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A non-deterministic approach for solving multi-objective linear programming

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Abstract: This study develops a solution approach of multi-objective linear programming problem ($molp$-problem) in non-deterministic way. Here each objective function of $molp$-problem corresponds a decision set in which each decision is described in virtue of three independent states : degree of acceptance, degree of hesitation and degree of rejection. By cultivation of experts' hesitancy independently, the decision making becomes more realistic and promising in today's complexity. Taking intersection of all these sets, optimal decision of $molp$-problem is drawn. The methodology is based on a principle that expert always wishes to elevate the acceptance part and put back the hesitation and rejection part of overall decision. A suitable solution algorithm is developed. The proposed model is applied on two practical fields to measure its competency. After comparing and analysing the outcomes with the existing approach, a validation on the efficiency of present framework is drawn.

Key words: Linear programming problem; function of acceptance, hesitation and rejection; neutrosophic set; multi-objective optimization.

1. Introduction

In the present real scenario, the information regarding any fact are incomplete and imprecise in nature due to ill human observation and consequently, making decision in respective ground is almost impossible in a straight way. Experts have to undergo through a confused state almost in every cases. There are a lot of parameters involved in a decision making and these affect the decision directly or indirectly with different degrees of presence. For instance, ruined economic structure of a society may call an obstacle in the smooth run of a nearer market or the political turmoil in a duration may inhibit the economic development of a country, though the other factors are as good as. These are two examples of indirect factor. Again, in a production house, all products may not be of standard quality to be produced in the market or all quality products may not be sold at a unique price. Similarly, due to varying market, the price of raw material or the maintenance cost of factory is not fixed. Hence it is clear that decision making should not be deterministic (i.e., the approach taken should not be in classical sense). It needs some special management to reach at probable fair outcome. Most of the optimization model in real state direct the multi-objective linear programming problem ($molp$-problem) and objectives are there conflicting in nature. Naturally, a compromise solution is reached and it fulfills all objectives with a degree of satisfaction. Probability theory, fuzzy set theory and its extensions are all reliable mathematical tools in that concern. Probability theory is based on classical set and is applied only on random process. Fuzzy set theory [30] tells about how much an element belongs to a set and it is measured on [0,1]. Then non-belonging of the element to this set is automatically described by the complement of belonging. Intuitionistic fuzzy set (ifs) theory [2] admits the degree of belonging and non-belonging of an element simultaneously, and these are

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individually measured on [0,1] so that their sum is restricted on [0,1] also. But there are certain real facts where we may get the tri-component outcomes as occurred in casting of poll by an elector, forecasting on the result of a sports event and in many decision makings. Such facts can not be analysed perfectly by fuzzy set or ifs due to their structural characteristics. Under neutrosophic set (nts) theory [26, 27], an element is designed by three independent characters namely acceptance, hesitation, rejection so that their sum lies in $[^{-}0, 3^{+}]$. The middle character of nts brings an opportunity to the researchers to handle the real life decision making problem in a more precise way. Here, expert has a flexibility to stand independently at a neutral zone between yes and no in making any decision. ifs only tackle incomplete information not indeterminate, but nts tackle both. So nts theory is the more generalized thought for modelling optimization and decision making based on uncertainty.

Researchers developed different optimization techniques in fuzzy and intuitionistic fuzzy sense for decision making with uncertainty. Zimmermann [31, 32] first took an attempt to solve the fuzzy multi-objective programming. Tang and Wang [28] developed a model of fuzzy nonlinear programming problems in manufacturing systems and its optimization technique using genetic algorithm to find a family of solutions with acceptable membership degrees instead of having exact optimal solution. Angelov [1] used ifs theory to solve optimization problem. A fuzzy optimization method for multi criteria decision making problem based on the inclusion degrees of ifs was proposed by Luo and Yu [20]. Nagorgani [21] presented a structure of intuitionistic fuzzy linear programming problem (lp-problem) and its solution approach in which decision variable, the coefficients of objective function and constraints were all taken as triangular intuitionistic fuzzy number. Based on ifs representation, Dubey et al. [14] designed an lp-problem under interval uncertainty by taking non-membership function with respect to three different viewpoints *viz.*, optimistic, pessimistic, and mixed. These formations along with their indeterminacy factors provides a S-shaped membership functions in the fuzzy counterparts of the intuitionistic fuzzy linear programming models. Bharatiand and Singh [12] took the intersection of various intuitionistic fuzzy decision sets corresponding to each objective function towards solving a $molp$-problem. A solution approach of nonlinear programming problem over ifs was managed by Singh and Yadav [25] using the concept of component wise optimization technique. A model of multi-objective optimization problem was developed by Firoz [16] where the uncertain parameters were described as intuitionistic fuzzy numbers. To solve this model, he developed an interactive neutrosophic programming approach. Pramanik [22] extended Zimmermann's approach to solve a neutrosophic $molp$-problem. Bera and Mahapatra [4–11] modified the structure of lp-problem by use of neutrosophic numbers in several directions to mitigate the complexity of decision making with uncertainty and applied these in different practical fields to ensure its' competency. Khalifa and Kumar [17] presented a structure of fully neutrosophic lp-problem and its solution approach. Several models of lp-problem were designed over ifs and nts by Loganathan and Lalitha [19], Edalatpanah [15], Basumatary and Broumi [3], Khatter [18], Das and Dash [13] and others. They proposed different ranking functions to convert these models into its' crisp forms. Rahaman et al [23] brought a decision making approach based on fuzzy parameterized hypersoft set theory.

This study develops a solution algorithm of $molp$-problem in non-deterministic approach. Each objective function of $molp$-problem corresponds a neutrosophic decision set. Then, by taking the intersection of all these sets, optimal decision set of $molp$-problem is reached. The methodology is based on a principle that expert always wishes to elevate the acceptance part and put back the hesitation and rejection part of overall decision. The efficiency of proposed model is examined on two practical fields. The design of study is made as follow.

Some useful results are highlighted in Section 2. The proposed model and an efficient algorithm towards solving of an $molp$-problem are drawn in Section 3. Section 4 illustrates the methodology by two real life examples. A proper validation is drawn after comparing the outcomes in existing frame. A brief note of the study, its limitation and future direction are stated in Section 5.

2. Preliminaries

We recall some necessary definitions and results to make out the main thought.

Definition 2.1. [12] A crisp *molp*-problem for k number of objective functions, l number of constraints and n decision variables is designed as :

$$\begin{aligned} & \text{optimize } g_s(x), \quad 1 \le s \le k \\ \text{subject to} \quad & \rho_i(x) = \sigma_i, \qquad 1 \le i \le l \\ & x = (x_1, x_2, \cdots, x_n) \ge 0 \end{aligned} \tag{1}$$

A solution x_0 (say) will be a complete optimal for the problem (1), if $g_s(x_0) \ge g_s(x)$ (for maximize function) and $g_s(x_0) \le g_s(x)$ (for minimize function) for all x and $1 \le s \le k$. But such solution x_0 which will optimize all objective functions simultaneously, generally does not exist, more specifically if the objectives are conflicting in nature. To overcome it, the Pareto optimality concept was welcome.

x_0 is a Pareto optimal solution of the *molp*-problem (1), if $g_s(x_0) \ge g_s(x)$ for all s and $g_s(x_0) > g_s(x)$ for atleast one s (for maximize function). For minimize function, it is $g_s(x_0) \le g_s(x)$ for all s and $g_s(x_0) < g_s(x)$ for atleast one s.

Definition 2.2. [31] Zimmermann designed the *molp*-problem (1) in fuzzy climate when all objectives are to be maximized as :

$$\begin{aligned} & \text{Find } x = (x_1, x_2, \cdots, x_n) \\ \text{subject to} \quad & g_s(x) \succeq U_s, \quad 1 \le s \le k \\ & \rho_i(x) = \sigma_i, \quad 1 \le i \le l \\ & x = (x_1, x_2, \cdots, x_n) \ge 0 \end{aligned} \tag{2}$$

Here U_s be the lowest permitted value and objective functions are all taken as fuzzy constraints. If μ_s be the membership value corresponding to objective function $g_s(x)$ for $1 \le s \le k$, then the feasible solution of the system is characterized by $\overline{\mu} = \min\{\mu_s : 1 \le s \le k\}$. The final decision is now obtained by solving the following problem in crisp sense :

$$\begin{aligned} & \max \overline{\mu} \\ \text{subject to} \quad & \mu_s(x) \ge \overline{\mu}, \quad 1 \le s \le k \\ & \rho_i(x) = \sigma_i, \quad 1 \le i \le l \\ & x = (x_1, x_2, \cdots, x_n) \ge 0 \end{aligned} \tag{3}$$

Definition 2.3. [26, 27, 29] An *nts* $\widetilde{T}$ defines an object y of the universe X in virtue of three independent characters namely acceptance $(\lambda_{\widetilde{T}})$, hesitation $(\zeta_{\widetilde{T}})$ and rejection $(\xi_{\widetilde{T}})$. Thus $\widetilde{T}$ is displayed as : $\widetilde{T} = \{< y, (\lambda_{\widetilde{T}}(y), \zeta_{\widetilde{T}}(y), \xi_{\widetilde{T}}(y)) >: y \in X\}$ where $\lambda_{\widetilde{T}}, \zeta_{\widetilde{T}}, \xi_{\widetilde{T}}$ are standard or non-standard subset of $]^-0, 1^+[$ with $^-0 \le \lambda_{\widetilde{T}}(y) + \zeta_{\widetilde{T}}(y) + \xi_{\widetilde{T}}(y) \le 3^+$.

When the three components of an nts are standard subset of $]^{-}0, 1^{+}[$ (i.e., the member of $[0,1]$ only), it is called single valued neutrosophic (SV-nts) set. Thus an SV-nts set $\widetilde{P}$ is designed as : $\widetilde{P} = \{< y, (\lambda_{\widetilde{P}}(y), \zeta_{\widetilde{P}}(y), \xi_{\widetilde{P}}(y)) >: y \in X\}$ where $\lambda_{\widetilde{P}}, \zeta_{\widetilde{P}}, \xi_{\widetilde{P}} : X \to [0,1]$ and $0 \leq \lambda_{\widetilde{P}}(y) + \zeta_{\widetilde{P}}(y) + \xi_{\widetilde{P}}(y) \leq 3$.

Definition 2.4. [4] The intersection of two nts $\widetilde{M}, \widetilde{N}$ defined over the common universe X is denoted by $\widetilde{M} \cap \widetilde{N} = \widetilde{V}$ and is defined by : $\widetilde{V} = \{< x, \lambda_{\widetilde{V}}(x), \zeta_{\widetilde{V}}(x), \xi_{\widetilde{V}}(x) >: x \in X\}$ where $\lambda_{\widetilde{V}}(x) = \lambda_{\widetilde{M}}(x) * \lambda_{\widetilde{N}}(x), \zeta_{\widetilde{V}}(x) = \zeta_{\widetilde{M}}(x) \diamond \zeta_{\widetilde{N}}(x), \xi_{\widetilde{V}}(x) = \xi_{\widetilde{M}}(x) \diamond \xi_{\widetilde{N}}(x)$. '$*$'and '$\diamond$'respectively refers t - norm and t - conorm. $a * b = \min\{a, b\}$ and $a \diamond b = \max\{a, b\}$ are two examples of t - norm and t - conorm respectively.

3. Proposed model for solving *molp*-problem

The solution procedure of an *molp*-problem is managed under neutrosophic environment in a distinct style. A suitable algorithm is also designed in order to that.

Proposition 3.1. *(Solution approach) In neutrosophic environment, expert wishes to elevate the acceptance part and put back the hesitation and rejection part of objective value and constraints both. To attain it, the problem is solved first as a single objective under the given constraints for each objective function. All objective values are then worked out with respect to each solution obtained as displayed by Table [**?**] (for the problem (1)). Here $g_s(x_s)^*$ is the optimal value with respect to the decision variable set x_s obtained from s^{th} single*

Table 1. Table for the values of objective functions.

Decision variable	g_1	g_2	$\cdots$	g_k
$x_1 = (x_1^1, \cdots, x_n^1)$	$g_1(x_1)^*$	$g_2(x_1)$	$\cdots$	$g_k(x_1)$
$x_2 = (x_1^2, \cdots, x_n^2)$	$g_1(x_2)$	$g_2(x_2)^*$	$\cdots$	$g_k(x_2)$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
$x_k = (x_1^k, \cdots, x_n^k)$	$g_1(x_k)$	$g_2(x_k)$	$\cdots$	$g_k(x_k)^*$

objective problem for $1 \leq s \leq k$. With the values calculated corresponding to each column of Table 1, the functions for three neutrosophic components are set (see the algorithm part for detail). The molp-problem (1) is then arranged in neutrosophic atmosphere as follows :

$$\begin{aligned} & \max \lambda_r(x), \ \min \zeta_r(x), \ \min \xi_r(x) \\ \textit{subject to} \quad & \lambda_r(x) \geq \zeta_r(x), \ \lambda_r(x) \geq \xi_r(x), \\ & \zeta_r(x) \geq 0, \ \xi_r(x) \geq 0, \\ & \lambda_r(x) + \zeta_r(x) + \xi_r(x) \leq 3, \\ & x = (x_1, x_2, \cdots, x_n) \geq 0 \\ & \textit{for } 1 \leq r \leq k + l \textit{ and } x \in X \end{aligned} \tag{4}$$

where $\lambda_r(x), \zeta_r(x), \xi_r(x)$ respectively represent the acceptance, hesitation, rejection part of the solution x to the r^{th} nts and X be the full solution set.

Let $\widetilde{O}$ and $\widetilde{C}$ be the integrated neutrosophic objective and constraints respectively. The decision set is *then defined by the intersection of these i.e.,*

$$\begin{aligned}
\widetilde{O} \cap \widetilde{C} &= \{< x, \min(\lambda_{\widetilde{O}}(x), \lambda_{\widetilde{C}}(x)), \max(\zeta_{\widetilde{O}}(x), \zeta_{\widetilde{C}}(x)), \max(\xi_{\widetilde{O}}(x), \xi_{\widetilde{C}}(x)) >: x \in X\} \\
&\quad \textit{where} \\
\widetilde{O} &= \{< x, \lambda_{\widetilde{O}}(x), \zeta_{\widetilde{O}}(x), \xi_{\widetilde{O}}(x) >: x \in X\} \\
&= \{< x, \min_s \lambda_s^g(x), \max_s \zeta_s^g(x), \max_s \xi_s^g(x) >: x \in X\}, \\
\widetilde{C} &= \{< x, \lambda_{\widetilde{C}}(x), \zeta_{\widetilde{C}}(x), \xi_{\widetilde{C}}(x) >: x \in X\} \\
&= \{< x, \min_i \lambda_i^\rho(x), \max_i \zeta_i^\rho(x), \max_i \xi_i^\rho(x) >: x \in X\} \quad \textit{and} \\
&\quad \min(\lambda_{\widetilde{O}}(x), \lambda_{\widetilde{C}}(x)) = \min_{r=1}^{k+l} \lambda_r(x) = \widetilde{\lambda}(x) \quad \textit{(say)}, \\
&\quad \max(\zeta_{\widetilde{O}}(x), \zeta_{\widetilde{C}}(x)) = \max_{r=1}^{k+l} \zeta_r(x) = \widetilde{\zeta}(x) \quad \textit{(say)}, \\
&\quad \max(\xi_{\widetilde{O}}(x), \xi_{\widetilde{C}}(x)) = \max_{r=1}^{k+l} \xi_r(x) = \widetilde{\xi}(x) \quad \textit{(say)}.
\end{aligned}$$

Here $\widetilde{\lambda}$, $\widetilde{\zeta}$, $\widetilde{\xi}$ *respectively indicate the acceptance, hesitation, rejection component of neutrosophic decision set of the problem. Now for this decision set,*

$$\widetilde{\lambda}(x) \le \lambda_r(x), \quad \widetilde{\zeta}(x) \ge \zeta_r(x), \quad \widetilde{\xi}(x) \ge \xi_r(x) \quad \textit{for} \quad 1 \le r \le k+l$$

Hence it can be presented by the following inequalities for the lowest state of acceptance 'a', highest state of hesitation 'b' and highest state of rejection 'c' related to objective and constraints as :

$$\begin{aligned}
& a \le \lambda_r(x), \quad b \ge \zeta_r(x), \quad c \ge \xi_r(x), \quad \textit{for} \quad 1 \le r \le k+l \\
& a \ge b, \quad a \ge c, \\
& b \ge 0, \quad c \ge 0, \\
& 0 \le a+b+c \le 3
\end{aligned}$$

To optimize in neutrosophic ground, the problem (1) is then finally designed as :

$$\begin{aligned}
& \max(a-b-c) \qquad (5) \\
\textit{subject to} \quad & a \le \lambda_r(x), \quad b \ge \zeta_r(x), \quad c \ge \xi_r(x), \quad \textit{for} \quad 1 \le r \le k+l \\
& a \ge b, \quad a \ge c, \\
& b \ge 0, \quad c \ge 0, \\
& 0 \le a+b+c \le 3, \\
& x = (x_1, x_2, \cdots, x_n) \ge 0
\end{aligned}$$

Now using simplex method, the system (5) provides the solution of problem (1).

Proposition 3.2. *(Solution algorithm)*
The solution of problem is managed here using the steps stated below.
Step 1 : *Solve the problem as a single objective with respect to all given constraints for the first objective*

function. Evaluate decision variables and the value of respective objective function.
Step 2 : *Calculate the value of remaining objective functions of the problem with these decision variables.*
Step 3 : *Perform the Step 1 and Step 2 for other objective functions of the problem similarly.*
Step 4 : *Draw a table (Table [1]) to put all objective values obtained in Step 1, Step 2, Step 3 at a glance.*
Step 5 : *Find the higher* (H^s) *and lower* (L^s) *value corresponding to each column of objective function* $g_s, 1 \leq s \leq k$ *in the Table [1].*
Step 6 : *When objective function is of maximization, for all* $1 \leq s \leq k$, *set*

$$\begin{aligned}
&H_\lambda^s = \max\{g_s(x_j) : 1 \leq j \leq k\},\ L_\lambda^s = \min\{g_s(x_j) : 1 \leq j \leq k\} \text{ for acceptance} \\
&H_\zeta^s = H_\lambda^s - \frac{t}{1+t}(H_\lambda^s - L_\lambda^s),\ L_\zeta^s = L_\lambda^s,\ t \in (0,1) \text{ for hesitation} \\
&H_\xi^s = H_\lambda^s - t(H_\lambda^s - L_\lambda^s),\ L_\xi^s = L_\lambda^s,\ \text{ for rejection.}
\end{aligned}$$

When objective function is of minimization, for all $1 \leq s \leq k$, *set*

$$\begin{aligned}
&H_\lambda^s = \max\{g_s(x_j) : 1 \leq j \leq k\},\ L_\lambda^s = \min\{g_s(x_j) : 1 \leq j \leq k\} \text{ for acceptance} \\
&H_\zeta^s = H_\lambda^s,\ L_\zeta^s = L_\lambda^s - \frac{t}{1+t}(H_\lambda^s - L_\lambda^s),\ t \in (0,1) \text{ for hesitation} \\
&H_\xi^s = H_\lambda^s,\ L_\xi^s = L_\lambda^s - t(H_\lambda^s - L_\lambda^s),\ \text{ for rejection.}
\end{aligned}$$

Step 7 : *Construct the function of acceptance* $\lambda_s(g_s)$, *hesitation* $\zeta_s(g_s)$ *and rejection* $\xi_s(g_s)$ *for each objective function* g_s, $1 \leq s \leq k$ *as follows :*

$$\lambda_s(g_s(x)) = \begin{cases} 0, & \text{while} \quad g_s(x) \leq L_\lambda^s \\ \frac{g_s(x) - L_\lambda^s}{H_\lambda^s - L_\lambda^s}, & \text{while} \quad L_\lambda^s \leq g_s(x) \leq H_\lambda^s \\ 1, & \text{while} \quad g_s(x) \geq H_\lambda^s. \end{cases}$$

$$\zeta_s(g_s(x)) = \begin{cases} 1, & \text{while} \quad g_s(x) \leq L_\zeta^s \\ \frac{H_\zeta^s - g_s(x)}{H_\zeta^s - L_\zeta^s}, & \text{while} \quad L_\zeta^s \leq g_s(x) \leq H_\zeta^s \\ 0, & \text{while} \quad g_s(x) \geq H_\zeta^s. \end{cases}$$

$$\xi_s(g_s(x)) = \begin{cases} 1, & \text{while} \quad g_s(x) \leq L_\xi^s \\ \frac{H_\xi^s - g_s(x)}{H_\xi^s - L_\xi^s}, & \text{while} \quad L_\xi^s \leq g_s(x) \leq H_\xi^s \\ 0, & \text{while} \quad g_s(x) \geq H_\xi^s. \end{cases}$$

Step 8 : *Draw a crisp lp-problem equivalent to the molp-problem (1) as :*

$$
\begin{aligned}
& \max(a-b-c) \qquad (6)\\
\textit{subject to,}\quad & \lambda_s(g_s(x)) \geq a,\\
& \zeta_s(g_s(x)) \leq b,\\
& \xi_s(g_s(x)) \leq c,\\
& 0 \leq b \leq a,\\
& 0 \leq c \leq a,\\
& 0 \leq a+b+c \leq 3,\\
& \rho_i(x) = \sigma_i,\\
& 1 \leq i \leq l,\\
& 1 \leq s \leq k,\\
& x \geq 0.
\end{aligned}
$$

Step 9 : *Solve the problem (6) using any suitable usual method.*

Remark 3.1. *From algorithm, it is clear that different functions for two neutrosophic components namely hesitation and rejection are formed at different states of $t \in (0,1)$. Consequently, the objective functional value of system (6) i.e., $\max(a-b-c)$ is changed at different 't'. But, whatever the state of $t \in (0,1)$ is applied, the system (6) always provides optimal solution to the problem at each 't' and these solutions may be distinct (see the application part : Sec4), if the problem admits an optimal solution. Decision makers then have a flexibility to chose and implement the suitable optimal solution with a degree of satisfaction $(a-b-c)$ in the demand of situation arisen.*

Remark 3.2. *(Drawback of existing methods)*
There are different solution approaches of molp-problem with uncertainty under distinct atmosphere. Following drawbacks of existing literatures, we note.
1. Bharatiand and Singh [12] developed the solution algorithm of molp-problem in intuitionistic fuzzy sense where all objective functions are maximization in nature. No view on minimization character was sighted therein. Further as this study was driven under ifs atmosphere, experts' hesitancy were not included independently in the experimental data and thus there was a question on the fair outcome.
2. Singh and Yadav [25] used to practice the component wise optimization technique to optimize intuitionistic fuzzy non-linear programming problem in manufacturing industry. So uncertainty and vagueness was not treated precisely as ifs does not allow to practice the indeterminacy nature independently. The model was on single objective treatment. In real state, one always wishes to elevate the level of acceptance of a decision and lower down its level of rejection. No attempt on this angle was seen, and thus the matter of experts' satisfaction was omitted there.
3. The notion of molp-problem in neutrosophic state was studied by Pramanik [22]. But he only considered the set of maximize objective functions. Further, no insight on the construction of neutrosophic components for the coefficients of objective function was given and the model was not demonstrated practically. So a clear picture of this study was not displayed to the beginners.
4. A solution approach of multi-objective intuitionistic fuzzy non-linear programming problem was brought by

Loganathan and Lalitha [19]. All intuitionistic fuzzy numbers were defuzzified therein to solve the problem in conventional way. So no new angle was seen in this attempt.

Remark 3.3. *(Impact of present study)*
The methodology to draw a conclusion of molp-problem designed in this text is reliable, realistic and timeliness. Followings arguments support this truth.
1. In the solution approach, the decision set is defined by taking intersection of objective functions and the set of constraints in neutrosophic sense. Then expert's hesitancy is treated independently and it is not possible in fuzzy or intuitionistic fuzzy state. Thus experts can take their opinions towards any decision making in a more flexible way.
2. There is a scope to elevate the degree of acceptance of a decision and to lower down its degree of hesitation and rejection. Thus the methodology does not only provide us a decision of an molp-problem but also brings a perfection to that.
3. The method provides the set of solutions of a molp-problem with different degrees of satisfaction to the experts. Thus there will be an opportunity to set a strategy of problem to the experts in their own rights.
4. The method is also applicable for a molp-problem with the objective functions in conflicting nature (i.e., maximization and minimization character). Thus the limitation of existing approaches (where objective functions are of maximization characters only) are overcome.

4. Application of proposed approach

The competency of proposed solution approach is examined here. Two real life examples of *molp*-problem are drawn and are solved with the help of solution methodology presented in this study. The outcome is analyzed and compared in existing frame.

Example 4.1. *A pharmaceutical company is going to develop a low energy high protein nutritional supplement in three forms (B_1 : Cereals, B_2 : Dry powder, B_3 : Liquid) under the supervision of a panel of nutritionists. These are produced by the variation of quantity of content e.g., energy, protein, carbohydrate, fat, vitamin, mineral. The current portfolio estimated by the panel is now available by Table 2. Based on the information, evaluate the unit of nutritional supplement required for an adult in a month which the company can provide to customers at a minimum cost keeping the maximum profit but assuring the quality. Let, the unit of nutrition*

Table 2. Estimation of various subjects related to production.

Nutrient	Unit	/100 gm	B_1	B_2	B_3	Requirement / month
Energy	kcal		170	200	185	≤ 18000
Protein	gm		10	11	9	≥ 950
Carbohyd	gm		70	79	75	≤ 7250
Fat	gm		2.5	2	1.5	≤ 185
Vitamin C	mg		130	145	135	≥ 13000
Iron	mg		35	40	38	≤ 3700
Unit profit	Rs		21	24	22.5	max
Unit cost	Rs		58	67	60	min
Quality	percent		80	77	82	max

B_1 , B_2 , B_3 required for an adult in a month be x_1, x_2, x_3 respectively. Mathematically, the problem is then

designed as :

$$\max\ \{g_1 = 21x_1 + 24x_2 + 22.5x_3\}$$
$$\min\ \{g_2 = 58x_1 + 67x_2 + 60x_3\}$$
$$\max\ \{g_3 = 80x_1 + 77x_2 + 82x_3\}$$

$$\begin{aligned} \textit{subject to} \quad & 170x_1 + 200x_2 + 185x_3 \leq 18000 \\ & 10x_1 + 11x_2 + 9x_3 \geq 950 \\ & 70x_1 + 79x_2 + 75x_3 \leq 7250 \\ & 2.5x_1 + 2x_2 + 1.5x_3 \leq 185 \\ & 130x_1 + 145x_2 + 135x_3 \geq 13000 \\ & 35x_1 + 40x_2 + 38x_3 \leq 3700 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

The proposed approach is now illustrated step-wise as stated in solution algorithm.

Step 1, Step 2, Step 3, Step 4 :

Solution with respect to g_1 *is :* $x_1 = 23.10345, x_2 = 34.72906, x_3 = 38.52217$.

Solution with respect to g_2 *is :* $x_1 = 24.37500, x_2 = 38.12500, x_3 = 31.87500$.

Solution with respect to g_3 *is :* $x_1 = 27.20000, x_2 = 24.00000, x_3 = 46.00000$.

The objective values with respect to these solutions are provided by Table 3. ***Step 5 :*** *The higher and lower values of objective functions are as follows.*

Table 3. Objective values with respect to the solutions.

(x_1, x_2, x_3)	g_1	g_2	g_3
(23.10345, 34.72906, 38.52217)	2185.42*	5978.18	7681.23
(24.37500, 38.12500, 31.87500)	2144.06	5880.63*	7499.38
(27.20000, 24.00000, 46.00000)	2182.20	5945.60	7796*

$$H^1 = 2185.42 \qquad H^2 = 5978.18 \qquad H^3 = 7796$$
$$L^1 = 2144.06 \qquad L^2 = 5880.63 \qquad L^3 = 7499.38$$

Step 6 : *The higher and lower values of three neutrosophic components corresponding to each objective function are set as follows. For objective function* g_1,

$$H^1_\lambda = 2185.42, \quad L^1_\lambda = 2144.06$$
$$H^1_\zeta = 2185.42 - 41.36 \times \frac{t}{1+t}, \quad L^1_\zeta = 2144.06,\ t \in (0,1)$$
$$H^1_\xi = 2185.42 - 41.36 \times t, \quad L^1_\xi = 2144.06$$

For objective function g_2 ,

$$H_\lambda^2 = 5978.18, \quad L_\lambda^2 = 5880.63$$
$$H_\zeta^2 = 5978.18, \quad L_\zeta^2 = 5880.63 - 97.55 \times \frac{t}{1+t}, \; t \in (0,1)$$
$$H_\xi^2 = 5978.18, \quad L_\xi^2 = 5880.63 - 97.55 \times t.$$

For objective function g_3 ,

$$H_\lambda^3 = 7796, \quad L_\lambda^3 = 7499.38$$
$$H_\zeta^3 = 7796 - 296.62 \times \frac{t}{1+t}, \quad L_\zeta^3 = 7499.38, \; t \in (0,1)$$
$$H_\xi^3 = 7796 - 296.62 \times t, \quad L_\xi^3 = 7499.38$$

Step 7 : *Followings are the functions of acceptance, hesitation and rejection for three objective functions.*

$$\lambda_1(g_1) = \frac{g_1 - 2144.06}{41.36}, \; \zeta_1(g_1) = \frac{2185.42 - 41.36 \times \frac{t}{1+t} - g_1}{41.36 \times \frac{1}{1+t}}, \; \xi_1(g_1) = \frac{2185.42 - 41.36 \times t - g_1}{41.36 \times (1-t)}$$
$$\lambda_2(g_2) = \frac{g_2 - 5880.63}{97.55}, \; \zeta_2(g_2) = \frac{5978.18 - g_2}{97.55 \times \frac{1+2t}{1+t}}, \quad \xi_2(g_2) = \frac{5978.18 - g_2}{97.55 \times (1+t)}$$
$$\lambda_3(g_3) = \frac{g_3 - 7499.38}{296.62}, \; \zeta_3(g_3) = \frac{7796 - 296.62 \times \frac{t}{1+t} - g_3}{296.62 \times \frac{1}{1+t}}, \; \xi_3(g_3) = \frac{7796 - 296.62 \times t - g_3}{296.62 \times (1-t)}$$

Step 8 : *The molp-problem is now converted into a crisp lp-problem as follows :*

$$\max = a - b - c$$
$$21x_1 + 24x_2 + 22.5x_3 - 2144.06 \geq 41.36 \times a,$$
$$2185.42 - 41.36 \times \frac{t}{1+t} - (21x_1 + 24x_2 + 22.5x_3) \leq 41.36 \times \frac{b}{1+t},$$
$$2185.42 - 41.36 \times t - (21x_1 + 24x_2 + 22.5x_3) \leq 41.36 \times c \times (1-t),$$
$$58x_1 + 67x_2 + 60x_3 - 5880.63 \geq 97.55 \times a,$$
$$5978.18 - (58x_1 + 67x_2 + 60x_3) \leq 97.55 \times \frac{b \times (1+2t)}{1+t},$$
$$5978.18 - (58x_1 + 67x_2 + 60x_3) \leq 97.55 \times c \times (1+t),$$
$$80x_1 + 77x_2 + 82x_3 - 7499.38 \geq 296.62 \times a,$$
$$7796 - 296.62 \times \frac{t}{1+t} - (80x_1 + 77x_2 + 82x_3) \leq 296.62 \times \frac{b}{1+t},$$
$$7796 - 296.62 \times t - (80x_1 + 77x_2 + 82x_3) \leq 296.62 \times c \times (1-t),$$
$$b \leq a,$$
$$c \leq a,$$
$$a + b + c \leq 3,$$
$$0 \leq b,$$
$$0 \leq c,$$
$$170x_1 + 200x_2 + 185x_3 \leq 18000,$$
$$10x_1 + 11x_2 + 9x_3 \geq 950,$$
$$70x_1 + 79x_2 + 75x_3 \leq 7250,$$
$$2.5x_1 + 2x_2 + 1.5x_3 \leq 185,$$
$$130x_1 + 145x_2 + 135x_3 \geq 13000,$$
$$35x_1 + 40x_2 + 38x_3 \leq 3700,$$
$$x_1, x_2, x_3 \geq 0,$$
$$0.1 \leq t \leq 0.9;$$

Step 9 : *For different t, several optimal stages are shown in Table 4 using software LINGO. Corresponding final optimal values for objective functions are given in Table 5 for different t.*

Proposition 4.1. *(Analysis of results and validation of methodology)*
The developed method provides the set of solutions with different degrees of satisfaction to the experts. Then experts may have a flexibility to choose a suitable optimal solution in demand of situation arisen. For the present problem, it is seen from Table 4 that the difference between acceptance part of a decision set and its sum of hesitation and rejection part goes towards perfection (i.e., $a - b - c$ goes towards 1) as t tends to 1. More clearly, this perfection arises when the higher value (lower value) of both hesitation and rejection part of a decision set are far from the higher value (lower value) of acceptance part for maximize (minimize) objective (see the Step 6 of algorithm). It is also seen that as t raises, both hesitation and rejection part of a decision gradually decrease whatever the state of acceptance value is. Thus the model directs the experts not only to find a decision of an

Table 4. Several stages for optimal solutions of Example [?] in nts.

t	x_1	x_2	x_3	a	b	c	$a-b-c$
0.1	24.92621	29.95515	41.84944	0.7852400	0.1362360	0.1351101	0.5138940
0.2	24.54591	30.95119	41.15523	0.7493202	0.1008157	0.9801529E-01	0.5504892
0.3	24.17727	31.91668	40.48232	0.7145019	0.7114758E-01	0.6735866E-01	0.5759956
0.4	23.82759	32.83250	39.84401	0.6814746	0.4593561E-01	0.4218576E-01	0.5933532
0.5	23.49967	33.69133	39.24543	0.6505027	0.2424595E-01	0.2155196E-01	0.6047048
0.6	23.19400	34.49190	38.68747	0.6216321	0.5388669E-02	0.4630888E-02	0.6116125
0.7	23.10345	34.72906	38.52217	0.6130791	0.1931560E-04	0.1604063E-04	0.6130438
0.8	23.10345	34.72906	38.52217	0.6130791	0.1887859E-04	0.1514949E-04	0.6130451
0.9	23.10345	34.72906	38.52217	0.6130791	0.1850402E-04	0.1435215E-04	0.6130463

Table 5. Optimal values for objective functions

t	$\max g_1$	$\min g_2$	$\max g_3$
0.1	2183.99	5963.68	7732.30
0.2	2184.29	5966.71	7721.64
0.3	2184.58	5969.64	7711.32
0.4	2184.85	5972.42	7701.52
0.5	2185.11	5975.03	7692.33
0.6	2185.35	5977.45	7683.77
0.7	2185.42	5978.18	7681.23
0.8	2185.42	5978.18	7681.23
0.9	2185.42	5978.18	7681.23

$molp$-problem but also to make a perfection of that. Moreover as the decision making in uncertain climate, by instinct, is full of hesitancy and indeterminacy, so hesitancy of experts should be cultivated more precisely on setting a decision. Therefore outcome of a decision making over nts is much better than in fuzzy or ifs climate.

Further, let us consider the problem stated by Roy and Das [24] and also the methodology designed therein. For that problem, a comparison of outcomes obtained by two methodologies is now drawn in Table 6. In earlier method, infeasible solutions are appeared at some stages, and setting of neutrosophic components for each objective function require two indicators (i.e., t and λ). This increases the number of stages for optimality, and then it may bring a lot of confusion to decision maker to opt the suitable one. Again, no any development for a $molp$-problem with minimize objective function is seen there. They made an attempt to elevate the trustiness, ambiguity of a decision and to lower down its falsity status. In practical arena, one always tries to get a reliable end over its ambiguity and falsity. All these regards ensure the superiority of present study.

Example 4.2. *A jeweller wishes to make three types of ornaments viz. necklace, bracelet and ear ring for her business purpose. To these, she likes to use lesser quantity of costly metal (pure gold, silver) and greater quantity of cheap metal (palladium, nickel). The following Table 7 provides all necessary information. With respect to provided data, find out the number of ornaments to be produced by jeweller monthly to attain the maximum profit by minimizing the metal cost and making charge.*

Suppose, the number of necklace, bracelet and ring to be made per month be x_1, x_2, x_3 respectively.

Table 6. Comparison of outcomes for the problem defined in [24].

Methodology →	Roy and Das [24]	Bera and Mahapatra
Objective value	1.391557	0.4864383
a	0.5308708	0.5308708
b	0.4135885	0.4443252E-01
c	0.5308708	0.0
x	65.25714	65.25714
y	26.91871	26.91871
z	49.83236	49.83236
t	0.8	0.8
λ	0.1 to 0.9	NA
Objective value	1.123483	0.3271771
a	0.5308708	0.5308708
b	0.6174164E-01	0.2036938
c	0.5308708	0.0
x	65.25714	65.25714
y	26.91871	26.91871
z	49.83236	49.83236
t	0.5	0.5
λ	0.1 to 0.9	NA
Objective value		0.1588747
a		0.5308708
b		0.2567809
c		0.1152153
x	infeasible solution	65.25714
y	found	26.91871
z		49.83236
t	0.4	0.4
λ	0.1 to 0.9	NA
Objective value		-0.2061023E-01
a		0.5308708
b		0.3098679
c		0.2416131
x	infeasible solution	65.25714
y	found	26.91871
z		49.83236
t	0.3	0.3
λ	0.1 to 0.9	NA

Table 7. Estimation of various subjects related to production.

Metal	Unit	1 Necklace	2 Bracelet	2 Ear ring	Availability / month
Gold	gm	10	10.5	4	≤ 810
Silver	gm	9.8	11	2.5	≤ 770
Nickel	gm	10	8	3	≥ 675
Palladium	gm	8	9	3	≥ 650
Unit profit	Rs	9800	10000	3500	max
Metal cost	Rs	48750	52000	17000	min
Making charge	Rs	5800	5400	2800	min

Mathematically, the problem is then designed as :

$$\max\ \{g_1 = 9800x_1 + 10000x_2 + 3500x_3\}$$
$$\min\ \{g_2 = 48750x_1 + 52000x_2 + 17000x_3\}$$
$$\min\ \{g_3 = 5800x_1 + 5400x_2 + 2800x_3\}$$

$$\begin{aligned} \textit{subject to} \quad & 10x_1 + 10.5x_2 + 4x_3 \leq 810 \\ & 9.8x_1 + 11x_2 + 2.5x_3 \leq 770 \\ & 10x_1 + 8x_2 + 3x_3 \geq 675 \\ & 8x_1 + 9x_2 + 3x_3 \geq 650 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

Solution with 1st objective is : $x_1 = 60.91549, x_2 = 10.70423, x_3 = 22.11268$.
Solution with 2nd objective is : $x_1 = 25.68182, x_2 = 26.36364, x_3 = 69.09091$.
Solution with 3rd objective is : $x_1 = 31.16776, x_2 = 37.33553, x_3 = 21.54605$.
The objective values with respect to these solutions are provided by Table 8.

Table 8. Objective values with respect to the solutions.

(x_1, x_2, x_3)	g_1	g_2	g_3
(60.91549, 10.70423, 22.11268)	781408.50*	3899119.88	473028.19
(25.68182, 26.36364, 69.09091)	757136.42	3796159*	484772.76
(31.16776, 37.33553, 21.54605)	754210.52	3825600.32	442713.80*

Several optimal stages are shown in Table 9 with different t *using software LINGO. Hence,* $\max g_1 = 776110.16$, $\min g_2 = 3876644.70$, $\min g_3 = 475591.82$ *for all* t.

Table 9. Several stages for optimal solutions of Example 4.2

t	x_1	x_2	x_3	a	b	c	$a-b-c$
0.1	53.22448	14.12245	32.36736	0.7817131	0.2000963	0.1984426	0.3831741
0.2	53.22448	14.12245	32.36736	0.7817131	0.1871031	0.1819057	0.4127043
0.3	53.22448	14.12245	32.36736	0.7817131	0.1773581	0.1679130	0.4364420
0.4	53.22448	14.12245	32.36736	0.7817131	0.1697787	0.1559192	0.4560152
0.5	53.22448	14.12245	32.36736	0.7817131	0.1637152	0.1455246	0.4724733
0.6	53.22448	14.12245	32.36736	0.7817131	0.1587541	0.1364293	0.4865297
0.7	53.22448	14.12245	32.36736	0.7817131	0.1546199	0.1284041	0.4986892
0.8	53.22448	14.12245	32.36736	0.7817131	0.1511217	0.1212705	0.5093209
0.9	53.22448	14.12245	32.36736	0.7817131	0.1481233	0.1148878	0.5187020

5. Conclusion

This study finds an non-deterministic solution approach of *molp*-problem. To emphasize the hesitancy of experts in decision making, the methodology is developed over *nts* theory. Here an *molp*-problem is primarily treated as a number of single objective crisp *lp*-problem under the asset of provided constraints. Each objective function corresponds a neutrosophic decision set. The intersection of all these set admits an optimal decision set of *molp*-problem. A suitable algorithm is furnished to sketch this solution approach. It is illustrated on two real grounds. The approach directs the experts not only to find a decision of an *molp*-problem but also to make a perfection of that by elevating its acceptance level and lower down its hesitancy and rejection level. A comparative analysis on the outcomes of application field over *nts*, fuzzy and *ifs* environment is also done. In today's uncertain state, this model ensures the trustiness of the worked out solution over its ambiguity and falsities.

But we welcome a software based algorithm and flow chart to attain a quick outcome of problem. We are also looking forward to solve multi-objective nonlinear programming problems under this setting in near future.

6. Competing Interests

Authors certify that they have no affiliations with or involvement in any organization or entity with any financial or non-financial interest in the subject matter or materials discussed in this manuscript.

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