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Remarks on n_m-pre-continuous maps

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Abstract: In this article, n_m-pre-open sets, n_m-pre-continuous, $n_m - T_0$-spaces, n_m-precompact and almost n_m-precompact in neutrosophic pre-minimal structure spaces are introduced and several of their properties and characterizations are established.

Key words: n_m-pre-open, n_m-pre-continuous, $n_m - T_0$-spaces, n_m-T_1-spaces, $n_m - T_2$-spaces, n_m-pre-closed graph, n_m-compact, n_m-precompact and almost n_m-precompact

1. Introduction

The contribution of mathematics to the present-day technology in reaching to a fast trend cannot be ignored. The theories presented differently from classical methods in studies such as fuzzy set [14], intuitionistic set [5], intuitionistic fuzzy set [4] , soft set [9], neutrosophic set [12, 13], etc., have great importance in this contribution of mathematics in recent years. Neutrosophic set is described by three functions : a membership function, indeterminacy function and a nonmembership function that are independently related. The theories of neutrosophic set have achieved greater success in various areas such as medical diagnosis, database, topology, image processing and decision making problems. While the neutrosophic set is a powerful tool in dealing with indeterminate and inconsistent data, the theory of rough set is a powerful mathematical tool to deal with incompleteness.

V. Popa and T. Noiri [10] introduced the notion of minimal structure which is a generalization of a topology on a given nonempty set. And they introduced the notion of $\mathcal{M}$-continuous functions as functions defined between minimal structures. M. Karthika et al [8] introduced and studied neutrosophic minimal structure spaces. S. Ganesan et al [6] introduced and studied n_m-α-open sets in neutrosophic minimal structure spaces.

The main objective of this study is to introduce a new hybrid intelligent structure called neutrosophic pre-minimal continuous. The significance of introducing hybrid structures is that the computational techniques, based on any one of these structures alone, will not always yield the best results but a fusion of two or more of them can often give better results.

The rest of this chapter is organized as follows. Some preliminary concepts required in our work are briefly recalled in Section 2. In Section 3, the chapter with some properties on n_m-T_0-spaces, n_m-T_1-spaces, n_m-T_2-spaces, n_m-pre-closed graph, n_m-compact, n_m-precompact and almost n_m-precompact.

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2. Preliminaries

Definition 2.1. [10] A subfamily m_x of the power set $\wp$(X) of a nonempty set X is called a minimal structure (in short, m-structure) on X if $\emptyset \in m_x$ and X $\in m_x$. By (X, m_x), we denote a nonempty set X with a minimal structure m_x on X and call it an m-space.
Each member of m_x is said to be m_x-open (or in short, m-open) and the complement of an m_x-open set is said to be m_x-closed (or in short, m-closed).

Definition 2.2. [12, 13] A neutrosophic set (in short ns) K on a set X $\neq \emptyset$ is defined by K = $\{\prec$ a, P_K(a), Q_K(a), R_K(a) $\succ$: a $\in$ X$\}$ where P_K : X $\to$ [0,1], Q_K : X $\to$ [0,1] and R_K : X $\to$ [0,1] denotes the membership of an object, indeterminacy and non-membership of an object, for each a $\in$ X to K, respectively and 0 $\leq$ P_K(a) + Q_K(a) + R_K(a) $\leq$ 3 for each a $\in$ X.

Definition 2.3. [8] Let the neutrosophic minimal structure space over a universal set X be denoted by n_m. n_m is said to be neutrosophic minimal structure space (in short, nms) over X if it satisfying following the axiom: $0_\sim$, $1_\sim$ $\in$ n_m. A family of neutrosophic minimal structure space is denoted by (X, N_{mX}).

Definition 2.4. [7] Let (X, N_{mX}) be a nms. Then, X is said to be n_m-$T_{\frac{1}{2}}$ if every n_mg-closed in X is n_m-closed.

Theorem 2.1. *[7] Let (X, N_{mX}) be a nms. Then, X is a n_m-$T_{\frac{1}{2}}$ if and only if {x} is n_m-closed or n_m-open, for each $x \in X$.*

3. n_m-pre-continuous map

Definition 3.1. Let (X, N_{mX}) be a nms and A $\leq$ X. A subset A of X is called an n_m-pre-open set if A $\leq$ $n_m$int($n_m$cl(A)).
The complement of an n_m-pre-open set is called an n_m-pre-closed set.

Definition 3.2. A map f : (X, N_{mX}) $\to$ (Y, N_{mY}) is called neutrosophic pre-minimal continuous (in short, n_m-pre-continuous) if f^{-1}(V) is a n_m-pre-open set in X, for each n_m-open set V in Y.

Lemma 3.1. 1. *Every neutrosophic minimal continuous is n_m-pre-continuous but the conversely.*

2. *Every n_m-α-continuous is n_m-pre-continuous but not conversely.*

Definition 3.3. Let (X, N_{mX}) be a nms. Then X is said to be

1. n_m-T_0 if for each pair of distinct points x and y in X, there exist a n_m-open set U such that either x $\in$ U and y $\notin$ U or x $\notin$ U and y $\in$ U.
2. n_m-T_1 if for each pair of distinct points x and y in X, there exist two n_m-open sets U and V such that either x $\in$ U but y $\notin$ U and y $\in$ V but x $\notin$ V.
3. n_m-T_2 if for each distinct points x and y of X, there exist two disjoint n_m-open sets U, V such that x $\in$ U and y $\in$ V.

Theorem 3.1. *Let (X, N_{mX}) be a nms. Then, X is n_m-T_0 if and only if for each pair of distinct points x, y of X, $n_m$cl({x}) $\neq$ $n_m$cl({y}).*

Proof:
Necessity. Let X be n_m-T_0 and x, y be any two distinct points of X. Then, there exists a n_m-open set U containing x or y, say x but not y. Then, X $\setminus$ U is a n_m-closed set which does not contain x but contains y. Since $n_m$cl({y}) is the smallest n_m-closed set containing y, then $n_m$cl({y}) $\leq$ X $\setminus$ U and therefore x $\notin$ $n_m$cl({y}). Consequently $n_m$cl({x}) $\neq$ $n_m$cl({y}).
Sufficiency. Suppose that x, y $\in$ X, x $\neq$ y and $n_m$cl({x}) $\neq$ $n_m$cl({y}). Let z be a point of X such that z $\in$ $n_m$cl({x}) but z $\notin$ $n_m$cl({y}). We claim that $x\# \notin$ $n_m$cl({y}). For, if x $\in$ $n_m$cl({y}) then $n_m$cl({x}) $\leq$ $n_m$cl({y}). This contradicts the fact that z $\notin$ $n_m$cl({y}). Consequently x belongs to the n_m-open set X $\setminus$ $n_m$cl({y}) to which y does not belong.

Theorem 3.2. *Let (X, N_{mX}) be a nms. Then, X is n_m-T_1 if and only if the singletons are n_m-closed sets.*

Proof. Let X be n_m-T_1 and x any point of X. Suppose y $\in$ X $\setminus$ {x}, then x $\neq$ y and so there exists a n_m-open set U such that y $\in$ U but x $\notin$ U. Consequently y $\in$ U $\leq$ X $\setminus$ {x}, that is X $\setminus$ {x} = max {U : y $\in$ X $\setminus$ {x}} which is n_m-open.
Conversely, suppose {r} is n_m-closed for every r $\in$ X. Let x, y $\in$ X with x $\neq$ y. Now, x $\neq$ y implies y $\in$ X $\setminus$ {x}. Hence, X $\setminus$ {x} is a n_m-open set contains y but not x. Similarly X $\setminus$ {y} is a n_m-open set contains x but not y. Accordingly X is n_m-T_1. □

Theorem 3.3. *Let (X, N_{mX}) be a nms. Then, the following statements are equivalent:*

1. *X is n_m-T_2.*
2. *Let $x \in X$. For each $y \neq x$, there exists a n_m-open set U containing x such that $y \notin n_m cl(U)$.*
3. *For each $x \in X$, max $\{n_m cl(U) : U \in N_{mX}$ and $x \in U\} = \{x\}$.*

Proof. (1) ⇒ (2): Since X is n_m-T_2, then there exist disjoint n_m-open sets U and V containing x and y respectively. So, U $\leq$ X $\setminus$ V. Therefore, $n_m$cl(U) $\leq$ X $\setminus$ V. So, y $\notin$ $n_m$cl(U).
(2) ⇒ (3): If possible for some y $\neq$ x, we have y $\in$ $n_m$cl(U) for every n_m-open set U containing x, which then contradicts (2).
(3) ⇒ (1): Let x, y $\in$ X and x $\neq$ y. Then, there exists a n_m-open set U containing x such that y $\notin$ $n_m$cl(U). Let V = X $\setminus$ $n_m$cl(U), then y $\in$ V, x $\in$ U and U $\cap$ V = $\emptyset$. Thus, X is n_m-T_2. □

Theorem 3.4. *Let (X, N_{mX}) be a nms. Then, then the following statements are hold:*

1. *Every n_m-T_2-space is n_m-T_1.*
2. *Every n_m-T_1-space is n_m-$T_{\frac{1}{2}}$*
3. *Every n_m-$T_{\frac{1}{2}}$-space is n_m-T_0.*

Proof. 1. The proof is straightforward from the definitions.

2. The proof is obvious by Theorem 3.2.

3. Let x and y be any two distinct points of X. By Theorem 2.1, the singleton set {x} is n_m-closed or n_m-open.
   (i) If {x} is n_m-closed, then X \ {x} is n_m-open. So y ∈ X \ {x} and x ∉ X \ {x}. Therefore, we have X is n_m-T_0.
   (ii) If {x} is n_m-open, then x ∈ {x} and y ∉ {x}. Therefore, we have X is n_m-T_0.

□

Definition 3.4. Let f : X → Y be a map on two nms (X, N_{mX}) and (Y, N_{mY}). Then f has an n_m-pre-closed graph if for each (x, y) ∈ (X × Y) − G(f), there exist an n_m-pre-open set U containing x and an n_m-open set V containing y such that (U × V) ∩ G(f) = ∅.

Lemma 3.2. *Let f : X → Y be a map on two nms (X, N_{mX}) and (Y, N_{mY}). Then f has an n_m-pre-closed graph if and only if for each (x, y) ∈ (X × Y) − G(f), there exist an n_m-pre-open set U containing x and an n_m-open set V containing y such that f(U) ∩ V = ∅.*

Theorem 3.5. *Let f : X → Y be a map on two nms (X, N_{mX}) and (Y, N_{mY}). If f is n_m-precontinuous and (Y, N_{mY}) is n_m-T_2, then G(f) is an n_m-pre-closed graph.*

Proof. Let (x, y) ∈ (X × Y) − G(f) then f(x) ≠ y. Since Y is n_m-T_2, there are disjoint n_m-open sets U, V such that f(x) ∈ U, y ∈ V. Then for f(x) ∈ U, by n_m-precontinuity, there exists an n_m-pre-open set G containing x such that f(G) ≤ U. Consequently, there exist an n_m-open set V and n_m-pre-open set G containing y, x respectively, such that f(G) ∩ V = ∅. Therefore, by Lemma 3.2, G(f) is n_m-pre-closed. □

Definition 3.5. Let (X, N_{mX}) be a nms. Then X is said to be n_m-pre-T_2 if for any distinct points x and y of X, there exist disjoint n_m-preopen sets U, V such that x ∈ U and y ∈ V.

Theorem 3.6. *Let f : X → Y be a map on two nms (X, N_{mX}) and (Y, N_{mY}). If f is an injective and n_m-precontinuous map and if Y is n_m-T_2, then X is n_m-pre-T_2.*

Proof. Obvious. □

Theorem 3.7. *Let f : X → Y be a map on two nms (X, N_{mX}) and (Y, N_{mY}). If f is an injective and n_m-precontinuous map with an n_m-pre-closed graph, then X is n_m-pre-T_2.*

Proof. Let x_1 and x_2 be any distinct points of X. Then $f(x_1) \neq f(x_2)$, so $(x_1, f(x_2)) \in (X \times Y) - G(f)$. Since the graph G(f) is n_m-pre-closed, there exist an n_m-pre-open set U containing x_1 and V ∈ N_{mY} containing $f(x_2)$ such that f(U) ∩ V = ∅. Since f is n_m-precontinuous, $f^{-1}(V)$ is an n_m-pre-open set containing x_2 such that $U \cap f^{-1}(V) = \emptyset$. Hence X is n_m-pre-T_2. □

Definition 3.6. (X, N_{mX}) be a nms and A ≤ X, A is said to be n_m-compact (resp. almost n_m-compact) relative to A if every collection $\{U_i : i \in \Delta\}$ of n_m-open subsets of X such that $A \leq \max\{U_i : i \in \Delta\}$, there exists a finite subset Δ_0 of Δ such that $A \leq \max\{U_j : j \in \Delta_0\}$ (resp. $A \leq \max\{n_m cl(U_j) : j \in \Delta_0\}$). (X, N_{mX}) be a nms and A ≤ X, A is said to be n_m-compact (resp. almost n_m-compact) if A is n_m-compact (resp. almost n_m-compact) as a neutrosophic minimal subspace of X.

Definition 3.7. (X, N_{mX}) be a nms and A $\leq$ X, A is said to be n_m-precompact (resp. almost n_m-precompact) relative to A if every collection $\{U_\delta : \delta \in \Delta\}$ of n_m-pre-open subsets of X such that A $\leq$ max $\{U_\delta : \delta \in \Delta\}$, there exists a finite subset Ω of Δ such that A $\leq$ max $\{U_\omega : \omega \in \Omega\}$ (resp. A $\leq$ max $\{n_m pcl(U_\omega) : \omega \in \Omega\}$). (X, N_{mX}) be a nms and A $\leq$ X, A is said to be n_m-precompact (resp. almost n_m-precompact) if A is n_m-precompact (resp. almost n_m-precompact) as a neutrosophic minimal subspace of X.

Theorem 3.8. *Let f : X $\to$ Y be a map on two nms (X, N_{mX}) and (Y, N_{mY}). If A is an n_m-precompact set, then f(A) is n_m-compact.*

Proof. Obvious. □

Conclusion

We presented several definitions, properties, explanations and examples inspired from the concept of neutrosophic minimal pre-closed sets and neutrosophic minimal pre-continuous maps. The results of this study may be help in many researches.

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