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An Overview on μ_N Strongly Nowhere Dense Sets

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Abstract: In this article we have introduced some new types of sets such as μ_N strongly dense, μ_N strongly nowhere dense, μ_N strongly first category sets, μ_N strongly nowhere residual sets and their attributes are explained briefly. Also by making use of these we have retrieved μ_N strongly Baire space and its properties are to be described.

Key words: μ_N strongly dense, μ_N strongly nowhere dense, μ_N strongly first category sets, μ_N strongly nowhere residual sets

1. Introduction

Zadeh's concept of fuzziness has a huge impact on all fields of mathematics. C.L.Chang[3] later combined the ideas of fuzziness with topological spaces, laying the groundwork for the theory of fuzzy topological spaces. K.T.Attanasov[1] discovered intuitionistic fuzzy sets, and with his friend Stoeva[2], he expanded his research to reveal a generalisation to intuitionistic L-fuzzy sets. F.Smarandache[7] directed his attention to the degree of indeterminacy and proposed the neutrosophic sets. Following that, A.A.Salama and Albowi[13] discovered the neutrosophic topological spaces using neutrosophic sets. We[12] created Generalized topological spaces via neutrosophic sets using all of the works as inspiration and named it as TS. In μ_N TS the concept of Baire space was putforth by us and here we extended our research ideas into the strong natures of μ_N Baire space.

2. Necessities

Definition 2.1. [14] Let X be a non-empty fixed set. A Neutrosophic set [NS for short] A is an object having the form $A = \{< x, \mu_A(x), \sigma_A(x), \gamma_A(x) >: x \in X\}$ where $\mu_A(x)$, $\sigma_A(x)$ and $\gamma_A(x)$ which represents the degree of membership function, the degree of indeterminacy and the degree of non-membership function respectively of each element $x \in X$ to the set A.

Remark 2.1. *[14] Every intuitionistic fuzzy set A is a non empty set in X is obviously on Neutrosophic sets having the form $A = \{< \mu_A(x), 1 - (\mu_A(x) + \sigma_A(x)), \gamma_A(x) >: x \in X\}$. Since our main purpose is to construct the tools for developing Neutrosophic Set and Neutrosophic topology , we must introduce the neutrosophic sets 0_N and 1_N in X as follows: 0_N may be defined as follows*
(0_1) $0_N = \{< x, 0, 0, 1 >: x \in X$

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1_N *may be defined as follows*
$(\ 1_1\ )\ 1_N\ =\ \{<\ x,\ 1,\ 0,\ 0\ >:\ x\ \in\ X$

Definition 2.2. [14] Let $A\ =\ \{<\ \mu_A,\ \sigma_A,\ \gamma_A\ >\}$ be a NS on X, then the complement of the set A [$C\ (\ A\ )$ for short] may be defined as three kinds of complements :
$(\ C_1\ )\ C\ (\ A\ )\ =\ A\ =\ \{<\ x,\ 1-\mu_A\ (\ x\ ),\ 1-\sigma_A\ (\ x\ ),\ 1-\gamma_A\ (\ x\ )\ >:\ x\ \in\ X\}$

Definition 2.3. [14] Let X be a non-empty set and neutrosophic sets A and B in the form $A\ =\ \{<\ x,\ \mu_A\ (\ x\ ),$ $\sigma_A\ (\ x\ ),\ \gamma_A\ (\ x\ )\ >:\ x\ \in\ X\}$ and $B\ =\ \{<\ x, \mu_B\ (\ x\ ),\ \sigma_B\ (\ x\ ),\ \gamma_B\ (\ x\ )\ >:\ x\ \in\ X\}$. Then we may consider two possibilities for definitions for subsets $(\ A\ \subseteq\ B\ )$.
$A\ \subseteq\ B$ may be defined as :
$(A\ \subseteq\ B)\ \iff\ \mu_A\ (\ x\ ) \leq \mu_B\ (\ x\ ),\ \sigma_A\ (\ x\ ) \leq \sigma_B\ (\ x\ ),\ \gamma_A\ (\ x\ ) \geq \gamma_B\ (\ x\ )\ \forall\ x\ \in\ X$

Proposition 2.1. *[14] For any neutrosophic set A, the following conditions holds:*
$0_N\ \subseteq\ A,\ 0_N\ \subseteq\ 0_N$
$A\ \subseteq\ 1_N,\ 1_N\ \subseteq\ 1_N$

Definition 2.4. [14] Let X be a non empty set and $A\ =\ \{<\ x,\ \mu_A\ (\ x\ ),\ \sigma_A\ (\ x\ ),$ $\gamma_A\ (\ x\ )\ >:\ x\ \in\ X\}$ $B\ =\ \{<\ x,\ \mu_B\ (\ x\ ),\ \sigma_B\ (\ x\ ),\ \gamma_B\ (\ x\ )\ >:\ x\ \in\ X\}$ are NSs. Then $A\ \cap\ B$ may be defined as :
$(\ I_1\ )\ A\ \cap\ B\ =<\ x,\ \mu_A\ (\ x\ )\ \wedge\ \mu_B\ (\ x\ ),\ \sigma_A\ (\ x\ )\ \wedge\ \sigma_B\ (\ x\ ),\ \gamma_A\ (\ x\ )\ \vee\ \gamma_B\ (\ x\ )\ >$
$A\ \cup\ B$ may be defined as :
$(\ I_1\ )\ A\ \cup\ B\ =<\ x,\ \mu_A\ (\ x\ )\ \vee\ \mu_B\ (\ x\ ),\ \sigma_A\ (\ x\ )\ \vee\ \sigma_B\ (\ x\ ),\ \gamma_A\ (\ x\ )\ \wedge\ \gamma_B\ (\ x\ )\ >$

Definition 2.5. [12] A μ_N topology is a non - empty set X is a family of neutrosophic subsets in X satisfying the following axioms:
$(\mu_{N_1})0_N \in \mu_N$
(μ_{N_2}) union of any number of μ_N open set is μ_N open.

Remark 2.2. *[12] The elements of μ_N are μ_N-open sets and their complement is called μ_N closed sets.*

Definition 2.6. [12] The μ_N- Closure of A is the intersection of all μ_N closed sets containing A.

Definition 2.7. [12] The μ_N- Interior of A is the union of all μ_N open sets contained in A.

Definition 2.8. [13] A neutrosophic set A in NTS is called neutrosophic dense if there exists no neutrosophic closed sets B in (X, T) such that $A \subset B \subset 1_N$.

Definition 2.9. [13] The μ_N Topological spaces is said to be μ_N Baire's Space if $\mu_N Int($ $\cup_{i=1}^{\infty} G_i) = 0_N$ where G_i's are μ_N nowhere dense set in (X, μ_N).

Proposition 2.2. *[13] Let (X, μ_N) be a μ_N TS. Then the following are equivalent.*

1. *(X, μ_N) is μ_N Baire's Space.*
2. *$\mu_N Int(A) = 0_N$, for all μ_N first category set in (X, μ_N).*
3. *$\mu_N Cl(A) = 1_N$, for every μ_N Residual set in (X, μ_N).*

3. μ_N Strongly Nowhere Dense sets:

Definition 3.1. Let (X,μ_N) be a μ_N Topological Space. A neutrosophic sets ζ defined on (X,μ_N) is called μ_N strongly nowhere dense set in (X,μ_N) if $\zeta \wedge \bar{\zeta}$ is a μ_N nowhere dense set in (X,μ_N). That is, ζ is a μ_N strongly nowhere dense set in (X,μ_N) if $\mu_N Int(\mu_N Cl(\zeta \cap \bar{\zeta})) = 0_N$ in (X,μ_N).

Proposition 3.1. *If ζ is a μ_N nowhere dense set in a μ_N Topological Space (X,μ_N), then ζ is a μ_N strongly nowhere dense set in (X,μ_N)*

Proof. Let ζ be a μ_N nowhere dense set in a μ_N Topological Space in (X,μ_N), then $\mu_N Int(\mu_N Cl(\zeta)) = 0_N$ in (X,μ_N). Since $\zeta \wedge \bar{\zeta} \subseteq \zeta$ in (X,μ_N). We obtain that $\mu_N Int(\mu_N Cl(\zeta \cap \bar{\zeta})) \subseteq \mu_N Int(\mu_N Cl(\zeta))$ and hence $\mu_N Int(\mu_N Cl(\zeta \cap \bar{\zeta})) \subseteq 0_N$. That is, $\mu_N Int(\mu_N Cl(\zeta \cap \bar{\zeta})) = 0_N$. Hence ζ is a μ_N strongly nowhere dense set in (X,μ_N). □

Proposition 3.2. *If $\bar{\zeta}$ is a μ_N nowhere dense set in a μ_N Topological Space in (X,μ_N), then ζ is a μ_N strongly nowhere dense set in (X,μ_N).*

Proof. Suppose that $\bar{\zeta}$ is a μ_N nowhere dense set in a μ_N Topological Space in (X,μ_N), then $\mu_N Int(\mu_N Cl(\bar{\zeta})) = 0_N$ in (X,μ_N). Since $\zeta \cap \bar{\zeta} \subseteq \bar{\zeta}$, $\mu_N Int(\mu_N Cl(\zeta \cap \bar{\zeta})) \subseteq \mu_N Int(\mu_N Cl(\bar{\zeta}))$ and hence $\mu_N Int(\mu_N Cl(\zeta \cap \bar{\zeta})) \subseteq 0_N$ that implies us $\mu_N Int(\mu_N Cl(\zeta \cap \bar{\zeta})) = 0_N$. Hence ζ is a μ_N strongly nowhere dense set in (X,μ_N). □

Proposition 3.3. *If $\mu_N Cl(\mu_N Int\bar{\zeta}) = 1_N$, for a neutrosophic set ζ defined on (X,μ_N), then ζ is a μ_N strongly nowhere dense set in (X,μ_N).*

Proof. Suppose that $\mu_N Cl(\mu_N Int\bar{\zeta}) = 1_N$ in (X,μ_N). Then we deduce that $\overline{\mu_N Cl(\mu_N Int\bar{\zeta})} = 0_N$ which implies us $\overline{\overline{\mu_N Int(\mu_N Cl\zeta)}} = 0_N$. We obtain $\mu_N Int(\mu_N Cl\zeta) = 0_N$. Thus, ζ is a μ_N nowhere dense set in (X,μ_N). By using proposition 3.2, ζ is a μ_N strongly nowhere dense set in (X,μ_N). □

Proposition 3.4. *If ζ is a μ_N strongly nowhere dense set in (X,μ_N), then $\bar{\zeta}$ is also a μ_N strongly nowhere dense set.*

Proof. Let ζ be a μ_N strongly nowhere dense set in (X,μ_N) which entails us that $\mu_N Int(\mu_N Cl(\zeta \cap \bar{\zeta})) = 0_N$ in (X,μ_N). Now $\mu_N Int(\mu_N Cl(\bar{\zeta} \cap \bar{\bar{\zeta}})) = \mu_N Int(\mu_N Cl(\bar{\zeta} \cap \zeta)) = 0_N$. This implies us $\bar{\zeta}$ is also a μ_N strongly nowhere dense set in (X,μ_N). □

Proposition 3.5. *If ζ is a μ_N nowhere dense set in μ_N Topological space then $\bar{\zeta}$ is μ_N strongly nowhere dense set in (X,μ_N).*

Proof. Let ζ be a μ_N nowhere dense set in (X,μ_N). Now by using Proposition 3.2 we get ζ is a μ_N strongly nowhere dense set in (X,μ_N) and by proposition 3.4 we obtain that $\bar{\zeta}$ is μ_N strongly nowhere dense set in (X,μ_N). □

Proposition 3.6. *If ζ is a μ_N strongly nowhere dense set in (X,μ_N), then $\mu_N Cl(\zeta \cup \bar{\zeta}) = 1_N$.*

Proof. Let ζ be a μ_N strongly nowhere dense set in (X, μ_N) then we obtain that $\mu_N Cl(\zeta \cup \bar{\zeta}) = 0_N \Rightarrow \mu_N Cl(\overline{\mu_N Int(\zeta \cap \bar{\zeta})}) = 1_N \Rightarrow \mu_N Cl(\mu_N Int(\overline{\zeta \cap \bar{\zeta}})) = 1_N$. But $\mu_N Cl(\mu_N Int(\bar{\zeta} \cup \zeta)) \subseteq \mu_N Cl(\bar{\zeta} \cup \zeta) \Rightarrow 1_N \subseteq \mu_N Cl(\bar{\zeta} \cup \zeta)$. Hence we get $\mu_N Cl(\zeta \cup \bar{\zeta}) = 1_N$. □

Proposition 3.7. *If $\mu_N Int\mathbf{P}$ is a μ_N dense set, for a neutrosophic set $\mathbf{P}$ defined on a μ_N TS (X, μ_N) then $\mathbf{P}$ is μ_N strongly nowhere dense set in (X, μ_N).*

Proof. Suppose that $\mu_N Int\mathbf{P}$ is a μ_N dense set in (X, μ_N), then $\mu_N Cl(\mu_N Int\mathbf{P}) = 1_N$ in (X, μ_N). Then $\overline{\mu_N Cl(\mu_N Int\mathbf{P})} = 0_N$. This implies us $\mu_N Int(\mu_N Cl\bar{\mathbf{P}}) = 0_N$ in (X, μ_N). Since $\mathbf{P} \cap \bar{\mathbf{P}} \subseteq \bar{\mathbf{P}}$ we deduce that $\mu_N Int(\mu_N Cl(\mathbf{P} \cap \bar{\mathbf{P}})) \subseteq \mu_N Int(\mu_N Cl\bar{\mathbf{P}})$ and hence $\mu_N Int(\mu_N Cl(\mathbf{P} \cap \mathbf{P})) \subseteq 0_N$. That is $\mu_N Int(\mu_N Cl(\mathbf{P} \cap \bar{\mathbf{P}})) = 0_N$ which implies us that $\mathbf{P}$ is μ_N strongly nowhere dense set in (X, μ_N). □

Proposition 3.8. *If $\mathbf{P}$ is a neutrosophic set defined on (X, μ_N) such that $\mu_N Int(\mu_N Fr(\mathbf{P})) = 0_N$ in a μ_N Topological space then $\mathbf{P}$ is a μ_N strongly nowhere dense set in (X, μ_N).*

Proof. Let $\mathbf{P}$ be a neutrosophic set defined on (X, μ_N) such that $\mu_N Int(\mu_N Fr(\mathbf{P})) = 0_N$. Since $\mu_N Fr(\mathbf{P}) = \mu_N Cl(\mathbf{P}) \cap \mu_N Cl(\bar{\mathbf{P}})$ and we know that $\mu_N Cl(\mathbf{P} \cap \bar{\mathbf{P}}) \subseteq \mu_N Cl(\mathbf{P}) \cap \mu_N Cl(\bar{\mathbf{P}})$. Now $\mu_N Cl(\mathbf{P} \cap \bar{\mathbf{P}}) \subseteq \mu_N Fr(\mathbf{P}) \Rightarrow \mu_N Int(\mu_N Cl(\mathbf{P} \cap \bar{\mathbf{P}})) \subseteq \mu_N Int(\mu_N Fr(\mathbf{P})) = 0_N$. Hence we get $mu_N Int(\mu_N Fr(\mathbf{P})) = 0_N$ that implies us $\mathbf{P}$ is a μ_N strongly nowhere dense set in (X, μ_N). □

Definition 3.2. A neutrosophic set in a μ_N Topological space is called μ_N simply open set in (X, μ_N) if $\mu_N Fr(\mathbf{P})$ is μ_N nowhere dense set in (X, μ_N). In otherwords, $\mathbf{P}$ is μ_N simply open set iff $\mu_N Int(\mu_N Cl(\mu_N Fr\mathbf{P})) = 0_N$ in (X, μ_N).

Proposition 3.9. *If $\mathbf{P}$ is a μ_N simply open set in a μ_N Topological space (X, μ_N), then $\mathbf{P}$ is a μ_N strongly nowhere dense set in (X, μ_N).*

Proof. Let $\mathbf{P}$ be a simply open set in (X, μ_N). Then $\mu_N Int(\mu_N Cl(\mu_N Fr\mathbf{P})) = 0_N$ in (X, μ_N). But $\mu_N Int(\mu_N Fr\mathbf{P}) \subseteq \mu_N Int\mu_N Cl(\mu_N Fr\mathbf{P})$. From this we obtain that $\mu_N Int(\mu_N Fr\mathbf{P}) = 0_N$ in (X, μ_N). Then by using proposition 3.8 we obtain that $\mathbf{P}$ is a μ_N strongly nowhere dense set in (X, μ_N). □

Remark 3.1. *The converse of the above proposition need not be true. This can be illustrated in the example given below.*

Example 3.1. *Let (X, μ_N) be a μ_N TS where $X = \{a, b\}$ and we define neutrosophic sets $\delta_1 = \{< 0.6, 0.4, 0.8 >< 0.8, 0.6, 0.9 >\}, \delta_2 = \{< 0.6, 0.3, 0.8 >< 0.9, 0.2, 0.7 >\}, \delta_3 = \{< 0.5, 0.4, 0.9 >< 0.7, 0.8, 0.9 >\}, \delta_4 = \{< 0.4, 0.6, 0.9 >< 0.6, 0.8, 0.9 >\}, \delta_5 = \{< 0.3, 0.7, 0.9 >< 0.5, 0.9, 0.9 >\}$ and $\mu_N = \{0_N, \delta_1, \delta_2, \delta_3, \delta_4\}$ be a μ_N TS here the μ_N simply open sets are $\{0_N, \delta_2, \delta_5, 1_N\}$ and the μ_N strongly nowhere dense sets are $\{0_N, \delta_2, \delta_3, \delta_5, 1_N\}$. Here δ_5 is μ_N strongly nowhere dense set in (X, μ_N) but not μ_N simply open set in (X, μ_N).*

Proposition 3.10. *If $\mathbf{P}$ is a μ_N closed set with $\mu_N Int(\mathbf{P}) = 0_N$ in a μ_N TS (X, μ_N), then $\mathbf{P}$ is a μ_N strongly nowhere dense set in (X, μ_N).*

Proof. Let $\mathbf{P}$ be a μ_N closed set with $\mu_N Int(\mathbf{P}) = 0_N$ in (X, μ_N). Then $\mu_N Int \mu_N Cl(\mu_N Cl\ \mathbf{P} \cap \mu_N Cl\ \overline{\mathbf{P}}) = \mu_N Int \mu_N Cl(\mathbf{P} \cap \overline{\mu_N Int\ \mathbf{P}}) = \mu_N Int \mu_N Cl(\mathbf{P} \cap \overline{0_N}) = \mu_N Int(\mu_N Cl\ \mathbf{P}) = \mu_N Int\ \mathbf{P} = 0_N$. Hence,we get $\mathbf{P}$ is a μ_N simply open set in (X, μ_N). By proposition 3.9 deduce that $\mathbf{P}$ is a μ_N strongly nowhere dense set in (X, μ_N). □

Proposition 3.11. *If $\mathbf{P}$ is a μ_N open set and μ_N dense set in a μ_N Topological space (X, μ_N), then $\mathbf{P}$ is a μ_N strongly nowhere dense set in (X, μ_N).*

Proof. Let $\mathbf{P}$ be a μ_N open and μ_N dense set in (X, μ_N). Then $\overline{\mathbf{P}}$ is a μ_N closed set with $\mu_N Int\ \overline{\mathbf{P}} = \overline{\mu_N Cl\ \mathbf{P}} = 0_N$ in (X, μ_N). Then by using proposition 3.10 we retrieve that $\overline{\mathbf{P}}$ is a μ_N strongly nowhere dense set in (X, μ_N) and by using the proposition 3.4 we obtain that $\overline{\overline{\mathbf{P}}}$ is a μ_N strongly nowhere dense set in (X, μ_N) which implies us that $\mathbf{P}$ is a μ_N strongly nowhere dense set in (X, μ_N). □

Proposition 3.12. *Every subset of μ_N strongly nowhere dense set is μ_N strongly nowhere dense set.*

Proof. Let $\mathbf{P}$ be a μ_N strongly nowhere dense set, then $\mu_N Int(\mu_N Cl(\mathbf{P} \cap \overline{\mathbf{P}}) = 0_N$. If $\zeta \subseteq \mathbf{P}$ we have $\zeta \cap \overline{\mathbf{p}} \subseteq \cap \overline{\mathbf{P}} \Rightarrow \mu_N Int(\mu_N Cl(\zeta \cap \overline{\mathbf{P}}) \subseteq \mu_N Int(\mu_N Cl(\mathbf{P} \cap \overline{\mathbf{P}}) = 0_N$. Therefore ζ is a μ_N strongly nowhere dense set. Hence, subset of μ_N strongly nowhere dense set is μ_N strongly nowhere dense set. □

Proposition 3.13. *A neutrosophic set is μ_N strongly nowhere dense set if and only $\mu_N Cl(\mu_N Int(\overline{\mathbf{P} \cap \overline{\mathbf{P}}})) = 1_N$.*

Proof. Suppose $\mathbf{P}$ is μ_N strongly nowhere dense then $\mu_N Int(\mu_N Cl(\mathbf{P} \cap \overline{\mathbf{P}})) = 0_N$. Now, $\mu_N Cl(\mu_N Int(\overline{\mathbf{P} \cap \overline{\mathbf{P}}}))$ $= \mu_N Cl(\mu_N Cl(\overline{\mathbf{P} \cap \overline{\mathbf{P}}})) = \overline{\mu_N Int(\mu_N Cl(\mathbf{P} \cap \overline{\mathbf{P}}))} = 1_N$. Conversely we assume that $\mu_N Cl(\mu_N Int(\overline{\mathbf{P} \cap \overline{\mathbf{P}}})) = 1_N$. On considering, $\mu_N Int(\mu_N Cl(\mathbf{P} \cap \overline{\mathbf{P}})) = \overline{\mu_N Cl(\mu_N Cl(\overline{\mathbf{P} \cap \overline{\mathbf{P}}}))} = \overline{\mu_N Cl(\mu_N Int(\overline{\mathbf{P} \cap \overline{\mathbf{P}}}))} = \mu_N Int(\mu_N Cl(\mathbf{P} \cap \overline{\mathbf{P}}) = 0_N$. Hence it is μ_N strongly nowhere dense set. □

Proposition 3.14. *If $\mathbf{P}$ is μ_N strongly nowhere dense set then $\mu_N Int(\mathbf{P} \cap \overline{\mathbf{P}}) = 0_N$.*

Proof. Suppose $\mathbf{P}$ is μ_N strongly nowhere dense then $\mu_N Int(\mu_N Cl(\mathbf{P} \cap \overline{\mathbf{P}}) = 0_N$. Now, $\mu_N Int(\mathbf{P} \cap \overline{\mathbf{P}}) \subseteq \mu_N Int(\mu_N Cl(\mathbf{P} \cap \overline{\mathbf{P}}) = 0_N$. Hence, $\mu_N Int(\mathbf{P} \cap \overline{\mathbf{P}}) = 0_N$. □

4. μ_N strongly first category sets in μ_N TS:

Definition 4.1. A neutrosophic set is said to be μ_N stongly first category set in μ_N TS if $\delta = \cup_{i=1}^{\infty} \delta_i$ where δ_i's are μ_N stongly nowhere dense sets. The left out sets are called as μ_N stongly second category sets. The complement of μ_N stongly first category sets are called μ_N stongly residual sets.

Example 4.1. *Let (X, μ_N) be a μ_N TS where $X = \{a, b\}$ and we define neutrosophic sets $L_1 = \{< 0.6, 0.4, 0.8 >< 0.8, 0.6, 0.9 >\}$, $L_2 = \{< 0.6, 0.3, 0.8 >< 0.9, 0.2, 0.7 >\}$, $L_3 = \{< 0.5, 0.4, 0.9 >< 0.7, 0.8, 0.9 >\}$, $L_4 = \{< 0.4, 0.6, 0.9 >< 0.6, 0.8, 0.9 >\}$, $L_5 = \{< 0.3, 0.7, 0.9 >< 0.5, 0.9, 0.9 >\}$ and $\mu_N = \{0_N, L_1, L_2, L_3, L_4\}$ be a μ_N TS. Here μ_N stongly first category set is $L_2 = \{< 0.6, 0.3, 0.8 >< 0.9, 0.2, 0.7 >\}$. The μ_N stongly second category sets are $0_N, 1_N, L_1, L_3, L_4$ and the μ_N stongly residual set is $\overline{L_2}$.*

Proposition 4.1. *If* $\mathbf{P}$ *is a* μ_N *first category set then* $\mathbf{P}$ *is* μ_N *stongly first category set.*

Proof. Let $\mathbf{P}$ be a μ_N first category set in a μ_N TS. Then $\mathbf{P} = \cup_{i=1}^{\infty}\mathbf{P}_i$ where $\mathbf{P}_i$'s are μ_N nowhere dense sets in μ_N TS. By making use of the fact "Every μ_N nowhere dense set is μ_N strongly nowhere dense set" we deduce that $\mathbf{P}_i$'s are μ_N strongly nowhere dense sets and hence $\mathbf{P} = \cup_{i=1}^{\infty}\mathbf{P}_i$ where $\mathbf{P}_i$'s are μ_N strongly nowhere dense sets. Therefore, $\mathbf{P}$ is μ_N strongly first category set. □

Remark 4.1. *Every* μ_N *strongly first category sets need not be* μ_N *first category set. It is exemplified below.*

Example 4.2. *Let* $X = \{a\}$ *and* $\mu_N = \{0_N, A, C, E\}$ *be a* μ_N *TS where* $0_N = \{< 0, 1, 1 >\}$, $A = \{< 0.7, 0.8, 0.9 >\}$, $B = \{< 0.3, 0.4, 0.6 >\}$, $C = \{< 0.9, 0.7, 0.6 >\}$, $1_N = \{< 1, 0, 0 >\}$. *Here the* mu_N *first category set is* $0_N = \{< 0, 1, 1 >\}$ *and the* μ_N *strongly first category set is* $C = \{< 0.9, 0.7, 0.6 >\}$. *From this clearly we deduce that the* μ_N *strongly first category sets need not be* μ_N *first category set.*

Proposition 4.2. *If* $\mathbf{P} = \cup_{i=1}^{\infty}\mathbf{P}_i$ *where* $\mathbf{P}_i$*'s are* μ_N *closed sets with* $\mu_N Int\mathbf{P} = 0_N$ *then* $\mathbf{P}$ *is a* μ_N *strongly first category set.*

Proof. Suppose $\mathbf{P} = \cup_{i=1}^{\infty}\mathbf{P}_i$ where $\mathbf{P}_i$'s are μ_N closed sets with $\mu_N Int\mathbf{P} = 0_N$ in a μ_N TS (X, μ_N). Then by the fact, "If $\mathbf{P}$ is μ_N closed in μ_N TS with $\mu_N Int(\mathbf{P}_i) = 0_N$ then $\mathbf{P}$ is μ_N strongly nowhere dense set". By making use of this theorem we deduce that $\mathbf{P}$ is μ_N closed in μ_N TS with $\mu_N Int(\mathbf{P}_i) = 0_N$. Thus, $\mathbf{P}$ is μ_N strongly nowhere dense set and then we have $\mathbf{P} = \cup_{i=1}^{\infty}\mathbf{P}_i$ where $\mathbf{P}_i$'s are μ_N strongly nowhere dense sets. Thereupon $\mathbf{P}$ is a μ_N strongly first category set. □

Theorem 4.1. *If* $\mathbf{P} = \cup_{i=1}^{\infty}\mathbf{P}_i$ *where* $\mu_N Int(\mu_N Fr(\mathbf{P}_i)) = 0_N$ *then* $\mathbf{P}$ *is* μ_N *strongly first category set.*

Proof. Assume that $\mathbf{P} = \cup_{i=1}^{\infty}\mathbf{P}_i$ where $\mu_N Int(\mu_N Fr(\mathbf{P}_i)) = 0_N$. By theorem,"If $\mu_N Int(\mu_N Fr(\mathbf{P})) = 0_N$ for a mu_N open set in μ_N TS then $\mathbf{P}$ is μ_N strongly nowhere dense set." By making use of this we obtain that $\mathbf{P}_i$'s are μ_N strongly nowhere dense sets. Therefore, $\mathbf{P} = \cup_{i=1}^{\infty}\mathbf{P}_i$ where $\mathbf{P}_i$'s are μ_N strongly nowhere dense sets and hence $\mathbf{P}$ is μ_N strongly first category set. □

Theorem 4.2. *If* $\mathbf{P} = \cup_{i=1}^{\infty}\mathbf{P}_i$ *where* $\mathbf{P}_i$*'s are* μ_N *open sets in* μ_N *TS then* $\mathbf{P}$ *is* μ_N *strongly first category set.*

Proof. Given that $\mathbf{P} = \cup_{i=1}^{\infty}\mathbf{P}_i$ where $\mathbf{P}_i$'s are μ_N open sets in μ_N TS and also μ_N dense set in μ_N TS. By theorem, " If $\mathbf{P}$ is a μ_N open set in μ_N TS and $\mathbf{P}$ is also μ_N dense set in (X, μ_N), then $\mathbf{P}$ is a μ_N strongly nowhere dense set in (X, μ_N). " By making use of this theorem we obtain that $\mathbf{P}_i$'s are μ_N strongly nowhere dense sets and $\mathbf{P} = \cup_{i=1}^{\infty}\mathbf{P}_i$, where $\mathbf{P}_i$'s are μ_N strongly nowhere dense sets. Thereupon we get $\mathbf{P}$ is μ_N strongly first category set. □

Theorem 4.3. *Every subset of* μ_N *strongly first category set is* μ_N *strongly first category set.*

Proof. Let $\mathbf{P}$ be a μ_N strongly first category set. Then $\mathbf{P} = \cup_{i=1}^{\infty}\mathbf{P}_i$ where $\mathbf{P}_i$'s are μ_N strongly nowhere dense sets. Suppose $\zeta \subseteq \mathbf{P} = \cup_{i=1}^{\infty}\mathbf{P}_i$. From this we deduce that $\zeta \subseteq \cup_{i=1}^{\infty}\mathbf{P}_i$ which implies us that $\zeta \subseteq \mathbf{P}$ for some μ_N strongly nowhere dense sets. By using proposition 3.14 we obtain that ζ is μ_N strongly first category set. □

Remark 4.2. *Superset of* μ_N *strongly first category set need not be* μ_N *strongly first category set. This can be explained in the below example.*

Example 4.3. *Let* $\mu_N = \{0_N, \tau_a, \tau_b\}$ *where* $0_N = \{< 0, 1, 1 >\}$, $\tau_a = \{< 0.1, 0.4, 0.6 >\}$, $\tau_b = \{< 0.2, 0.3, 0.5 >\}$, $\tau_c = \{< 0.6, 0.6, 0.1 >\}$, $\tau_d = \{< 0.5, 0.7, 0.2 >\}$. *Here,* $\tau_c = \{< 0.6, 0.6, 0.1 >\}$ *is* μ_N *strongly first category set but* $\overline{\tau_b} \supseteq \tau_c = \{< 0.6, 0.6, 0.1 >\}$ *but* $\overline{\tau_b} = \{< 0.5, 0.7, 0.2 >\}$ *is not* μ_N *strongly first category set.*

5. μ_N Strongly Baire Space:

Definition 5.1. A μ_N TS is called μ_N strongly Baire space if $\mu_N Cl(\cup_{i=1}^{\infty} \mathbf{P}_i) = 1_N$ where $\mathbf{P}_i$'s are μ_N strongly nowhere dense sets.

Example 5.1. *Let* $X = \{a\}$ *and* $\mu_N = \{0_N, A, C, E\}$ *be a* μ_N *TS where* $0_N = \{< 0, 1, 1 >\}$, $A = \{< 0.7, 0.8, 0.9 >\}$, $B = \{< 0.3, 0.4, 0.6 >\}$, $C = \{< 0.9, 0.7, 0.6 >\}$, $1_N = \{< 1, 0, 0 >\}$. *Here the* μ_N *first category set is* $0_N = \{< 0, 1, 1 >\}$ *and the* μ_N *strongly first category sets are* $C = \{< 0.9, 0.7, 0.6 >\}$ *and* $1_N = \{< 1, 0, 0 >\}$. $\mu_N Cl(1_N) = 1_N$. *Hence* (X, μ_N) *is a* μ_N *strongly Baire space.*

Theorem 5.1. *Let* (X, μ_N) *be a* μ_N *TS.Then the following statements are parallel in nature.*

1. (X, μ_N) *is a* μ_N *strongly Baire space.*
2. $\mu_N Cl(\mathbf{P}) = 1_N$, *for every* μ_N *strongly first category set.*
3. $\mu_N Int(\mathbf{P}) = 0_N$, *for every* μ_N *residual sets.*

Proof. $(i) \Rightarrow (ii)$. Let $\mathbf{P}$ be a μ_N strongly first category set in (X, μ_N). Then $\mathbf{P} = \cup_{i=1}^{\infty} \mathbf{P}_i$ where $\mathbf{P}_i$' s are μ_N strongly nowhere dense sets. Since (X, μ_N) is a μ_N strong Baire space we get $\mu_N Cl(\cup_{i=1}^{\infty} \mathbf{P}_i) = 1_N$. Hence, $\mu_N Cl(\mathbf{P}) = 1_N$.

$(ii) \Rightarrow (iii)$. Let $\mathbf{P}$ be a μ_N strongly residual set in (X, μ_N). Then we retrieve that $\overline{\mathbf{P}}$ is a μ_N strongly first category set in (X, μ_N). From (ii) we obtain that $\mu_N Cl(\overline{\mathbf{P}}) = 1_N \Rightarrow \overline{\mu_N Int(\mathbf{P})} = 1_N$. Hence, $\mu_N Int(\mathbf{P}) = 0_N$.

$(iii) \Rightarrow (i)$. Let $\mathbf{P}$ be a μ_N strongly first category set in (X, μ_N). Then $\mathbf{P} = \cup_{i=1}^{\infty} \mathbf{P}_i$ where $\mathbf{P}_i$' s are a μ_N strongly nowhere dense sets. We have if $\mathbf{P}$ is a μ_N strongly first category set then $\overline{\mathbf{P}}$ is a μ_N strongly residual set in (X, μ_N). Now by making use of (iii) we obtain that $\mu_N Int(\overline{\mathbf{P}}) = 0_N$ which gives us that $\overline{\mu_N Cl(\mathbf{P}_i)} = 0_N$. Therefore we get $\mu_N Cl(\mathbf{P}_i) = 1_N$ and hence $\mu_N Cl(\cup_{i=1}^{\infty} \mathbf{P}_i) = 1_N$ where $\mathbf{P}_i$' s are a μ_N strongly nowhere dense sets. Hence, (X, μ_N) is a μ_N strongly Baire space. □

Theorem 5.2. *If* $\{\mathbf{P}_i\}, i = 1$ *to* ∞ *is* μ_N *open set and* μ_N *dense set in* μ_N *TS then* (X, μ_N) *is a* μ_N *strongly Baire space.*

Proof. We know that, "If ζ is μ_N open set and μ_N dense then ζ is μ_N strongly nowhere dense sets". By making use of this fact we obtain that $\mathbf{P}_i$' s are μ_N strongly nowhere dense sets in (X, μ_N). Let $\xi = \cup_{i=1}^{\infty} \xi_i$ then ξ_i's are μ_N strongly first category sets. Now, $\mu_N Cl(\xi_i) = \mu_N Cl(\cup_{i=1}^{\infty} \xi_i) \supseteq \cup_{i=1}^{\infty} \mu_N Cl(\xi_i) = 1_N$. Hence, (X, μ_N) is a μ_N strongly Baire space. □

References

[1] Atanassov.K.T, Intuitionistic fuzzy sets, Fuzzy Sets and Systems, 20(1986), 87–96.

[2] Atanassov.K.and Stoeva.S,Intuitionistic fuzzy sets,Polish Syrup.on Interval & fuzzy mathematics,(August1983)23-26.

[3] Chang.C.L, Fuzzy topological spaces, Journal of Mathematical Analysis and Application, 24(1968), 183–190.

[4] Dogan Coker, An introduction to intuitionstic fuzzy topological spaces, Fuzzy Sets and Systems, 88(1997), 81–89

[5] Ekici.E and Etienne Kerre.K, On fuzzy contra continuities, Advances in Fuzzy Mathematics,2006,35-44

[6] FloretinSmarandache, Neutrosophy and Neutrosophic Logic, First International Conference on Neutrosophy, Neutrosophic Logic, Set, Probability, and Statistics University of New Mexico, Gallup, NM 87301,USA, 2002.

[7] FloretinSmarandache, NeutrosophicSet:- A Generalization of Intuitionistic Fuzzy set, Journal of DefenseResourses Management, 1(2010),107–116.

[8] FloretinSmarandache, A Unifying Field in Logic: Neutrosophic Logic. Neutrosophy, Neutrosophic set, Neutrosophic Probability. Ameican Research Press, Rehoboth, NM,1999.

[9] Iswarya .P,Dr.K.Bageerathi,A Study on neutrosophic Frontier and neutrosophic semi frontier in Neutrosophic topological spaces.Neutrosophic sets and systems, vol . 16,2017.

[10] Dhavaseelan.R, S.Jafari, Narmada Devi.R, Hanif Page.Md.Neutrosophic Baire's Space, Neutrosophic Sets and Systems, Vol 16.2017

[11] Dhavaseelan.R, Narmada Devi.R, S.Jafari, Characterization of Neutrosophic Nowhere Dense Sets,International Journal Of Mathematical Archive,Vol.9,No.3,2018

[12] Raksha Ben .N, Hari Siva Annam.G, Generalized topological spaces via neutrosophic Sets, J.Math.Comput.Sci.,11(2021), 716-734.

[13] Raksha Ben N, Hari Siva Annam.G, μ_N Dense sets and its nature,South East Asian Journal of Mathematics and Mathematical Sciences, Vol 17,No.2(2021),68-81.

[14] Salama A.A and Alblowi S.A, Neutrosophic set and Neutrosophic topological space,ISOR J. Mathematics, 3(4)(2012), 31–35.

[15] Zadeh.L.A, Fuzzy set, Inform and Control, 8(1965), 338– 353.