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Nano weakly $g^{\#}$-closed sets in nano topological spaces

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Abstract: We define and study the concepts of nano weakly $g^{\#}$-closed sets by using the concept of nano $g^{\#}$-closed set in nano topological spaces. Further, we discuss the notions such as nano weakly $g^{\#}$-continuous functions, nano weakly $g^{\#}$-open functions and nano weakly $g^{\#}$-closed functions in this paper.

Key words: $nwg^{\#}$-closed, $nwg^{\#}$-continuous, $ng^{\#}$-compact and $nwg^{\#}$-compact

1. Introduction

Lellis Thivagar, et al [6] was the main brain behind developing the concept of nano topology. It is constructed in terms of lower and upper approximations and boundary region of a subset of a universe. The elements of the nano topology are called the nano open sets. He further established the weak forms of nano topology, nano extremally disconnected space, nano topology in $\check{C}$ech rough closure space, nano topology via neutrosophic sets so on. Recently several researcher were introduced and studied the new sets and functions in nano topological spaces for example[[10], [2] and [17]]. In this paper is to introduce and study the concepts of nano weakly $g^{\#}$-closed sets by using the concept of nano $g^{\#}$-closed set in nano topological spaces. Further, we discuss the some notions such as nano weakly $g^{\#}$-continuous functions, nano weakly $g^{\#}$-open functions and nano weakly $g^{\#}$-closed functions with suitable examples are given.

2. Preliminaries

Definition 2.1. [15] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$.

1. The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X) = \bigcup_{x\in U}\{R(x) : R(x) \subseteq X\}$, where R(x) denotes the equivalence class determined by x.
2. The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is, $U_R(X) = \bigcup_{x\in U}\{R(x) : R(x) \cap X \neq \phi\}$.

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3. The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not - X with respect to R and it is denoted by $B_R(X)$. That is, $B_R(X) = U_R(X) - L_R(X)$.

Definition 2.2. [6] Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then $\tau_R(X)$ satisfies the following axioms:

1. U and $\phi \in \tau_R(X)$,
2. The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$,
3. The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

Thus $\tau_R(X)$ is a topology on U called the nano topology with respect to X and $(U, \tau_R(X))$ is called the nano topological space. The elements of $\tau_R(X)$ are called nano-open sets (briefly n-open sets). The complement of a n-open set is called n-closed.

We denote a nano topological space (or) space by $(U, \mathcal{N})$, where $\mathcal{N} = \tau_R(X)$. The nano-interior and nano-closure of a subset A of U are denoted by $nint(A)$ and $ncl(A)$, respectively.

Definition 2.3. A subset H of a space $(U, \mathcal{N})$ is called

1. nano semi open [6] if $H \subseteq ncl(nint(H))$.
2. nano regular-open [6] if $H = nint(ncl(H))$.
3. nano β-open [18] if $H \subseteq ncl(nint(ncl(H)))$.
4. nano b-open [13] if $H \subseteq nint(ncl(H)) \cup ncl(nint(H))$.
5. nano π-open [1] if the finite union of nano regular-open sets.
6. nano α-open set [6] if $H \subseteq nint(ncl(nint(H)))$.

The complements of the above used sets are called their respective closed sets.

Definition 2.4. A subset H of a space $(U, \mathcal{N})$ is called

1. nano g-closed [3] if $ncl(H) \subseteq G$, whenever $H \subseteq G$ and G is nano open.
2. nano πg-closed [16] if $ncl(H) \subseteq G$, whenever $H \subseteq G$ and G is nano π-open.
3. nano $g\alpha$-closed [8] if $n\alpha cl(H) \subseteq G$ whenever $H \subseteq G$ and G is nano α-open.
4. nano αg-closed set [8] if n-$\alpha cl(H) \subseteq G$ whenever $H \subseteq G$ and G is nano open.
5. nano sg-closed set [4] if $nscl(H) \subseteq G$, whenever $H \subseteq G$ and G is nano semi open.

The complements of the above used sets are called their respective open sets.

Definition 2.5. A subset H of a nano topological space $(U, \mathcal{N})$ is called

1. nano regular closed [6] if $H = ncl(nint(H))$.
2. nano semi open set [6] if $H \subseteq ncl(nint(H))$.
3. nano rg-closed [20] if $ncl(H) \subseteq G$ whenever $H \subseteq G$ and G is n-regular open.

4. nano dense [19] if $ncl(H) = U$,
5. nano nowhere dense [7] if $nint(ncl(H)) = \phi$.
6. nano codense [11] if $U - H$ is n-dense.

Definition 2.6. A subset H of a space $(U, \mathcal{N})$ is called

1. nwg-closed set [9] if $ncl(nint(H) \subseteq V$ whenever $H \subseteq V$ and V is nano open.
2. $nrwg$-closed set [9] if $ncl(nint(H)) \subseteq U$ whenever $H \subseteq U$ and U is regular open.
3. nwI_{rg}-closed set [12] if $(nint(E))^* \subseteq F$ whenever $E \subseteq F$ and F is a n-regular open set in U.
4. $ng^{\#}$-closed set [5] if $ncl(H) \subseteq G$ whenever $H \subseteq G$ and G is $n\alpha g$-open in U.

In future nano topological spaces $(U, \tau_R(X))$ is referred as a space $(U, \tau_R(X))$.

3. Nano weakly $g^{\#}$-closed sets

We introduce the definition of nano weakly $g^{\#}$-closed sets in nano topological spaces and study the relationships of such sets.

Definition 3.1. A subset E of a space $(U, \tau_R(X))$ is called a nano weakly $g^{\#}$-closed (briefly, $nwg^{\#}$-closed) set if $ncl(nint(E)) \subseteq G$ whenever $E \subseteq G$ and G is $n\alpha g$-open in U.

Proposition 3.1. *Each $ng^{\#}$-closed set is $nwg^{\#}$-closed in a space $(U, \tau_R(X))$.*

Remark 3.1. *The converse of Proposition 3.1 is need not be true as seen from the following example.*

Example 3.1. *Let $U = \{h_1, h_2, h_3\}$ with $U/R = \{\{h_1, h_2\}, \{h_3\}\}$ and $X = \{h_1, h_2\}$. Then $\tau_R(X) = \{\phi, \{h_1, h_2\}, U\}$. Then the subset $\{h_1\}$ is $nwg^{\#}$-closed set but not a $ng^{\#}$-closed in $(U, \tau_R(X))$.*

Proposition 3.2. *Each $nwg^{\#}$-closed set is nwg-closed in a space $(U, \tau_R(X))$.*

Proof. Let E be any $nwg^{\#}$-closed set and G be any nano open set containing E. Then G is an $n\alpha g$-open set containing E. We have $ncl(nint(E)) \subseteq G$. Thus, E is nwg-closed.

Remark 3.2. *The converse of Proposition 3.2 is need not be true as seen from the following example.*

Example 3.2. *Let $U = \{h_1, h_2, h_3\}$ with $U/R = \{\{h_1\}, \{h_2, h_3\}\}$ and $X = \{h_1\}$. Then $\tau_R(X) = \{\phi, \{h_1\}, U\}$. Then the subset $\{h_1, h_2\}$ is nwg-closed set but not $nwg^{\#}$-closed in $(U, \tau_R(X))$.*

Proposition 3.3. *Each $nwg^{\#}$-closed set is $nw\pi g$-closed in a space $(U, \tau_R(X))$.*

Proof. Let E be any $nwg^{\#}$-closed set and G be any $n\pi$-open set containing E. Then G is a nsg-open set containing E. We have $ncl(nint(E)) \subseteq G$. Thus, E is $nw\pi g$-closed.

Remark 3.3. *The converse of Proposition 3.3 is need not be true as seen from the following example.*

Example 3.3. *In Example 3.2, then the subset $\{h_1, h_3\}$ is $nw\pi g$-closed but not $nwg^{\#}$-closed.*

Proposition 3.4. *Each $nwg^{\#}$-closed set is $nrwg$-closed in a space $(U, \tau_R(X))$.*

Proof. Let E be any $nwg^{\#}$-closed set and G be any nano regular open set containing E. Then G is an $n\alpha g$-open set containing E. We have $ncl(nint(E)) \subseteq G$. Thus, E is $nrwg$-closed.

Remark 3.4. *The converse of Proposition 3.4 is need not be true as seen from the following example.*

Example 3.4. *In Example 3.2, then the subset $\{h_1\}$ is $nrwg$-closed but not $nwg^{\#}$-closed.*

Theorem 3.1. *If a subset E of a space U is both nano closed and ng-closed, then it is $nwg^{\#}$-closed in U.*

Proof. Let E be a ng-closed set in U and G be any nano open set containing E. Then $G \supseteq ncl(E) \supseteq ncl(nint(ncl(E)))$. Since E is nano closed, $G \supseteq ncl(nint(E))$ and hence $nwg^{\#}$-closed in U.

Theorem 3.2. *If a subset E of a space U is both nano open and $nwg^{\#}$-closed, then it is nano closed.*

Proof. Since E is both nano open and $nwg^{\#}$-closed, $E \supseteq ncl(nint(E)) = ncl(E)$ and hence E is nano closed in U.

Corollary 3.1. *If a subset E of a space U is both nano open and $nwg^{\#}$-closed, then it is both nano regular open and nano regular closed in U.*

Theorem 3.3. *Let U be a space and $E \subseteq U$ be nano open. Then, E is $nwg^{\#}$-closed if and only if E is $ng^{\#}$-closed.*

Proof. Let E be $ng^{\#}$-closed. By Proposition 3.1, it is $nwg^{\#}$-closed. Conversely, let E be $nwg^{\#}$-closed. Since E is nano open, by Theorem 3.2, E is nano closed. Hence E is $ng^{\#}$-closed.

Theorem 3.4. *If a set E of U is $nwg^{\#}$-closed, then $ncl(nint(E)) - E$ contains no non-empty $n\alpha g$-closed set.*

Proof. Let H be an $n\alpha g$-closed set such that $H \subseteq ncl(nint(E)) - E$. Since H^c is $n\alpha g$-open and $E \subseteq H^c$, from the definition of $nwg^{\#}$-closedness it follows that $ncl(nint(E)) \subseteq H^c$. i.e., $H \subseteq (ncl(nint(E)))^c$. This implies that $H \subseteq (ncl(nint(E))) \cap (ncl(nint(E)))^c = \phi$.

Proposition 3.5. *If a subset E of a space U is nano nowhere dense, then it is $nwg^{\#}$-closed.*

Proof. Since $nint(E) \subseteq nint(ncl(E))$ and E is nano nowhere dense, $nint(E) = \phi$. Therefore $ncl(nint(E)) = \phi$ and hence E is $nwg^{\#}$-closed in U.

Remark 3.5. *The converse of Proposition 3.5 need not be true as seen in the following example.*

Example 3.5. *Let $U = \{h_1, h_2, h_3\}$ with $U/R = \{\{h_1\}, \{h_2, h_3\}\}$ and $X = \{h_1, h_2\}$. Then $\tau_R(X) = \{\phi, \{h_1\}, \{h_2, h_3\}, U\}$. Then the subset $\{h_1\}$ is $nwg^{\#}$-closed set but not nano nowhere dense in U.*

Remark 3.6. *The following examples show that the concepts of $nwg^{\#}$-closedness and the concepts of nano semi-closedness are independent of each other.*

Example 3.6. *In Example 3.1, then the subset $\{h_1, h_3\}$ is $nwg^{\#}$-closed set but not nano semi-closed in U.*

Example 3.7. *Let $U = \{h_1, h_2, h_3\}$ with $U/R = \{\{h_1\}, \{h_2\}, \{h_3\}\}$ and $X = \{h_1, h_2\}$. Then $\tau_R(X) = \{\phi, \{h_1\}, \{h_2\}, \{h_1, h_2\}, U\}$. Then the subset $\{h_1\}$ is nano semi-closed set but not $nwg^{\#}$-closed in U.*

Remark 3.7. *From the above discussions and known results, we obtain the following diagram, where $E \to F$ represents E implies F but not conversely.*

Diagram

$$nano\ closed \to nwg^{\#}\text{-}closed \to nwg\text{-}closed \to nw\pi g\text{-}closed \to nrwg\text{-}closed$$

Definition 3.2. A subset E of a space U is called $nwg^{\#}$-open set if E^c is $nwg^{\#}$-closed in U.

Proposition 3.6.

1. *Every $ng^{\#}$-open set is $nwg^{\#}$-open but not conversely.*
2. *Every ng-open set is $nwg^{\#}$-open but not conversely.*

Theorem 3.5. *A subset E of a space U is $nwg^{\#}$-open if $H \subseteq nint(ncl(E))$ whenever $H \subseteq E$ and H is $n\alpha g$-closed.*

Proof. Let E be any $nwg^{\#}$-open. Then E^c is $nwg^{\#}$-closed. Let H be an $n\alpha g$-closed set contained in E. Then H^c is an $n\alpha g$-open set containing E^c. Since E^c is $nwg^{\#}$-closed, we have $ncl(nint(E^c)) \subseteq H^c$. Therefore $H \subseteq nint(ncl(E))$.

Conversely, we suppose that $H \subseteq nint(ncl(E))$ whenever $H \subseteq E$ and H is $n\alpha g$-closed. Then H^c is an $n\alpha g$-open set containing E^c and $H^c \supseteq (nint(ncl(E)))^c$. It follows that $H^c \supseteq ncl(nint(E^c))$. Hence E^c is $nwg^{\#}$-closed and so E is $nwg^{\#}$-open.

Definition 3.3. Let $f : (U, \tau_R(X)) \to (V, \tau'_R(Y))$ be a function. Then f is said to be nano contra $g^{\#}$-continuous if the inverse image of every nano open set in V is $ng^{\#}$-closed set in U.

Theorem 3.6. *The following are equivalent for a function $f : (U, \tau_R(X)) \to (V, \tau'_R(Y))$,*

1. *f is nano contra $g^{\#}$-continuous.*
2. *the inverse image of every nano closed set of V is $ng^{\#}$-open in U.*

Proof. Let G be any nano closed set of V. Since $V \backslash G$ is nano open, then by (1), it follows that $f^{-1}(V \backslash G) = U \backslash f^{-1}(G)$ is $ng^{\#}$-closed. This shows that $f^{-1}(G)$ is $ng^{\#}$-open in U.

Converse part is similar.

4. Nano weakly $g^{\#}$-continuous functions

Definition 4.1. Let U and V be two nano topological spaces. A function $f : (U, \tau_R(X)) \to (V, \tau'_R(Y))$ is called nano weakly $g^{\#}$-continuous (briefly, $nwg^{\#}$-continuous) if $f^{-1}(G)$ is a $nwg^{\#}$-open set in U for each nano open set G of V.

Example 4.1. *Let $U = \{h_1, h_2, h_3\}$ with $U/R = \{\{h_1\}, \{h_2, h_3\}\}$ and $X = \{h_1, h_2\}$. Then $\tau_R(X) = \{\phi, \{h_1\}, \{h_2, h_3\}, U\}$ in U. Let $V = \{h_1, h_2, h_3\}$ with $V/R = \{\{h_1\}, \{h_2\}, \{h_3\}\}$ and $Y = \{h_1\}$. Then $\tau'_R(Y) = \{\phi, \{h_1\}, V\}$ in V. The function $f : (U, \tau_R(X)) \to (V, \tau'_R(Y))$ defined by $f(h_1) = h_2$, $f(h_2) = h_3$ and $f(h_3) = h_1$ is $nwg^{\#}$-continuous, because every subset of V is $nwg^{\#}$-closed in U.*

Theorem 4.1. *Each $ng^{\#}$-continuous function is $nwg^{\#}$-continuous.*

Proof. It follows from Theorem 3.1.

The converse of Theorem 4.1 need not be true as seen in the following example.

Example 4.2. *Let $U = \{h_1, h_2, h_3\}$ with $U/R = \{\{h_1\}, \{h_2, h_3\}\}$ and $X = \{h_1, h_2\}$. Then $\tau_R(X) = \{\phi, \{h_1\}, \{h_2, h_3\}, U\}$ in U. Let $V = \{h_1, h_2, h_3\}$ with $V/R = \{\{h_1\}, \{h_2\}, \{h_3\}\}$ and $Y = \{h_2\}$. Then $\tau'_R(Y) = \{\phi, \{h_2\}, V\}$ in V. Let $f : (U, \tau_R(X)) \to (V, \tau'_R(Y))$ be the identity function. Then f is $nwg^{\#}$-continuous but not $ng^{\#}$-continuous.*

Theorem 4.2. *A function $f : (U, \tau_R(X)) \to (V, \tau'_R(Y))$ is $nwg^{\#}$-continuous if and only if $f^{-1}(G)$ is a $nwg^{\#}$-closed set in U for each nano closed set G of V.*

Proof. Let G be any nano closed set of V. According to the assumption $f^{-1}(G^c) = U \backslash f^{-1}(G)$ is $nwg^{\#}$-open in U, so $f^{-1}(G)$ is $nwg^{\#}$-closed in U.

The converse can be proved in a similar manner.

Definition 4.2. A space U is said to be nano locally $g^{\#}$-indiscrete if every $ng^{\#}$-open set of U is nano closed in U.

Theorem 4.3. *Let $f : (U, \tau_R(X)) \to (V, \tau'_R(Y))$ be a function. If f is nano contra $g^{\#}$-continuous and U is nano locally $g^{\#}$-indiscrete, then f is nano continuous.*

Proof. Let H be a nano closed in V. Since f is nano contra $g^{\#}$-continuous, $f^{-1}(H)$ is $ng^{\#}$-open in U. Since U is nano locally $g^{\#}$-indiscrete, $f^{-1}(H)$ is nano closed in U. Hence f is nano continuous.

Theorem 4.4. *Let $f : (U, \tau_R(X)) \to (V, \tau'_R(Y))$ be a function. If f is nano contra $g^{\#}$-continuous and U is nano locally $g^{\#}$-indiscrete, then f is $nwg^{\#}$-continuous.*

Proof. Let $f : (U, \tau_R(X)) \to (V, \tau'_R(Y))$ be nano contra $g^{\#}$-continuous and U is nano locally $g^{\#}$-indiscrete. By Theorem 4.3, f is nano continuous, then f is $nwg^{\#}$-continuous.

Proposition 4.1. *If $f : (U, \tau_R(X)) \to (V, \tau'_R(Y))$ is perfectly nano continuous and $nwg^{\#}$-continuous, then it is nR-map.*

Proof. Let H be any nano regular open subset of V. According to the assumption, $f^{-1}(H)$ is both nano open and nano closed in U. Since $f^{-1}(H)$ is nano closed, it is $nwg^{\#}$-closed. We have $f^{-1}(H)$ is both nano open and $nwg^{\#}$-closed. Hence, by Corollary 3.1, it is nano regular open in U, so f is nR-map.

Definition 4.3. A space U is called $ng^{\#}$-compact if every cover of U by $ng^{\#}$-open sets has finite subcover.

Definition 4.4. A space U is nano weakly $g^{\#}$-compact (briefly, $nwg^{\#}$-compact) if every $nwg^{\#}$-open cover of U has a finite subcover.

Remark 4.1. *Each $nwg^{\#}$-compact space is $ng^{\#}$-compact.*

Theorem 4.5. *Let $f : (U, \tau_R(X)) \to (V, \tau'_R(Y))$ be surjective $nwg^{\#}$-continuous function. If U is $nwg^{\#}$-compact, then V is nano compact.*

Proof. Let $\{E_i : i \in I\}$ be an nano open cover of V. Then $\{f^{-1}(E_i) : i \in I\}$ is a $nwg^{\#}$-open cover in U. Since U is $nwg^{\#}$-compact, it has a finite subcover, say $\{f^{-1}(E_1),\ f^{-1}(E_2), ..., \ f^{-1}(E_n)\}$. Since f is surjective $\{E_1,\ E_2, ..., \ E_n\}$ is a finite subcover of V and hence V is nano compact.

Definition 4.5. A space U is nano weakly $g^{\#}$-connected (briefly, $nwg^{\#}$-connected) if U cannot be written as the disjoint union of two non-empty $nwg^{\#}$-open sets.

Theorem 4.6. *If a space U is $nwg^{\#}$-connected, then U is nano almost connected (resp. $ng^{\#}$-connected).*

Proof. It follows from the fact that each nano regular open set (resp. $ng^{\#}$-open set) is $nwg^{\#}$-open.

Theorem 4.7. *For a space U, the following statements are equivalent:*

1. *U is $nwg^{\#}$-connected.*
2. *The empty set ϕ and U are only subsets which are both $nwg^{\#}$-open and $nwg^{\#}$-closed.*
3. *Each $nwg^{\#}$-continuous function from U into a discrete space V which has at least two points is a constant function.*

Proof. $(1) \Rightarrow (2)$. Let $K \subseteq U$ be any proper subset, which is both $nwg^{\#}$-open and $nwg^{\#}$-closed. Its complement $U \backslash K$ is also $nwg^{\#}$-open and $nwg^{\#}$-closed. Then $U = K \cup (U \backslash K)$ is a disjoint union of two non-empty $nwg^{\#}$-open sets which is a contradiction with the fact that U is $nwg^{\#}$-connected. Hence, $K = \phi$ or U.

$(2) \Rightarrow (1)$. Let $U = E \cup F$ where $E \cap F = \phi$, $E \neq \phi$, $F \neq \phi$ and E, F are $nwg^{\#}$-open. Since $E = U \backslash F$, E is $nwg^{\#}$-closed. According to the assumption $E = \phi$, which is a contradiction.

$(2) \Rightarrow (3)$. Let $f : (U, \tau_R(X)) \rightarrow (V, \tau'_R(Y))$ be a $nwg^{\#}$-continuous function where V is a discrete space with at least two points. Then $f^{-1}(\{a\})$ is $nwg^{\#}$-closed and $nwg^{\#}$-open for each $a \in V$ and $U = \cup\{f^{-1}(\{a\}) : a \in V\}$. According to the assumption, $f^{-1}(\{a\}) = \phi$ or $f^{-1}(\{a\}) = U$. If $f^{-1}(\{a\}) = \phi$ for all $a \in V$, f will not be a function. Also there is no exist more than one $a \in V$ such that $f^{-1}(\{a\}) = U$. Hence, there exists only one $a \in V$ such that $f^{-1}(\{a\}) = U$ and $f^{-1}(\{a_1\}) = \phi$ where $a \neq a_1 \in V$. This shows that f is a constant function.

$(3) \Rightarrow (2)$. Let $K \neq \phi$ be both $nwg^{\#}$-open and $nwg^{\#}$-closed in U. Let $f : (U, \tau_R(X)) \rightarrow (V, \tau'_R(Y))$ be a $nwg^{\#}$-continuous function defined by $f(K) = \{c\}$ and $f(U \backslash K) = \{d\}$ where $c \neq d$. Since f is constant function we get $K = U$.

Theorem 4.8. *Let $f : (U, \tau_R(X)) \rightarrow (V, \tau'_R(Y))$ be a $nwg^{\#}$-continuous surjective function. If U is $nwg^{\#}$-connected, then V is nano connected.*

Proof. We suppose that V is not nano connected. Then $V = E \cup F$ where $E \cap F = \phi$, $E \neq \phi$, $F \neq \phi$ and E, F are nano open sets in V. Since f is $nwg^{\#}$-continuous surjective function, $U = f^{-1}(E) \cup f^{-1}(F)$ are disjoint union of two non-empty $nwg^{\#}$-open subsets. This is contradiction with the fact that U is $nwg^{\#}$-connected.

5. Nano weakly $g^{\#}$-open functions and nano weakly $g^{\#}$-closed functions

Definition 5.1. Let U and V be spaces. A function $f:(U,\tau_R(X))\to(V,\tau'_R(Y))$ is called nano weakly $g^{\#}$-open (briefly, $nwg^{\#}$-open) if $f(G)$ is a $nwg^{\#}$-open set in V for each nano open set G of U.

Definition 5.2. Let U and V be spaces. A function $f:(U,\tau_R(X))\to(V,\tau'_R(Y))$ is called nano weakly $g^{\#}$-closed (briefly, $nwg^{\#}$-closed) if $f(G)$ is a $nwg^{\#}$-closed set in V for each nano closed set G of U.

It is clear that an open function is $nwg^{\#}$-open and a nano closed function is $nwg^{\#}$-closed.

Theorem 5.1. *Let U and V be spaces. A function $f:(U,\tau_R(X))\to(V,\tau'_R(Y))$ is $nwg^{\#}$-closed if and only if for each subset G of V and for each nano open set H containing $f^{-1}(G)$ there exists a $nwg^{\#}$-open set S of V such that $G\subseteq S$ and $f^{-1}(S)\subseteq H$.*

Proof. Let G be any subset of V and let H be an nano open subset of U such that $f^{-1}(G)\subseteq H$. Then $S=V\backslash f(U\backslash H)$ is $nwg^{\#}$-open set containing G and $f^{-1}(S)\subseteq H$.

Conversely, let P be any nano closed subset of U. Then $f^{-1}(V\backslash f(P))\subseteq U\backslash P$ and $U\backslash P$ is nano open. According to the assumption, there exists a $nwg^{\#}$-open set S of V such that $V\backslash f(P)\subseteq S$ and $f^{-1}(S)\subseteq U\backslash P$. Then $P\subseteq U\backslash f^{-1}(S)$. From $V\backslash S\subseteq f(P)\subseteq f(U\backslash f^{-1}(S))\subseteq V\backslash S$ it follows that $f(P)=V\backslash S$, so $f(P)$ is $nwg^{\#}$-closed in V. Therefore f is a $nwg^{\#}$-closed function.

Remark 5.1. *The composition of two $nwg^{\#}$-closed functions need not be a $nwg^{\#}$-closed as we can see from the following example.*

Example 5.1. *Let $U=\{h_1,h_2,h_3\}$ with $U/R=\{\{h_1\},\{h_2,h_3\}\}$ and $X=\{h_1\}$. Then $\tau_R(X)=\{\phi,\{h_1\},U\}$. Let $V=\{h_1,h_2,h_3\}$ with $V/R=\{\{h_1\},\{h_2,h_3\}\}$ and $Y=\{h_1,h_2\}$. Then $\tau'_R(Y)=\{\phi,\{h_1\},\{h_2,h_3\},V\}$. Let $W=\{h_1,h_2,h_3\}$ with $W/R=\{\{h_1,h_2\},\{h_3\}\}$ and $Z=\{h_1,h_2\}$. Then $\tau''_R(Z)=\{\phi,\{h_1,h_2\},Z\}$. We define $f:(U,\tau_R(X))\to(V,\tau'_R(Y))$ by $f(h_1)=h_3$, $f(h_2)=h_2$ and $f(h_3)=h_1$ and let $g:(V,\tau'_R(Y))\to(W,\tau''_R(Z))$ be the identity function. Hence both f and g are $nwg^{\#}$-closed functions. For a closed set $K=\{h_2,h_3\}$, $(g\circ f)(K)=g(f(K))=g(\{h_1,h_2\})=\{h_1,h_2\}$ which is not $nwg^{\#}$-closed in W. Hence the composition of two $nwg^{\#}$-closed functions need not be a $nwg^{\#}$-closed.*

Theorem 5.2. *Let U, V and W be spaces. If $f:(U,\tau_R(X))\to(V,\tau'_R(Y))$ is a nano closed function and $g:(V,\tau'_R(Y))\to(W,\tau''_R(Z))$ is a $nwg^{\#}$-closed function, then $g\circ f:(U,\tau_R(X))\to(W,\tau''_R(Z))$ is a $nwg^{\#}$-closed function.*

Definition 5.3. A function $f:(U,\tau_R(X))\to(V,\tau'_R(Y))$ is called a nano weakly $g^{\#}$-irresolute (briefly, $nwg^{\#}$-irresolute) if $f^{-1}(G)$ is a $nwg^{\#}$-open set in U for each $nwg^{\#}$-open set G of V.

Example 5.2. *Let $U=\{h_1,h_2,h_3\}$ with $U/R=\{\{h_2\},\{h_1,h_3\}\}$ and $X=\{h_2,h_3\}$. Then $\mathcal{N}=\{\phi,\{h_2\},\{h_1,h_3\},U\}$. Let $V=\{h_1,h_2,h_3\}$ with $V/R=\{\{h_2\},\{h_1,h_3\}\}$ and $Y=\{h_2\}$. Then $\mathcal{N}'=\{\phi,\{h_2\},V\}$. Let $f:(U,\tau_R(X))\to(V,\tau'_R(Y))$ be the identity function. Then f is $nwg^{\#}$-irresolute.*

Remark 5.2. *The following examples show that the concepts of $n\alpha g$-irresoluteness and the concepts of $nwg^{\#}$-irresoluteness are independent of each other.*

Example 5.3. *Let* $U = \{h_1, h_2, h_3\}$ *with* $U/R = \{\{h_1, h_2\}, \{h_3\}\}$ *and* $X = \{h_1, h_2\}$. *Then* $\tau_R(X) = \{\phi, \{h_1, h_2\}, U\}$. *Let* $V = \{h_1, h_2, h_3\}$ *with* $V/R = \{\{h_1\}, \{h_2\}, \{h_3\}\}$ *and* $Y = \{h_1\}$. *Then* $\tau'_R(Y) = \{\phi, \{h_1\}, V\}$. *Let* $f : (U, \tau_R(X)) \to (V, \tau'_R(Y))$ *be the identity function. Then* f *is* $nwg^{\#}$*-irresolute but not* $n\alpha g$*-irresolute.*

Example 5.4. *Let* $U = \{h_1, h_2, h_3\}$ *with* $U/R = \{\{h_1\}, \{h_2\}, \{h_3\}\}$ *and* $X = \{h_1, h_2\}$. *Then* $\tau_R(X) = \{\phi, \{h_1\}, \{h_2\}, \{h_1, h_2\}, U\}$. *Let* $V = \{h_1, h_2, h_3\}$ *with* $V/R = \{\{h_1, h_2\}, \{h_3\}\}$ *and* $Y = \{h_1, h_2\}$. *Then* $\tau'_R(Y) = \{\phi, \{h_1, h_2\}, V\}$. *Let* $f : (U, \tau_R(X)) \to (V, \tau'_R(Y))$ *be the identity function. Then* f *is* $n\alpha g$*-irresolute but not* $nwg^{\#}$*-irresolute.*

Theorem 5.3. *Let* $f : (U, \tau_R(X)) \to (V, \tau'_R(Y))$ *and* $g : (V, \tau'_R(Y)) \to (W, \tau''_R(Z))$ *be functions such that* $g \circ f : (U, \tau_R(X)) \to (W, \tau''_R(Z))$ *is* $nwg^{\#}$*-closed function. Then the following statements hold:*

1. *if* f *is nano continuous and injective, then* g *is* $nwg^{\#}$*-closed.*
2. *if* g *is* $nwg^{\#}$*-irresolute and surjective, then* f *is* $nwg^{\#}$*-closed.*

Proof. (1). Let S be a nano closed set of V. Since $f^{-1}(S)$ is nano closed in U, we can conclude that $(g \circ f)(f^{-1}(S))$ is $nwg^{\#}$-closed in W. Hence $g(S)$ is $nwg^{\#}$-closed in W. Thus g is a $nwg^{\#}$-closed function.

(2). It can be proved in a similar manner as (1).

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