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A study on continuous functions in semi prime ideal space

V.Thamaraiselvi[1]*, P.Sivagami[2], G.Hari Siva Annam[3]
[1]Research scholar, Reg.No:19222102092016,
PG and Research Department of Mathematics, Kamaraj college,
Thoothukudi-628003, India. ORCID: 0000-0001-5441-4987
[2]PG and Research Department of Mathematics, Kamaraj college,
Thoothukudi-628003, India. ORCID: 0000-0002-2035-5306
[3]PG and Research Department of Mathematics, Kamaraj college,
Thoothukudi-628003, India. ORCID: 0000-0002-0561-1287

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Abstract: In this paper new type of continuous functions namely S-continuous introduced in semi prime ideal space and compare with continuous function in topological space and study some of their properties. Also we introduced strongly S-continuous and S-irresolute in semi prime ideal space and compared with S-continuous in semi prime ideal space. Totally S-continuous and contra S-continuous were introduced and discussed.

Key words: S-continuous, strongly S-continuous, totally S-continuous.

1. Introduction

Ideals in a topological space (X, τ) is treated in the classic text by Kuratowski [7]. An ideal I on a topological space (X, τ) is a non empty collection of subsets of X which satisfies (i) $A \in I$ and $B \subseteq A$ implies $B \in I$. (ii) $A \in I$ and $B \in I$ implies $A \cup B \in I$. A topological space together with an ideal I is called an ideal topological space and denoted by (X, τ, I). He also defined the local function for each subset of X with respect to an ideal I and τ. Given a topological space (X, τ) with an ideal I on X and if $\rho(X)$ is the set of all subsets of X, a set operator $(.)^* : \rho(X) \to \rho(X)$ called a local function of A with respect to τ and I is defined as follows: For $A \subseteq X$, $A^*(I, \tau) = \{x \in X / U \cap A \notin I \text{ for every } U \in r(x)\}$ where $r(x) = \{U \in \tau / x \in U\}$. A kuratowski closure operator $(cl)^*(.)$ for a topology $\tau^*(I, \tau)$ called *-topology finer than τ is defined by $(cl)^*(A) = A \cup A^*(I, \tau)$. We denote A^* for $A^*(I, \tau)$ and τ^* for $\tau^*(I, \tau)$. Further Vaidyanathaswamy [12] extended the study of ideals and local functions. The properties of the topology generated by the ideal I and τ, called the star topology which is finer than τ, denoted by τ^* are studied by Vaidyanathaswamy, Hashimoto, Hayashi and Samuels [4]. In 1990, Jankovic and Hamlet [3] in addition to their findings, consolidated all the results. A function $f : (X, \tau) \to (Y, \sigma)$ is said to be continuous if inverse image of every open in Y is open in X. Let $f : X \to Y)$ and $f : Y \to Z$ be two bijection mappings. Then $g \circ f : Y \to Z$ is also bijection mapping and $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$. In 2021 we introduced prime ideals in topological space and study some properties [10]. Also we introduced semi prime ideal space in a topological space [11]. In this research paper we introduced S-continuous, strongly S-continuous and S-irresolutu in semi prime ideal.space and discussed.

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*Correspondence: thamarai4208@gmail.com

2. Preliminaries

Definition 2.1. A semi prime ideal S on a topological space (X, τ) is a nonempty collection of subsets of X which satisfies (i) $A \in S$ and $B \subseteq A$ implies $B \in S$. (ii) $A \cap B \in S$ and $A \in S$ implies $B \in S$. The space (X, τ, S) is said to be a semi prime ideal space.

Definition 2.2. Given a topological space (X, τ) with a semi prime ideal S on X and if $\rho(X)$ is the set of all subsets of X, a set operator $(.)^*_S : \rho(X) \to \rho(X)$ is called a semi prime local function of A with respect to τ and S is defined as follows: For $A \subseteq X, A^*_S(S, \tau) = \{x \in U / U \cap A \notin S foreveryU \in r(x)\}$ where $r(x) = \{U \in \tau / x \in U$.

Definition 2.3. A subset A of a semi prime ideal space (X, τ, S) is said to be S-closed if $A^*_S \subseteq A$.

Definition 2.4. A subset A of a semi prime ideal space (X, τ, S) is S-open if its complement is S-closed.

Result 2.1. Every open is S-open.

Definition 2.5. A subset A of a semi prime ideal space is said to be S-clopen if it is both S-closed and S-open.

Definition 2.6. A function $f : (X, \tau) \to (Y, \sigma)$ is said to be totally continuous if inverse image of every open in Y is clopen in X.

Definition 2.7. A function $f : (X, \tau) \to (Y, \sigma)$ is said to be contra continuous if inverse image of every open in Y is closed in X.

Definition 2.8. Let (X, τ) be a topological space and let $x \in X$. Then the neighbourhood of x is an open set of X containing x.

3. Continuous functions in semi prime ideal space

In this section we defined S-continuous and discussed some properties.

Definition 3.1. A function $f : (X, \tau, S) \to (Y, \sigma)$ is said to ben S-continuous if inverse image of every open in Y is S-open in X.

Example 3.1. *Consider the semi prime ideal space* $X = \{a, b, c\}, \tau = \{\phi, \{a\}, X\}, S = \{\phi, \{a\}, \{b\}, \{a, b\}\}$ *and* $Y = \{a, b, c\}, \sigma = \{\phi, \{b\}, X\}$. *Define* $f : (X, \tau, S) \to (Y, \sigma)$ *by* $f(a) = b, f(b) = c, f(c) = a$. *Then f is S-continuous.*

Definition 3.2. A function $f : (X, \tau, S_1) \to (Y, \sigma, S_2)$ is said to be S-irresolute if the inverse image of every S-open in Y is S-open in X.

Example 3.2. *Consider the semi prime ideal spaces* $X = \{a, b, c\}$, $\tau = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, X\}, S = \{\phi, \{a\}, \{b\}, \{a, b\}$ *and* $Y = \{a, b, c\}, \sigma = \{\phi, \{b\}, X\}$. *Define* $f : (X, \tau, S) \to (Y, \sigma, S)$ *by* $f(a) = b, f(b) = c, f(c) = a$*.Then f is S-irresolute.*

Definition 3.3. A function $f : (X, \tau) \to (Y, \sigma, S)$ is said to be strongly S-continuous if inverse image of every S-open in Y is open in X.

Example 3.3. *Consider the semi prime ideal space* $X = \{a, b, c\}$, $\tau = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, X\}$ *, and* $Y = \{a, b, c\}$, $\sigma = \{\phi, \{b\}, X\}, S = \{\phi, \{a\}, \{b\}, \{a, b\}$. *Define* $f : (X, \tau) \to (Y, \sigma, S)$ *by* $f(a) = b, f(b) = c, f(c) = a$*.Then f is strongly S-continuous.*

Theorem 3.1. *Every continuous is S-continuous.*

Proof. Let $f:(X,\tau,S)\rightarrow(Y,\sigma)$ be a continuous function and let U be any open in Y. Since f is continuous, $f^{-1}(U)$ is open in X and hence $f^{-1}(U)$ is S-open in X. Therefore f is S-continuous. □

Remark 3.1. *The converse of the above theorem need not be true as shown in the following example.*

Example 3.4. *Consider the semi prime ideal space* $X=\{a,b,c\}$,
$\tau=\{\phi,\{a\},X\}$ *, and* $Y=\{a,b,c\}$,
$\sigma=\{\phi,\{a\},\{b,c\},X\},S=\{\phi,\{a\},\{b\},\{a,b\}$*. Define* $f:(X,\tau,S)\rightarrow(Y,\sigma)$ *by* $f(a)=a,f(b)=b,f(c)=c$*.Then f is S-continuous but not continuous..*

Theorem 3.2. *Every strongly S-continuous is continuous.*

Proof. Let $f:(X,\tau)\rightarrow(Y,\sigma,S)$ be strongly S-continuous and let U be any open set in Y. Since every open is S-open, U is S-open. Since f is strongly S-continuous, $f^{-1}(U)$ is open in X and hence f is continuous. □

Remark 3.2. *The converse of the above theorem need not be true as shown in the following example.*

Example 3.5. *Consider the semi prime ideal space* $X=\{a,b,c\}$,
$\tau=\{\phi,\{a\},\{b,c\},X\}$ *, and* $Y=\{a,b,c\}$,
$\sigma=\{\phi,\{a\},X\},S=\{\phi,\{a\},\{b\},\{a,b\}$*. Define* $f:(X,\tau)\rightarrow(Y,\sigma,S)$ *by* $f(a)=a,f(b)=b,f(c)=c$*.Then f is continuous but not strongly S-continuous.*

Theorem 3.3. *Every strongly S-continuous is S-continuous.*

Proof. Obvious. □

Remark 3.3. *The converse of the above theorem need not be true as shown in the following example.*

Example 3.6. *Consider the semi prime ideal space* $X=\{a,b,c\}$,
$\tau=\{\phi,\{a\},\{b,c\},X\}$, $S_1=\{\phi,\{a\},\{c\},\{a,c\}\}$ *and* $Y=\{a,b,c\}$,
$\sigma=\{\phi,\{a\},X\},S_2=\{\phi,\{a\},\{b\},\{a,b\}$*. Define* $f:(X,\tau,S_1)\rightarrow(Y,\sigma,S_2)$ *by* $f(a)=a,f(b)=b,f(c)=c$*.Then f is S-continuous but not strongly S-continuous.*

Theorem 3.4. *every irresolute is S-continuous.*

Proof. Let $f:(X,\tau,S_1\rightarrow(Y,\sigma,S_2$ be S-irresolute and U be open in Y. Since every open is S-open, U is open. Since f is S-irresolute, $f^{-1}(U)$ is S-open in X and hence f is S-continuous. □

Remark 3.4. *The converse of the above theorem need not be true as shown in the following example.*

Example 3.7. *Consider the semi prime ideal space* $X=\{a,b,c\}$,
$\tau=\{\phi,\{a\},\{b,c\},X\}$, $S_1=\{\phi,\{a\},\{c\},\{a,c\}\}$ *and* $Y=\{a,b,c\}$,
$\sigma=\{\phi,\{a\},X\},S_2=\{\phi,\{a\},\{b\},\{a,b\}$*. Define* $f:(X,\tau,S_1)\rightarrow(Y,\sigma,S_2)$ *by* $f(a)=a,f(b)=b,f(c)=c$*.Then f is S-continuous but not S-irresolute.*

Theorem 3.5. *Every strongly S-continuous is S-irresolute.*

Proof. Let $f : (X, \tau, S_1 \rightarrow (Y, \sigma, S_2$ be strongly S-continuous and U be S-open in Y. Since f is strongly S-continuous, $f^{-1}(U)$ is open in X. Since every open is S-open, $f^{-1}(U)$ is S-open in X and hence f is irresolute. □

Remark 3.5. *The converse of the above theorem need not be true as shown in the following example.*

Example 3.8. *Consider the semi prime ideal space* $X = \{a, b, c\}$,
$\tau = \{\phi, \{a\}, \{b, c\}, X\}$, $S_1 = \{\phi, \{a\}, \{c\}, \{a, c\}\}$ *and* $Y = \{a, b, c\}$,
$\sigma = \{\phi, \{a\}, X\}, S_2 = \{\phi, \{a\}, \{b\}, \{a, b\}$. *Define* $f : (X, \tau, S_1) \rightarrow (Y, \sigma, S_2)$ *by* $f(a) = a, f(b) = c, f(c) = b$. *Then f is S-irresolute but not strongly S-continuous.*

Example 3.9. *The concepts continuous and S-irresolute are independent to each other. For, consider the semi prime ideal space* $X = \{a, b, c\}$,
$\tau = \{\phi, \{a\}, \{b, c\}, X\}$, $S_1 = \{\phi, \{a\}, \{c\}, \{a, c\}\}$ *and* $Y = \{a, b, c\}$,
$\sigma = \{\phi, \{a\}, X\}, S_2 = \{\phi, \{a\}, \{b\}, \{a, b\}$. *Define* $f : (X, \tau, S_1) \rightarrow (Y, \sigma, S_2)$ *by* $f(a) = a, f(b) = b, f(c) = c$. *Then f is not S-irresolute but continuous.*
Consider the semi prime ideal space $X = \{a, b, c\}, \tau = \{\phi, \{a\}, X\}$,
$S_1 = \{\phi, \{a\}, \{b\}, \{a, b\}\}$ *and* $Y = \{a, b, c\}, \sigma = \{\phi, \{a\}, \{b, c\}, X\}$,
$S_2 = \{\phi, \{a\}, \{c\}, \{a, c\}$. *Define* $f : (X, \tau, S_1) \rightarrow (Y, \sigma, S_2)$ *by* $f(a) = a, f(b) = c, f(c) = b$. *Then f is S-irresolute but not continuous.*

Theorem 3.6. *Let* $f : (X, \tau, S_1) \rightarrow (Y, \sigma, S_2)$ *be a function. Then*

1. *f is S-continuous iff inverse image of every closed in Y is S-closed in X.*
2. *f is S-irresolute iff inverse image of every S-closed set in Y is S-closed in X.*
3. *f is strongly S-continuous iff inverse image of every S-closed in Y is closed in X*

Proof. Obvious □

Theorem 3.7. *Let X and Y be topological spaces and* $f : X \rightarrow Y$. *Then f is S-continuous iff for each* $x \in X$ *and each neighbourhood V of f(x), there is a S-neighbourhood U of x such that* $f(U) \subset V$.

Proof. Let V be neighbourhood in f(x). Since f is S-continuous, $f^{-1}(V)$ is S-open in X. Let $U = f^{-1}(V)$. Then U is S-open and hence S-neighbourhood of x which gives $f(U) \subset V$.
Conversely assume that for each $x \in X$ and each neighbourhood V of f(x), there is a S-neighbourhood U of x such that $f(U) \subset V$. Let B be open set in Y and let $x \in f^B$. Then $f(x) \in B$ and hence B is a neighbourhood of f(x). By assumption, there is a S-neighbourhood A_x of x such that $f(A_x) \subset B$. Then $A_x \subset f^{-1}(B)$ for each x. This gives $f^{-1}(B) = \cup A_x$. Since every A_x is S-open, $\cup A_x$ is S-open which gives $f^{-1}(B)$ is open and hence f is S-continuous. □

4. Contra and totally continuous in semi prime ideal space

In this section we defined contra S-continuous in semi prime ideal space and compared with continuous functions. Also we defined totally S-continuous functions and discussed.

Definition 4.1. A function $f : (X, \tau, S) \rightarrow (Y, \sigma)$ is said to be contra S-continuous if inverse image of every open in Y is S-closed in X.

Example 4.1. *Consider the semi prime ideal space* $X = \{a, b, c\}, \tau = \{\phi, \{a\}, X\}$, $S = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}$ *and* $Y = \{a, b, c\}, \sigma = \{\phi, \{b\}, X\}$. *Define* $f : (X, \tau, S) \to (Y, \sigma)$ *by* $f(a) = b, f(b) = c, f(c) = a$. *Then f is contra S-continuous.*

Definition 4.2. A function $f : (X, \tau, S) \to (Y, \sigma)$ is said to be totally S-continuous if inverse image of every open is S-clopen in X.

Example 4.2. *Consider the semi prime ideal space* $X = \{a, b, c\}, \tau = \{\phi, \{a\}, X\}$, $S = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}$ *and* $Y = \{a, b, c\}, \sigma = \{\phi, \{b\}, X\}$. *Define* $f : (X, \tau, S) \to (Y, \sigma)$ *by* $f(a) = b, f(b) = c, f(c) = a$. *Then f is totally S-continuous.*

Theorem 4.1. *Every totally S-continuous is contra S-continuous.*

Proof. Let $f : (X, \tau, S) \to (Y, \sigma)$ be totally S-continuous and U be open set in Y. Since f is totally S-continuous $f^{-1}(U)$ is S-clopen in X. Therefore $f^{-1}(U)$ is S-closed and hence f is contra S-continuous. □

Example 4.3. *The converse of above theorem is not always true. For Consider the semi prime ideal space* $X = \{a, b, c\}, \tau = \{\phi, \{b, c\}, X\}$, $S = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}$ *and* $Y = \{a, b, c\}, \sigma = \{\phi, \{a\}, X\}$. *Define* $f : (X, \tau, S) \to (Y, \sigma)$ *by* $f(a) = a, f(b) = b, f(c) = c$. *Then f is contra S-continuous but not totally S-continuous.*

Theorem 4.2. *Every contra continuous is contra S-continuous.*

Proof. Let $f : (X, \tau, S) \to (Y, \sigma)$ be contra continuous and U be open set in Y. Since f is contra continuous $f^{-1}(U)$ is S-closed in X. Since every closed is S-closed, $f^{-1}(U)$ is S-closed in X and hence f is contra S-continuous. □

Example 4.4. *The converse of above theorem is not always true. For Consider the semi prime ideal space* $X = \{a, b, c\}, \tau = \{\phi, \{a\}, X\}$, $S = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}$ *and* $Y = \{a, b, c\}, \sigma = \{\phi, \{b\}, X\}$. *Define* $f : (X, \tau, S) \to (Y, \sigma)$ *by* $f(a) = b, f(b) = c, f(c) = a$. *Then f is contra S-continuous but not contra continuous.*

Theorem 4.3. *Every totally continuous is totally S-continuous.*

Proof. Let $f : (X, \tau, S) \to (Y, \sigma)$ be totally continuous and U be open set in Y. Since f is totally continuous $f^{-1}(U)$ is clopen in X. Since every clopen is S-clopen, $f^{-1}(U)$ is S-clopen in X and hence f is totally S-continuous. □

Example 4.5. *The converse of above theorem need not be always true. For Consider the semi prime ideal space* $X = \{a, b, c\}, \tau = \{\phi, \{a\}, X\}$, $S = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}$ *and* $Y = \{a, b, c\}, \sigma = \{\phi, \{b\}, X\}$. *Define* $f : (X, \tau, S) \to (Y, \sigma)$ *by* $f(a) = b, f(b) = c, f(c) = a$. *Then f is totally S-continuous but not totally continuous.*

Theorem 4.4. *every totally continuous is contra S-continuous.*

Proof. Obvious. □

Example 4.6. *The converse of above theorem need not be always true.*
For consider the semi prime ideal space $X = \{a, b, c\}, \tau = \{\phi, \{b, c\}, X\}$,
$S = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}$ *and* $Y = \{a, b, c\}, \sigma = \{\phi, \{a\}, X\}$. *Define*
$f : (X, \tau, S) \to (Y, \sigma)$ *by* $f(a) = a, f(b) = b, f(c) = c$. *Then f is contra S-continuous but not totally continuous.*

Theorem 4.5. *Every totally S-continuous is S-continuous.*

Proof. Let $f : (X, \tau, S) \to (Y, \sigma)$ be totally S-continuous and U be open set in Y. Since f is totally S-continuous, $f^{-1}(U)$ is S-clopen in X and hence S-open in X. Therefore f is S-continuous. □

Example 4.7. *The converse of above theorem need not be always true.*
For consider the semi prime ideal space $X = \{a, b, c\}, \tau = \{\phi, \{b, c\}, X\}$,
$S = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}$ *and* $Y = \{a, b, c\}, \sigma = \{\phi, \{a\}, X\}$. *Define*
$f : (X, \tau, S) \to (Y, \sigma)$ *by* $f(a) = a, f(b) = b, f(c) = c$. *Then f is S-continuous but not totally S-continuous.*

5. composition of functions in semi prime ideal space

In this section we discussed about the composition of continuous functions in semi prime ideal space.

Theorem 5.1. *Let* $f : (X, \tau, S_1) \to (Y, \sigma, S_2)$ *and* $g : (Y, \sigma, S_2) \to (Z, \delta, S_3$ *be bijection mappings. If*

1. *f is strongly S-continuous and g is strongly S-continuous, then* $g \circ f$ *is strongly S-continuous.*
2. *f is strongly S-continuous and g is strongly S-continuous, then* $g \circ f$ *is continuous.*
3. *f is strongly S-continuous and g is strongly S-continuous, then* $g \circ f$ *is S-continuous.*
4. *f is strongly S-continuous and g is strongly S-continuous, then* $g \circ f$ *is S-irresolute.*

Proof.

1. Let V be S-open in Z. Since g is strongly S-continuous, $g^{-1}(V)$ is open in Y and hence $g^{-1}(V)$ is S-open in Y. Since f is strongly S-continuous, $f^{-1}(g^{-1}(V))$ is open in X. Since f and g are bijections, $(g \circ f)^{-1}(V)$ is open in X. Therefore $g \circ f$ is strongly S-continuous.

(2),(3) and (4) are obvious. □

Theorem 5.2. *Let* $f : (X, \tau, S_1) \to (Y, \sigma, S_2)$ *and* $g : (Y, \sigma, S_2) \to (Z, \delta, S_3$ *be bijection mappings. If*

1. *f is strongly S-continuous and g is S-irresolute, then* $g \circ f$ *is strongly S-continuous.*
2. *f is strongly S-continuous and g is S-irresolute, then* $g \circ f$ *is continuous.*
3. *f is strongly S-continuous and g is S-irresolute, then* $g \circ f$ *is S-continuous.*
4. *f is strongly S-continuous and g is S-irresolute, then* $g \circ f$ *is S-irresolute.*

Proof.

1. Let V be S-open in Z. Since g is S-irresolute, $g^{-1}(V)$ is S-open in Y. Since f is strongly S-continuous, $f^{-1}(g^{-1}(V))$ is open in X. Since f and g are bijections, $(g \circ f)^{-1}(V)$ is open in X. Therefore $g \circ f$ is strongly S-continuous.

(2),(3) and (4) are obvious. □

Theorem 5.3. *Let $f : (X, \tau, S_1) \to (Y, \sigma, S_2)$ and $g : (Y, \sigma, S_2) \to (Z, \delta, S_3$ be bijection mappings. If*

1. *f is strongly S-continuous and g is S-continuous, then $g \circ f$ is continuous.*
2. *f is strongly S-continuous and g is S-continuous, then $g \circ f$ is S-continuous.*
3. *f is strongly S-continuous and g is continuous, then $g \circ f$ is continuous.*
4. *f is strongly S-continuous and g is continuous, then $g \circ f$ is S-continuous.*

Proof.

1. Let V be open in Z. Since g is S-continuous, $g^{-1}(V)$ is S-open in. Since f is strongly S-continuous, $f^{-1}(g^{-1}(V))$ is open in X. Since f and g are bijections, $(g \circ f)^{-1}(V)$ is open in X. Therefore $g \circ f$ is continuous.

(2),(3) and (4) are obvious. □

Theorem 5.4. *Let $f : (X, \tau, S_1) \to (Y, \sigma, S_2)$ and $g : (Y, \sigma, S_2) \to (Z, \delta, S_3$ be bijection mappings. If*

1. *f is S-irresolute and g is S-continuous, then $g \circ f$ is continuous.*
2. *f is strongly S-continuous and g is S-irresolute, then $g \circ f$ is S-irresolute.*
3. *f is S-irresolute and g is strongly S-continuous, then $g \circ f$ is S-irresolute.*
4. *f is S-irresolute and g is strongly S-continuous, then $g \circ f$ is S-continuous.*
5. *f is S-irresolute and g is S-continuous, then $g \circ f$ is S-continuous.*
6. *f is S-irresolute and g is continuous, then $g \circ f$ is S-continuous.*

Proof. (1) Let V be S-open in Z. Since g is S-irresolute, $g^{-1}(V)$ is S-open in Y. Since f is S-irresolute, $f^{-1}(g^{-1}(V))$ is S-open in X. Since f and g are bijections, $(g \circ f)^{-1}(V)$ is S-open in X. Therefore $g \circ f$ is S-irresolute.
(2),(3) and (4) are obvious.
(5) Let V be S-open in Z. Since g is S-continuous, $g^{-1}(V)$ is S-open in Y. Since f is S-irresolute, $f^{-1}(g^{-1}(V))$ is S-open in X. Since f and g are bijections, $(g \circ f)^{-1}(V)$ is S-open in X. Therefore $g \circ f$ is S-continuous.
(6) obvious. □

Theorem 5.5. *Let $f : (X, \tau, S_1) \to (Y, \sigma, S_2)$ and $g : (Y, \sigma, S_2) \to (Z, \delta, S_3)$ be bijection mappings. If*

1. *f is S-continuous and g is strongly S-continuous, then $g \circ f$ is S-irresolute.*
2. *f is S-continuous and g is strongly S-continuous, then $g \circ f$ is S-continuous.*
3. *f is S-continuous and g is continuous, then $g \circ f$ is S-continuous.*

Proof. (1) Let V be S-open in Z. Since g is strongly S-continuous, $g^{-1}(V)$ is open in Y. Since f is S-continuous, $f^{-1}(g^{-1}(V))$ is S-open in X. Since f and g are bijections, $(g \circ f)^{-1}(V)$ is S-open in X. Therefore $g \circ f$ is S-irresolute.
(2) Obvious.
(3) Let V be open in Z. Since g is continuous, $g^{-1}(V)$ is open in Y. Since f is S-continuous, $f^{-1}(g^{-1}(V))$ is S-open in X. Since f and g are bijections, $(g \circ f)^{-1}(V)$ is S-open in X. Therefore $g \circ f$ is S-continuous. □

Theorem 5.6. *Let $f:(X,\tau,S_1)\to(Y,\sigma,S_2)$ and $g:(Y,\sigma,S_2)\to(Z,\delta,S_3)$ be bijection mappings. If*

1. *f is continuous and g is strongly continuous, then $g\circ f$ is S-continuous.*
2. *f is continuous and g is strongly S-continuous, then $g\circ f$ is strongly S- continuous.*
3. *f is S-continuous and g is strongly S-continuous, then $g\circ f$ is S-irresolute.*

Proof.

1. Let V be open in Z. Since g is continuous, $g^{-1}(V)$ is open in Y. Since f is continuous $f^{-1}(g^{-1}(V))$ is open in X.Since f and g are bijections, $(g\circ f)^{-1}(V)$ is open in X and hence $(g\circ f)^{-1}(V)$ is S-open in X. Therefore $g\circ f$ is S-continuous.
2. Let V be S-open in Z. Since g is strongly S-continuous, $g^{-1}(V)$ is open in Y. Since f is continuous $f^{-1}(g^{-1}(V))$ is open in X.Since f and g are bijections, $(g\circ f)^{-1}(V)$ is open in X. Therefore $g\circ f$ is strongly S-continuous.
3. obvious.

□

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