S}_{p^\star}$-nI-closed) if its complement is strongly nano pre*-nI-open.
3. nano $\beta^\star$-I-open (resp. $\beta^\star$-nI-open) set if $O \subseteq C_n(I_n^\star(C_n(O)))$.
4. strongly semi*-I-open (resp. $\mathbb{S}_{s^\star}$-nI-open) if $O \subseteq C_n^\star(I_n^\star(O))$.

Example 3.1. *Let $U = \{o_1, o_2, o_3, o_4\}$ with $U/R = \{\{o_2\}, \{o_4\}, \{o_1, o_3\}\}$ and $X = \{o_3, o_4\}$. Then the nano topology $\mathcal{N} = \{\phi, \{o_4\}, \{o_1, o_3\}, \{o_1, o_3, o_4\}, U\}$ and $I = \{\phi, \{o_3\}\}$. Clearly the set $\{\phi, \{o_2\}, \{o_3\}, \{o_2, o_3\}, U\}$ is $\mathbb{S}_{p^\star}$-nI-open.*

Remark 3.1. *Let $(U, \mathcal{N}, I)$ be an ideal nonotopological space,*

1. *If O is pre-nI-open set, then O is $\mathbb{S}_{p^\star}$-nI-open.*
2. *If O is $\mathbb{S}_{p^\star}$-nI-open set, then O is a pre*-nI-open.*
3. *$\mathbb{S}_{p^\star}$-nI-open sets and b-nI-open sets are independent.*
4. *$\mathbb{S}_{p^\star}$-nI-open sets and np-open sets are independent.*

Example 3.2. *In an Example 3.1,*

1. *the set $\{o_2\}$ is $\mathbb{S}_{p^\star}$-nI-open but not pre-nI-open set.*
2. *the set $\{o_2, o_3, o_4\}$ is pre*-nI-open but not $\mathbb{S}_{p^\star}$-nI-open.*
3. *the set $\{o_2\}$ is $\mathbb{S}_{p^\star}$-nI-open but not b-nI-open.*
4. *the set $\{o_1, o_3\}$ is b-nI-open but not $\mathbb{S}_{p^\star}$-nI-open.*
5. *the set $\{o_2\}$ is $\mathbb{S}_{p^\star}$-nI-open but not np-open.*
6. *the set $\{o_4\}$ is np-open but not $\mathbb{S}_{p^\star}$-nI-open.*

Theorem 3.1. *Let $(U, \mathcal{N}, I)$ be an ideal nano topological space then, O is $\mathbb{S}_{p^\star}$-nI-open $\iff$ there exists $\mathbb{S}_{p^\star}$-nI-open K such that $O \subseteq K \subseteq C_n^\star(O)$.*

Proof.

Let O be a $\mathbb{S}_{p^\star}$-nI-open, then $O \subseteq I_n^\star(C_n^\star(O))$. We put $K = I_n^\star(C_n^\star(O))$, which is a $n\star$-open set. Therefore $K = I_n^\star(K) \subseteq I_n^\star(C_n^\star(K))$ be a $\mathbb{S}_{p^\star}$-nI-open set Such that $O \subseteq K = I_n^\star(C_n^\star(K)) \subseteq C_n^\star(O)$.

Conversely, if K is $\mathbb{S}_{p^\star}$-nI-open set such that $O \subseteq K \subseteq C_n^\star(O)$, taking $n\star$-closure, then $C_n^\star(O) \subseteq C_n^\star(K)$.

On the other hand $O \subseteq K \subseteq I_n^\star(C_n^\star(K)) \subseteq I_n^\star(C_n^\star(O))$.

Hence O is $\mathbb{S}_{p^\star}$-nI-open. □

Corollary 3.1. *Let $(U, \mathcal{N}, I)$ be an ideal nano topological space, then O is a $\mathbb{S}_{p^\star}$-nI-open set $\iff$ there exists an n-open set $O \subseteq K \subseteq C_n^\star(O)$.*

Proof.

Obvious. □

Proposition 3.1. *Let $(U,\mathcal{N},I)$ be an ideal nano topological space.*

1. *If O is $\mathbb{S}_{p^\star}$-nI-open set, then $C_n^\star(O)$ is a $\mathbb{S}_{s^\star}$-nI-open set.*
2. *If O is $\mathbb{S}_{s^\star}$-nI-open, then $I_n^\star(O)$ is $\mathbb{S}_{p^\star}$-nI-open set.*

Proof.

1. Let O be $\mathbb{S}_{p^\star}$-nI-open. Then $O \subseteq I_n^\star(C_n^\star(O))$ and $C_n^\star(O) \subseteq C_n^\star(I_n^\star(C_n^\star(O)))$. This implies $C_n^\star(O)$ is a $\mathbb{S}_{s^\star}$-nI-open.
2. Let O be $\mathbb{S}_{s^\star}$-nI-open, then $O \subseteq C_n^\star(I_n^\star(O))) \Longrightarrow I_n^\star(O) \subseteq I_n^\star(C_n^\star(I_n^\star(O)))$. This implies $I_n^\star(O)$ is $\mathbb{S}_{p^\star}$-nI-open.

□

Theorem 3.2. *Let $(U,\mathcal{N},I)$ be an ideal nano topological space, where I is n-codense then the following hold:*

1. *If O $\mathbb{S}_{p^\star}$-nI-open, then O is a strong β-nI-open.*
2. *If O is $\mathbb{S}_{p^\star}$-nI-open, then O is a $n\beta$-open.*
3. *If O is $\mathbb{S}_{p^\star}$-nI-open, then O is a weakly semi-nI-open.*
4. *If O is $\mathbb{S}_{p^\star}$-nI-open, then O is a np-open.*

Proof.

It is obvious.

□

Remark 3.2. *The reverse of the Theorem 3.2 is not true in general as shown in the following Examples.*

Example 3.3. *In an Example 3.1*

1. *the set $\{o_1\}$ is strong β-nI-open but not $\mathbb{S}_{p^\star}$-nI-open.*
2. *the set $\{o_1, o_2, o_3\}$ is $n\beta$-open but not $\mathbb{S}_{p^\star}$-nI-open.*
3. *the set $\{o_1, o_2, o_4\}$ is weakly semi-nI-open but not $\mathbb{S}_{p^\star}$-nI-open.*

Theorem 3.3. *Let $(U,\mathcal{N},I)$ be an ideal nano topological space, such that every n-open set is $n\star$-closed, then every strong β-nI-open set is $\mathbb{S}_{p^\star}$-nI-open.*

Proof.

Let O is a strong β-nI-open, then $O \subseteq C_n^\star(I_n(C_n^\star(O)))$. Since $I_n(C_n^\star(O))$ is n-open, by hypothesis $I_n(C_n^\star(O)) = C_n^\star(I_n(C_n^\star(O)))$. So $O \subseteq C_n^\star(I_n(C_n^\star(O))) = I_n(C_n^\star(O) \subseteq I_n^\star(C_n^\star(O))$. Hence O is $\mathbb{S}_{p^\star}$-nI-open. □

Theorem 3.4. *Let $(U,\mathcal{N},I)$ be an ideal nano topological space. If O is $n\star$-perfect, then the following hold:*

1. *If O is $\mathbb{S}_{p^\star}$-nI-open, then O is almost strong-nI-open.*

2. *O is a $\mathbb{S}_{p^\star}$-nI-open $\iff$ it is nI-open set.*

Proof.

1. Let O is $\mathbb{S}_{p^\star}$-nI-open , then $O \subseteq I_n^\star(C_n^\star(A)) = I_n(C_n^\star(O)) \subseteq C_n^\star(I_n(C_n^\star(O))) = C_n^\star(I_n(O_n^\star))$. This implies O is almost strong-nI-open.
2. Let O is a $\mathbb{S}_{p^\star}$-nI-open, then $o \subseteq I_n^\star(C_n^\star(O)) \subseteq I_n^\star(C_n(O)) = I_n(O_n^\star)$. Hence O is nI-open.
   Conversely, if O is nI-open, then $O \subseteq I_n(O_n^\star) \subseteq I_n^\star(O_n^\star) = I_n^\star(C_n^\star(O))$. Hence O is $\mathbb{S}_{p^\star}$-nI-open.

□

Corollary 3.2. *Let $(U, \mathcal{N}, I)$ be an ideal nano topological space, If O is $n\star$-perfect, then every pre*-nI-open set is $\mathbb{S}_{p^\star}$-nI-open.*

Proof.

Let O is pre*-nI-open set, since it is $n\star$-perfect, then $O \subseteq I_n^\star(C_n(O)) = I_n^\star(C_n^\star(O))$. Hence O is $\mathbb{S}_{p^\star}$-nI-open. □

Proposition 3.2. *In an ideal nano space, every nI-open set is $\mathbb{S}_{p^\star}$-nI-open.*

Proof.

If O is nI-open, then $O \subseteq int(O_n^\star) \subseteq int(O_n^\star \cup O) \subseteq I_n^\star(C_n^\star(A))$. Hence O is $\mathbb{S}_{p^\star}$-nI-open. □

Theorem 3.5. *Let $(U, \mathcal{N}, I)$ be an ideal nano topological space, where I is n-codense, then the following are equivalent:*

1. *O is pre*-nI-open.*
2. *O is $\mathbb{S}_{p^\star}$-nI-open.*

Proof.

It is obvious. □

Theorem 3.6. *Let $(U, \mathcal{N}, I)$ be an ideal nano topological space and $O \subseteq U$ be a np-open and ns-closed. Then O is $\mathbb{S}_{p^\star}$-nI-open.*

Proof.

Let O is np-open, then $O \subseteq I_n(C_n(O))$. Since O is ns-closed then $I_n(C_n(O)) = I_n(O)$, now $O \subseteq I_n(O) \subseteq I_n^\star(C_n^\star(O))$. Hence O is $\mathbb{S}_{p^\star}$-nI-open.

□

Theorem 3.7. *Let $(U, \mathcal{N}, I)$ be an ideal nano topological space and $O \subseteq U$ be $\mathbb{S}_{p^\star}$-nI-open and $n\star$-closed. Then O is $\mathbb{S}_{s^\star}$-nI-open.*

Proof.

Let O is $\mathbb{S}_{p^\star}$-nI-open, then $O \subseteq I_n^\star(C_n^\star(O))$. Since $O$is $n\star$-closed then $I_n^\star(C_n^\star(O)) = I_n^\star(O)$. Now $O \subseteq I_n^\star(O) \subseteq C_n^\star(I_n^\star(O))$. Hence O is $\mathbb{S}_{s^\star}$-nI-open.

□

Theorem 3.8. *Let $(U,\mathcal{N},I)$ be an ideal nano topological space, and $U \subseteq U$, then the followings hold:*

1. *O is $\mathbb{S}_{p^\star}$-nI-open set, if O is both weakly semi-nI-open and $\mathbb{S}_{s^\star}$-nI-set.*
2. *O is $\mathbb{S}_{p^\star}$-nI-open set, if O is both semi-nI-open set and $t^{\#}$-nI-set.*

Proof.

1. Let O is weakly semi-nI-open set, then $O \subseteq C_n^\star(I_n(C_n(O)))$. Since O is $\mathbb{S}_{s^\star}$-nI-set then, $I_n(O) = C_n^\star(I_n(C_n(O)))$. Now $O \subseteq I_n(O) \subseteq I_n^\star(C_n^\star(O))$. Hence O is $\mathbb{S}_{p^\star}$-nI-open.
2. Let O is semi-nI-open set, then $O \subseteq C_n^\star(I_n(O))$. Since O is $t^{\#}$-nI-set then, $I_n(O) = C_n^\star(I_n(O))$. Now $O \subseteq I_n(O) \subseteq I_n^\star(C_n^\star(O))$. Hence O is $\mathbb{S}_{p^\star}$-nI-open.

□

Theorem 3.9. *Let $(U,\mathcal{N},I)$ be an ideal nano topological space. O is a $\mathbb{S}_{p^\star}$-nI-open set if O is both pre*-nI-open and n-closed.*

Proof.

Let O is pre*-nI-open set, then $O \subseteq I_n^\star(C_n(O))$. Since O is n-closed set, then $O \subseteq I_n^\star(C_n(O)) = I_n^\star(O) \subseteq I_n^\star(C_n^\star(O))$. Hence O is $\mathbb{S}_{p^\star}$-nI-open. □

Theorem 3.10. *Let $(U,\mathcal{N},I)$ be an ideal nano topological space, $O, K \subseteq U$.*

1. *If O is $\mathbb{S}_{p^\star}$-nI-open and K is np-open set, then $O \cup K$ is pre*-nI-open.*
2. *If O is $\mathbb{S}_{p^\star}$-nI-open and K is a weakly semi-nI-open set, then $O \cup K$ is $\beta^\star$-nI-open.*

Proof.

1. Let O is $\mathbb{S}_{p^\star}$-nI-open then $O \subseteq I_n^\star(C_n^\star(O))$, and K is a np-open then $K \subseteq I_n(C_n(K))$.
   Now : $O \cup K \subseteq I_n^\star(C_n^\star(O)) \cup I_n(C_n(K)) \subseteq I_n^\star(C_n(O)) \cup I_n^\star(C_n(K)) \subseteq I_n^\star(C_n(O \cup K))$. Hence $O \cup K$ is a pre*-nI-open set.
2. Let O is $\mathbb{S}_{p^\star}$-nI-open, then $O \subseteq I_n^\star(C_n^\star(O))$, K is weakly semi-nI-open then $K \subseteq C_n^\star(I_n(C_n(K)))$
   Now : $O \cup K \subseteq I_n^\star(C_n^\star(O)) \cup C_n^\star(I_n(C_n(K))) \subseteq C_n(I_n^\star(C_n(O))) \cup C_n(I_n^\star(C_n(K))) = C_n(I_n^\star(C_n(O)) \cup I_n^\star(C_n(K))) \subseteq C_n(I_n^\star(C_n(O \cup K)))$. Hence $O \cup K$ is a $\beta^\star$-nI-open set.

□

Theorem 3.11. *Let $(U,\mathcal{N},I)$ be an ideal nano topological space then the following conditions are equivalent.*

1. *O is $\mathbb{S}_{s^\star}$-nI-open and $\mathbb{S}_{p^\star}$-nI-open.*
2. *I is n-codense then O is α-nI-open.*

Proof.

$(1) \Longrightarrow (2)$: Let O is $\mathbb{S}_{s^\star}$-nI-open and $\mathbb{S}_{p^\star}$-nI-open,

we have $O \subseteq I_n^\star(C_n^\star(O)) \subseteq I_n^\star(C_n^\star(C_n^\star(I_n^\star(O)))) = I_n^\star(C_n^\star(I_n^\star(O))) = I_n(C_n^\star(I_n(O)))$. Hence O is α-nI-open.

$(2) \Longrightarrow (1)$: It is obvious.

□

Theorem 3.12. *A subset O of a space $(U, \mathcal{N}, I)$ is said to be an $\mathbb{S}_{p^\star}$-nI-closed $\Longleftrightarrow$ $C_n^\star(I_n^\star(O)) \subseteq O$.*

Proof.

Let O be $\mathbb{S}_{p^\star}$-nI-closed, then $(U - O)$ is a $\mathbb{S}_{p^\star}$-nI-open and hence $(U - O) \subseteq I_n^\star(C_n^\star(U - O)) = U - C_n^\star(I_n^\star(O))$. Therefore, we obtain $C_n^\star(I_n^\star(O)) \subseteq O$.

Conversely, let $C_n^\star(I_n^\star(O)) \subseteq O$, then $(U - O) \subseteq I_n^\star(C_n^\star(U - O))$ and hence $(U - O)$ is $\mathbb{S}_{p^\star}$-nI-open. Therefore, O is $\mathbb{S}_{p^\star}$-nI-closed.

□

Theorem 3.13. *Let $(U, \mathcal{N}, I)$ be an ideal nano topological space, if I is n-codense, then O is an $\mathbb{S}_{p^\star}$-nI-closed $\Longleftrightarrow$ $C_n^\star(I_n^\star(O)) \subseteq O$.*

Proof.

Let O be a $\mathbb{S}_{p^\star}$-nI-closed set of U, then $O \supseteq C_n^\star(I_n^\star(O)) = C_n^\star(I_n^\star(O))$.

Conversely, let O be any subset of U, such that $O \supseteq C_n^\star(I_n^\star(O))$. This implies that $O \supseteq C_n^\star(I_n^\star(O))$, i.e., O is $\mathbb{S}_{p^\star}$-nI-closed. □

Theorem 3.14. *A subset O of a space $(U, \mathcal{N}, I)$ is said to be $\mathbb{S}_{p^\star}$-nI-closed $\Longleftrightarrow$ there exists a $\mathbb{S}_{p^\star}$-nI-closed set K such that $I_n^\star(O) \subseteq K \subseteq O$.*

Proof.

Let O be an $\mathbb{S}_{p^\star}$-nI-closed, then $C_n^\star(I_n^\star(O)) \subseteq O$. We put $K = C_n^\star(I_n^\star(O))$ be a $n\star$-closed set. i.e, K is $\mathbb{S}_{p^\star}$-nI-closed and $I_n^\star(O) \subseteq C_n^\star(I_n^\star(O)) = K \subseteq A$.

Conversely, if K is $\mathbb{S}_{p^\star}$-nI-closed set such that $I_n^\star(O) \subseteq K \subseteq O$, then $I_n^\star(O) = I_n^\star(K)$.

On the other hand, $C_n^\star(I_n^\star(K) \subseteq K$ and hence $O \supseteq K \supseteq C_n^\star(I_n^\star(K)) = C_n^\star(I_n^\star(O)$. Thus $O \supseteq C_n^\star(I_n^\star(O))$. Hence O is $\mathbb{S}_{p^\star}$-nI-closed.

□

Corollary 3.3. *A subset O of a space $(U, \mathcal{N}, I)$ is $\mathbb{S}_{p^\star}$-nI-closed set $\Longleftrightarrow$ there exists a $n\star$-closed set B such that $I_n^\star(O) \subseteq K \subseteq O$.*

Theorem 3.15. *Let $(U, \mathcal{N}, I)$ be an ideal nano topological space, $O, K \subseteq U$. Then $O \cap K$ is a $\mathbb{S}_{p^\star}$-nI-closed set, if O is $\mathbb{S}_{p^\star}$-nI-closed and K is np-closed set.*

Proof.

It is proved similarly by Theorem 3.10(1). □

Theorem 3.16. *Let $(U, \mathcal{N}, I)$ be an ideal nano topological space, then each pre-nI-regular set in U is $\mathbb{S}_{p^\star}$-nI-open and $\mathbb{S}_{p^\star}$-nI-closed set.*

Proof.

It follows from the fact that every pre-nI-regular set is pre-nI-open and pre-nI-closed. This implies that it is $\mathbb{S}_{p^\star}$-nI-open and $\mathbb{S}_{p^\star}$-nI-closed. □

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