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Nano binary generalized closed sets

Jasmine Elizabeth J[1*] and Hari Siva Annam G[2]
[1]Department of Mathematics, Kamaraj College, Thoothukudi-628003,(TN), India. (Part Time Research Scholar[19122102092008], Manonmaniam Sundaranar University, Tirunelveli-627012, (TN), India.)
[2]PG and Research Department of Mathematics,Kamaraj College,Thoothukudi-628003, (TN), India.

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Abstract: The purpose of this paper is to introduce and study the nano binary mildly generalized closed sets, nano binary weakly generalized closed sets and nano binary strongly generalized closed sets in nano binary topological spaces. Also studied their characterizations and properties.

Key words: weakly N_B g − closed,mildly N_B g − closed,strongly N_B g − closed.

1. Introduction

M. Lellis Thivagar [1] instigated the idea of nano topological space with respect to a subset X of a universe U. S. Nithyanantha Jothi et al. [2] instigated the idea of binary topological spaces. By combining these two ideas Dr. G. Hari Siva Annam and J. Jasmine Elizabeth [3] instigated nano binary topological spaces. In this paper we have instigated nano binary mildly generalized closed sets, nano binary weakly generalized closed sets and nano binary strongly generalized closed sets in nano binary topological spaces. Also studied their properties and characterizations with suitable examples.

2. Preliminaries

Definition 2.1. [3] Let (U_1, U_2) be the universe, R be an equivalence on (U_1, U_2) and $\tau_R(X_1, X_2) = \{(U_1, U_2), (\phi, \phi), L_R(X_1, X_2), U_R(X_1, X_2), B_R(X_1, X_2)\}$ where $(X_1, X_2) \subseteq (U_1, U_2)$. Then by the property $\tau_R(X_1, X_2)$ satisfies the following axioms

1.(U_1, U_2) and $(\phi, \phi) \in \tau_R(X_1, X_2)$

2. The union of the elements of any sub collection of $\tau_R(X_1, X_2)$ is in $\tau_R(X_1, X_2)$

3. The intersection of the elements of any finite sub collection of $\tau_R(X_1, X_2)$ is in $\tau_R(X_1, X_2)$. That is, $\tau_R(X_1, X_2)$ is a topology on (U_1, U_2) called the nano binary topology on (U_1, U_2) with respect to (X_1, X_2). We call $(U_1, U_2,\ \tau_R(X_1, X_2))$ as the nano binary topological spaces. The elements of $\tau_R(X_1, X_2)$ are called as nano binary open sets and it is denoted by N_B open sets. Their complements are called N_B closed sets.

Definition 2.2. [3] If $(U_1, U_2, \tau_R(X_1, X_2))$ is a nano binary topological spaces with respect to (X_1, X_2) and if $(H_1, H_2) \subseteq (U_1, U_2)$, then the nano binary interior of (H_1, H_2) is defined as the union of all N_B open subsets of (H_1, H_2). That is, $(H_1, H_2)^{1^o} = \cup\{H_{1_\alpha} : (H_{1_\alpha}, H_{2_\alpha})$ is nano binary open and $(H_{1_\alpha}, H_{2_\alpha})$

*Correspondence: jasmineelizabeth89@gmail.com

$\subseteq (H_1, H_2)\}$ and $(H_1, H_2)^{2^o} = \cup\{H_{2_\alpha} : (H_{1_\alpha}, H_{2_\alpha})$ is nano binary open and $(H_{1_\alpha}, H_{2_\alpha}) \subseteq (H_1, H_2)\}$. The ordered pair $((H_1, H_2)^{1^o}, (H_1, H_2)^{2^o})$ is called the nano binary interior of (H_1, H_2), denoted by $N_B^\circ(H_1, H_2)$. That is, $N_B^\circ(H_1, H_2)$ is the largest N_B open subset of (H_1, H_2). The nano binary closure of (H_1, H_2) is defined as the intersection of all N_B closed sets containing (H_1, H_2). That is, $(H_1, H_2)^{1^*} = \cap\{H_{1_\alpha} : (H_{1_\alpha}, H_{2_\alpha})$ is nano binary closed and $(H_1, H_2) \subseteq (H_{1_\alpha}, H_{2_\alpha})\}$ and $(H_1, H_2)^{2^*} = \cap\{H_{2_\alpha} : (H_{1_\alpha}, H_{2_\alpha})$ is nano binary closed and $(H_1, H_2) \subseteq (H_{1_\alpha}, H_{2_\alpha})\}$. The ordered pair $((H_1, H_2)^{1^*}, (H_1, H_2)^{2^*})$ is called the nano binary closure of (H_1, H_2), denoted by $\overline{N_B}(H_1, H_2)$. That is, $\overline{N_B}(H_1, H_2)$ is the smallest N_B closed set containing (H_1, H_2).

Definition 2.3. [3] Let $(U_1, U_2, \tau_R(X_1, X_2))$ be a nano binary topological space. A subset (H_1, H_2) of $(U_1, U_2, \tau_R(X_1, X_2))$ is called nano binary generalized closed (N_B g-closed)set if $\overline{N_B}(\varrho_1, \varrho_2) \subseteq (\mathcal{L}_1, \mathcal{L}_2)$ whenever $(\varrho_1, \varrho_2) \subseteq (\mathcal{L}_1, \mathcal{L}_2)$ and $(\mathcal{L}_1, \mathcal{L}_2)$ is N_B open.

Definition 2.4. [3] A subset (ϱ_1, ϱ_2) of (U_1, U_2) is said to be nano binary generalized open (N_B g-open) if its complement is N_B g-closed.

Definition 2.5. [3] The intersection of all N_B g-closed sets containing (ϱ_1, ϱ_2) is said to be nano binary generalized closure of (ϱ_1, ϱ_2).

Definition 2.6. [3] The union of all N_B g- open sets contained in (ϱ_1, ϱ_2) is said to be nano binary generalized interior of (ϱ_1, ϱ_2).

Definition 2.7. [3] A nano binary generalized closure is denoted by $\overline{N_B}^*$ and nano binary generalized interior is denoted by $N_B^{\circ *}$.

Result 2.1. [3] 1. Every N_B open set is N_B g-open.
2. Every N_B closed set is N_B g-closed.

Note 2.1. The contrary of the prior result is not true as shown in the following example.

Example 2.1. *Let* $U_1 = \{o, p, q\}, U_2 = \{7, 8\}$ *with* $(U_1, U_2)/R = \{(\{o, q\}, \{7\}), (\{p\}, \{8\})\}$ *and* $(X_1, X_2) = (\{o, p\}, \{8\})$. *Then* $\tau_R(X_1, X_2) = \{(\phi, \phi), (U_1, U_2), (\{o, q\}, \{7\}), (\{p\}, \{8\})\}$. *Let* $(Y_1, Y_2) = (\{o, q\}, \{7\})$ *and* $(H_1, H_2) = (\{q\}, \{7\})$. *Then* $\overline{N_B}(H_1, H_2) = (\{o, q\}, \{7\}) \subseteq (Y_1, Y_2)$. *Therefore,* (H_1, H_2) *is* N_B *g-closed but not* N_B *closed.*

3. Nano Binary Generalized Closed Set

Theorem 3.1. *A subset* $(\mathcal{L}_1, \mathcal{L}_2)$ *of a* N_B *topological space* $(U_1, U_2, \tau_R(X_1, X_2))$ *is* N_B *g-closed if* $\overline{N_B}(\mathcal{L}_1, \mathcal{L}_2) - (\mathcal{L}_1, \mathcal{L}_2)$ *contains no non-empty* N_B *g-closed set.*

Proof: *Suppose* $(\mathcal{L}_1, \mathcal{L}_2)$ *is* N_B *g-closed. Then* $\overline{N_B}(\mathcal{L}_1, \mathcal{L}_2) \subseteq (Y_1, Y_2)$, *where* $(\mathcal{L}_1, \mathcal{L}_2) \subseteq (Y_1, Y_2)$ *and* (Y_1, Y_2) *is* N_B*-open. Let* (κ_1, κ_2) *be a* N_B *g-closed subset of* $\overline{N_B}(\mathcal{L}_1, \mathcal{L}_2) - (\mathcal{L}_1, \mathcal{L}_2)$. *Then* $(\mathcal{L}_1, \mathcal{L}_2) \subseteq ((U_1, U_2) - (\kappa_1, \kappa_2))$ *and* $((U_1, U_2) - (\kappa_1, \kappa_2))$ *is* N_B *g-open. Since* $(\mathcal{L}_1, \mathcal{L}_2)$ *is* N_B *g-closed,* $\overline{N_B}(\mathcal{L}_1, \mathcal{L}_2) \subseteq (U_1, U_2) - (\kappa_1, \kappa_2) \Rightarrow (\kappa_1, \kappa_2) \subseteq (U_1, U_2) - \overline{N_B}(\mathcal{L}_1, \mathcal{L}_2)$. *That is* $(\kappa_1, \kappa_2) \subseteq \overline{N_B}(\mathcal{L}_1, \mathcal{L}_2)$ *and* $(\kappa_1, \kappa_2) \subseteq (U_1, U_2) - \overline{N_B}(\mathcal{L}_1, \mathcal{L}_2) \Rightarrow (\phi, \phi)$. *Therefore,* (κ_1, κ_2) *is empty.*

Theorem 3.2. *If* (ϱ_1, ϱ_2) *and* (υ_1, υ_2) *are* N_B *g-closed, then* $(\varrho_1, \varrho_2) \cup (\upsilon_1, \upsilon_2)$ *is* N_B *g-closed.*

Proof: *Let* (ϱ_1, ϱ_2) *and* (υ_1, υ_2) *be* N_B *g-closed sets. Then* $\overline{N_B}(\varrho_1, \varrho_2) \subseteq (Y_1^{'}, Y_2^{'})$, *where* $(\varrho_1, \varrho_2) \subseteq (Y_1^{'}, Y_2^{'})$

and $(Y_1^{'}, Y_2^{'})$ is N_B-open and $\overline{N_B}(\upsilon_1, \upsilon_2) \subseteq (Y_1^{"}, Y_2^{"})$, where $(\upsilon_1, \upsilon_2) \subseteq (Y_1^{"}, Y_2^{"})$ and $(Y_1^{"}, Y_2^{"})$ is N_B-open. Since (ϱ_1, ϱ_2) and (υ_1, υ_2) are subsets of $(Y_1^{'}, Y_2^{'})$ and $(Y_1^{"}, Y_2^{"})$, $(\varrho_1, \varrho_2) \cup (\upsilon_1, \upsilon_2)$ is a subset $(Y_1^{'}, Y_2^{'}) \cup (Y_1^{"}, Y_2^{"})$, which is N_B -open. Then $\overline{N_B}((\varrho_1, \varrho_2) \cup (\upsilon_1, \upsilon_2)) = \overline{N_B}(\varrho_1, \varrho_2) \cup \overline{N_B}(\upsilon_1, \upsilon_2) \subseteq (Y_1^{'}, Y_2^{'}) \cup (Y_1^{"}, Y_2^{"})$. Therefore, $(\varrho_1, \varrho_2) \cup (\upsilon_1, \upsilon_2)$ is N_B g-closed.

Remark 3.1. *The intersection of two N_B g-closed set is again an N_B g-closed set as previous.*

Theorem 3.3. *If $(\mathcal{L}_1, \mathcal{L}_2)$ is N_B g-closed and $(\mathcal{L}_1, \mathcal{L}_2) \subseteq (\mathcal{B}_1, \mathcal{B}_2) \subseteq \overline{N_B}(\mathcal{L}_1, \mathcal{L}_2)$, then $(\mathcal{B}_1, \mathcal{B}_2)$ is N_B g-closed.*
Proof: *Let $(\mathcal{B}_1, \mathcal{B}_2) \subseteq (Y_1, Y_2)$, where (Y_1, Y_2) is N_B-open. Then $(\mathcal{L}_1, \mathcal{L}_2) \subseteq (\mathcal{B}_1, \mathcal{B}_2) \Rightarrow (\mathcal{L}_1, \mathcal{L}_2) \subseteq (Y_1, Y_2)$. Since $\mathcal{L}_1, \mathcal{L}_2)$ is N_B g-closed, $\overline{N_B}(\mathcal{L}_1, \mathcal{L}_2) \subseteq (Y_1, Y_2)$. Also $(\mathcal{B}_1, \mathcal{B}_2) \subseteq \overline{N_B}(\mathcal{L}_1, \mathcal{L}_2) \Rightarrow \overline{N_B}(\mathcal{B}_1, \mathcal{B}_2) \subseteq \overline{N_B}(\mathcal{L}_1, \mathcal{L}_2) \Rightarrow \overline{N_B}(\mathcal{B}_1, \mathcal{B}_2)$
$\subseteq (Y_1, Y_2)$. Therefore, $(\mathcal{B}_1, \mathcal{B}_2)$ is N_B g-closed.*

Theorem 3.4. *An N_B g-closed set $(\mathcal{L}_1, \mathcal{L}_2)$ is N_B closed if and only if $\overline{N_B}(\mathcal{L}_1, \mathcal{L}_2)$
$- (\mathcal{L}_1, \mathcal{L}_2)$ is N_B closed.*
Proof: *Let $(\mathcal{L}_1, \mathcal{L}_2)$ be N_B closed. Then $\overline{N_B}(\mathcal{L}_1, \mathcal{L}_2) = (\mathcal{L}_1, \mathcal{L}_2) \Rightarrow \overline{N_B}(\mathcal{L}_1, \mathcal{L}_2) - (\mathcal{L}_1, \mathcal{L}_2) = (\phi, \phi)$, which is N_B closed. Conversely, $\overline{N_B}(\mathcal{L}_1, \mathcal{L}_2) - (\mathcal{L}_1, \mathcal{L}_2)$ is N_B closed. Then $\overline{N_B}(\mathcal{L}_1, \mathcal{L}_2) - (\mathcal{L}_1, \mathcal{L}_2) = (\phi, \phi)$, since $(\mathcal{L}_1, \mathcal{L}_2)$ is N_B g-closed. Therefore, $\overline{N_B}(\mathcal{L}_1, \mathcal{L}_2) = (\mathcal{L}_1, \mathcal{L}_2)$ and hence $(\mathcal{L}_1, \mathcal{L}_2)$ is N_B closed.*

Definition 3.1. In N_B topological space $(U_1, U_2, \tau_R(X_1, X_2))$, a subset (ϱ_1, ϱ_2) of (U_1, U_2) is said to be

1. weakly N_B g-closed if $\overline{N_B}({N_B}^{\circ}(\varrho_1, \varrho_2)) \subseteq (\mathcal{G}_1, \mathcal{G}_2)$ whenever $(\varrho_1, \varrho_2) \subseteq (\mathcal{G}_1, \mathcal{G}_2)$ and $(\mathcal{G}_1, \mathcal{G}_2)$ is N_B -open in (U_1, U_2).
2. mildly N_B g-closed if $\overline{N_B}({N_B}^{\circ}(\varrho_1, \varrho_2)) \subseteq (\mathcal{G}_1, \mathcal{G}_2)$ whenever $(\varrho_1, \varrho_2) \subseteq (\mathcal{G}_1, \mathcal{G}_2)$ and $(\mathcal{G}_1, \mathcal{G}_2)$ is N_B g-open in (U_1, U_2).
3. strongly N_B g-closed if $\overline{N_B}(\varrho_1, \varrho_2) \subseteq (\mathcal{G}_1, \mathcal{G}_2)$ whenever $(\varrho_1, \varrho_2) \subseteq (\mathcal{G}_1, \mathcal{G}_2)$ and $(\mathcal{G}_1, \mathcal{G}_2)$ is N_B g-open in (U_1, U_2).

The complements of the above mentioned sets are called their respective N_B g-open sets.

Theorem 3.5. *In N_B topological space $(U_1, U_2, \tau_R(X_1, X_2))$, a subset (ϱ_1, ϱ_2) of (U_1, U_2) is mildly N_B g-closed if and only if $\overline{N_B}({N_B}^{\circ}(\varrho_1, \varrho_2)) \subseteq (\varrho_1, \varrho_2)$.*
Proof: *If $\overline{N_B}({N_B}^{\circ}(\varrho_1, \varrho_2)) \nsubseteq (\varrho_1, \varrho_2)$, there exists $\{(x_1, x_2)\} \in (U_1, U_2)$ such that $\{(x_1, x_2)\} \in \overline{N_B}({N_B}^{\circ}(\varrho_1, \varrho_2)) - (\varrho_1, \varrho_2)$. Then $\{(x_1, x_2)\} \in \overline{N_B}({N_B}^{\circ}$
$(\varrho_1, \varrho_2)) - (\varrho_1, \varrho_2) \subseteq (U_1, U_2) - (\varrho_1, \varrho_2)$ and so $(\varrho_1, \varrho_2) \subseteq (U_1, U_2) - \{(x_1, x_2)\}$, where $(U_1, U_2) - \{(x_1, x_2)\}$ is N_B g -open being N_B -open. Thus $(\varrho_1, \varrho_2) \subseteq (U_1, U_2) - (x_1, x_2)$, where $(U_1, U_2) - \{(x_1, x_2)\}$ is N_B g -open. But $\overline{N_B}({N_B}^{\circ}(\varrho_1, \varrho_2))$
$\nsubseteq (U_1, U_2) - \{(x_1, x_2)\}$, since $\{(x_1, x_2)\} \in \overline{N_B}({N_B}^{\circ}(\varrho_1, \varrho_2)) \Rightarrow (\varrho_1, \varrho_2)$ is not mildly N_B g-closed. Suppose, let $\overline{N_B}({N_B}^{\circ}(\varrho_1, \varrho_2)) \subseteq (\varrho_1, \varrho_2)$ and $(\mathcal{G}_1, \mathcal{G}_2)$ be any N_B g-open set such that $(\varrho_1, \varrho_2) \subseteq (\mathcal{G}_1, \mathcal{G}_2)$. Then $\overline{N_B}({N_B}^{\circ}(\varrho_1, \varrho_2)) \subseteq (\varrho_1, \varrho_2) \subseteq (\mathcal{G}_1, \mathcal{G}_2)$. Therefore, (ϱ_1, ϱ_2) is mildly N_B g-closed.*

Theorem 3.6. *In N_B topological space $(U_1, U_2, \tau_R(X_1, X_2))$, a subset (ϱ_1, ϱ_2) of (U_1, U_2) is weakly N_B g-closed if and only if $\overline{N_B}({N_B}^\circ(\varrho_1, \varrho_2)) \subseteq (\varrho_1, \varrho_2)$.*

Proof: *If $\overline{N_B}({N_B}^\circ(\varrho_1, \varrho_2)) \nsubseteq (\varrho_1, \varrho_2)$, there exists $(x_1, x_2) \in (U_1, U_2)$ such that $(x_1, x_2) \in \overline{N_B}({N_B}^\circ(\varrho_1, \varrho_2)) - (\varrho_1, \varrho_2)$. Then $(x_1, x_2) \in \overline{N_B}({N_B}^\circ(\varrho_1, \varrho_2)) - (\varrho_1, \varrho_2) \subseteq (U_1, U_2) - (\varrho_1, \varrho_2)$. Thus $(\varrho_1, \varrho_2) \subseteq (U_1, U_2) - (x_1, x_2)$, where $(U_1, U_2) - (x_1, x_2)$ is N_B -open. But $\overline{N_B}({N_B}^\circ(\varrho_1, \varrho_2)) \nsubseteq (U_1, U_2) - (x_1, x_2)$, since $(x_1, x_2) \in \overline{N_B}({N_B}^\circ(\varrho_1, \varrho_2)) \Rightarrow (\varrho_1, \varrho_2)$ is not weakly N_B g-closed. Suppose, let $\overline{N_B}({N_B}^\circ(\varrho_1, \varrho_2)) \subseteq (\varrho_1, \varrho_2)$ and $(\mathcal{G}_1, \mathcal{G}_2)$ be any N_B open set such that $(\varrho_1, \varrho_2) \subseteq (\mathcal{G}_1, \mathcal{G}_2)$. Then $\overline{N_B}({N_B}^\circ(\varrho_1, \varrho_2)) \subseteq (\varrho_1, \varrho_2) \subseteq (\mathcal{G}_1, \mathcal{G}_2)$. Therefore, (ϱ_1, ϱ_2) is weakly N_B g-closed.*

Theorem 3.7. *In N_B topological space $(U_1, U_2, \tau_R(X_1, X_2))$, a subset (ϱ_1, ϱ_2) of (U_1, U_2) is mildly N_B g-closed if and only if (ϱ_1, ϱ_2) is weakly N_B g-closed.*

Proof: *Proof follows from theorem 3.7 and theorem 3.8.*

Theorem 3.8. *In N_B topological space $(U_1, U_2, \tau_R(X_1, X_2))$, a subset (ϱ_1, ϱ_2) of (U_1, U_2) is strongly N_B g-closed if and only if $\overline{N_B}(\varrho_1, \varrho_2) \subseteq (\varrho_1, \varrho_2)$.*

Proof: *If $\overline{N_B}(\varrho_1, \varrho_2) \nsubseteq (\varrho_1, \varrho_2)$, there exists $(x_1, x_2) \in (U_1, U_2)$ such that $(x_1, x_2) \in \overline{N_B}(\varrho_1, \varrho_2) - (\varrho_1, \varrho_2)$. Then $(x_1, x_2) \in \overline{N_B}(\varrho_1, \varrho_2) - (\varrho_1, \varrho_2) \subseteq (U_1, U_2) - (\varrho_1, \varrho_2)$. Thus $(\varrho_1, \varrho_2) \subseteq (U_1, U_2) - (x_1, x_2)$, where $(U_1, U_2) - (x_1, x_2)$ is N_B g -open being N_B -open. But $\overline{N_B}(\varrho_1, \varrho_2) \nsubseteq (U_1, U_2) - (x_1, x_2)$, since $(x_1, x_2) \in \overline{N_B}(\varrho_1, \varrho_2) \Rightarrow (\varrho_1, \varrho_2)$ is not strongly N_B g-closed. Suppose, let $\overline{N_B}(\varrho_1, \varrho_2) \subseteq (\varrho_1, \varrho_2)$ and $(\mathcal{G}_1, \mathcal{G}_2)$ be any N_B g-open set such that $(\varrho_1, \varrho_2) \subseteq (\mathcal{G}_1, \mathcal{G}_2)$. Then $\overline{N_B}(\varrho_1, \varrho_2) \subseteq (\varrho_1, \varrho_2) \subseteq (\mathcal{G}_1, \mathcal{G}_2)$. Therefore, (ϱ_1, ϱ_2) is strongly N_B g-closed.*

Theorem 3.9. *In N_B topological space $(U_1, U_2, \tau_R(X_1, X_2))$, a subset (ϱ_1, ϱ_2) of (U_1, U_2) is N_B g-closed if and only if $\overline{N_B}(\varrho_1, \varrho_2) \subseteq (\varrho_1, \varrho_2)$.*

Proof: *If $\overline{N_B}(\varrho_1, \varrho_2) \nsubseteq (\varrho_1, \varrho_2)$, there exists $(x_1, x_2) \in (U_1, U_2)$ such that $(x_1, x_2) \in \overline{N_B}(\varrho_1, \varrho_2) - (\varrho_1, \varrho_2)$. Then $(x_1, x_2) \in \overline{N_B}(\varrho_1, \varrho_2) - (\varrho_1, \varrho_2) \subseteq (U_1, U_2) - (\varrho_1, \varrho_2)$. Thus $(\varrho_1, \varrho_2) \subseteq (U_1, U_2) - (x_1, x_2)$, where $(U_1, U_2) - (x_1, x_2)$ is N_B -open. But $\overline{N_B}(\varrho_1, \varrho_2) \nsubseteq (U_1, U_2) - (x_1, x_2)$, since $(x_1, x_2) \in \overline{N_B}(\varrho_1, \varrho_2) \Rightarrow (\varrho_1, \varrho_2)$ is not N_B g-closed. Suppose, let $\overline{N_B}(\varrho_1, \varrho_2) \subseteq (\varrho_1, \varrho_2)$ and $(\mathcal{G}_1, \mathcal{G}_2)$ be any N_B -open set such that $(\varrho_1, \varrho_2) \subseteq (\mathcal{G}_1, \mathcal{G}_2)$. Then $\overline{N_B}(\varrho_1, \varrho_2)$ $\subseteq (\varrho_1, \varrho_2) \subseteq (\varrho_1, \varrho_2)$. Therefore, (ϱ_1, ϱ_2) is N_B g-closed.*

Theorem 3.10. *In N_B topological space $(U_1, U_2, \tau_R(X_1, X_2))$, a subset (ϱ_1, ϱ_2) of (U_1, U_2) is strongly N_B g-closed if and only if (ϱ_1, ϱ_2) is N_B g-closed.*

Proof: *Proof follows from the prior theorems.*

Theorem 3.11. *In N_B topological space $(U_1, U_2, \tau_R(X_1, X_2))$*

1. *Every N_B g -closed is weakly N_B g-closed.*
2. *Every strongly N_B g -closed is mildly N_B g-closed.*

Proof: *The proof is obvious.*

Remark 3.2. *The converses of the above theorem are not true as shown in the following examples.*

Example 3.1. *Let* $U_1 = \{o, p, q\}, U_2 = \{8, 9\}$ *with* $(U_1, U_2)/R = \{(\{o, p\}, \{9\}), (\{q\}, \{8\})\}$ *and* $(X_1, X_2) = (\{p\}, \{9\})$. *Then* $\tau_R(X_1, X_2) = \{(\phi, \phi), (U_1, U_2), (\{o, p\}, \{9\})\}$. *Here* $(\{p\}, \{9\})$ *is weakly* N_B *g-closed but not* N_B *g-closed. Since* $\overline{N_B}{N_B}^\circ(\{p\}, \{9\}) = \overline{N_B}(\phi, \phi) = (\phi, \phi) \subseteq (\{o, p\}, \{9\}) = (\mathcal{G}_1, \mathcal{G}_2)$ *but* $\overline{N_B}(\{p\}, \{9\}) = (U_1, U_2) \nsubseteq (\{o, p\}, \{9\})$.

Example 3.2. *Let* $U_1 = \{s, t, u\}, U_2 = \{e, f\}$ *with* $(U_1, U_2)/R = \{(\{s, u\}, \{e\}), (\{t\}, \{f\})\}$ *and* $(X_1, X_2) = (\{s, t\}, \{f\})$. *Then* $\tau_R(X_1, X_2) = \{(\phi, \phi), (U_1, U_2), (\{s, u\}, \{e\}), (\{t\}, \{f\})\}$. *Take* $(\mathcal{G}_1, \mathcal{G}_2) = (\{t, u\}, \{e, f\})$ *is* N_B *g-open and* $(H_1, H_2) = (\{u\}, \{e\})$. *Here* $\overline{N_B}{N_B}^\circ(\{u\}, \{e\}) = \overline{N_B}(\phi, \phi) = (\phi, \phi) \subseteq (\mathcal{G}_1, \mathcal{G}_2)$ *but*$\overline{N_B}(\{u\}, \{e\}) = (\{s, u\}, \{e\}) \nsubseteq (\mathcal{G}_1, \mathcal{G}_2)$. *Therefore,* $(\{u\}, \{e\})$ *is mildly* N_B *g-closed but not strongly* N_B *g-closed.*

Result 3.1. In N_B topological space $(U_1, U_2, \tau_R(X_1, X_2))$ for a subset (ϱ_1, ϱ_2) of (U_1, U_2), then the following properties are equivalent

1. (ϱ_1, ϱ_2) is mildly N_B g-closed.
2. $\overline{N_B}({N_B}^\circ(\varrho_1, \varrho_2)) - (\varrho_1, \varrho_2) = (\phi, \phi)$.
3. $\overline{N_B}({N_B}^\circ(\varrho_1, \varrho_2)) \subseteq (\varrho_1, \varrho_2)$
4. (ϱ_1, ϱ_2) is N_B pre -closed.

Theorem 3.12. *In* N_B *topological space* $(U_1, U_2, \tau_R(X_1, X_2))$ *if* (ϱ_1, ϱ_2) *is mildly* N_B *g-closed, then* $(\varrho_1, \varrho_2) \cup ((U_1, U_2) - \overline{N_B}({N_B}^\circ(\varrho_1, \varrho_2)))$ *is mildly* N_B *g-closed.*

Proof: *Since* (ϱ_1, ϱ_2) *is mildly* N_B *g-closed,* $\overline{N_B}({N_B}^\circ(\varrho_1, \varrho_2)) \subseteq (\varrho_1, \varrho_2)$. *Then* $(U_1, U_2) - (\varrho_1, \varrho_2) \subseteq (U_1, U_2) - \overline{N_B}({N_B}^\circ(\varrho_1, \varrho_2)) \Rightarrow (\varrho_1, \varrho_2) \cup ((U_1, U_2) - (\varrho_1, \varrho_2)) \subseteq (\varrho_1, \varrho_2) \cup ((U_1, U_2) - \overline{N_B}({N_B}^\circ(\varrho_1, \varrho_2))) \Rightarrow (U_1, U_2) \subseteq (\varrho_1, \varrho_2) \cup ((U_1, U_2) - \overline{N_B}({N_B}^\circ(\varrho_1, \varrho_2)))$. *Also* $(\varrho_1, \varrho_2) \cup ((U_1, U_2) - \overline{N_B}({N_B}^\circ(\varrho_1, \varrho_2))) \subseteq (U_1, U_2)$. *Therefore,* $(\varrho_1, \varrho_2) \cup ((U_1, U_2) - \overline{N_B}({N_B}^\circ(\varrho_1, \varrho_2))) = (U_1, U_2)$. *Hence* $(\varrho_1, \varrho_2) \cup ((U_1, U_2) - \overline{N_B}({N_B}^\circ(\varrho_1, \varrho_2)))$ *is mildly* N_B *g-closed.*

Theorem 3.13. *In* N_B *topological space* $(U_1, U_2, \tau_R(X_1, X_2))$ *every* N_B *closed set is mildly* N_B *g-closed.*

Proof: *Let* (ϱ_1, ϱ_2) *be* N_B *closed set. Let* $(\mathcal{G}_1, \mathcal{G}_2)$ *be any* N_B *g-open subset of* (U_1, U_2) *such that* $(\varrho_1, \varrho_2) \subseteq (\mathcal{G}_1, \mathcal{G}_2)$. *Then* $\overline{N_B}({N_B}^\circ(\varrho_1, \varrho_2)) \subseteq \overline{N_B}(\varrho_1, \varrho_2) = (\varrho_1, \varrho_2) \subseteq (\mathcal{G}_1, \mathcal{G}_2)$. *Therefore,* (ϱ_1, ϱ_2) *is mildly* N_B *g-closed.*

Remark 3.3. *The reverse of the above theorem is not true as shown in the following example.*

Example 3.3. *In example 2.11,* $(\{q\}, \{7\})$ *is mildly* N_B *g-closed but not* N_B *closed.*

Theorem 3.14. *In a* N_B *topological space* $(U_1, U_2, \tau_R(X_1, X_2))$, *if* (ϱ_1, ϱ_2) *is mildly* N_B *g-closed and* $(\mathcal{G}_1, \mathcal{G}_2)$ *is a subset such that* $(\varrho_1, \varrho_2) \subseteq (\mathcal{G}_1, \mathcal{G}_2) \subseteq \overline{N_B}({N_B}^\circ(\varrho_1, \varrho_2))$, *then* $(\mathcal{G}_1, \mathcal{G}_2)$ *is mildly* N_B *g-closed.*

Proof: *Since* (ϱ_1, ϱ_2) *is mildly* N_B *g-closed,* $\overline{N_B}({N_B}^\circ(\varrho_1, \varrho_2)) \subseteq (\varrho_1, \varrho_2)$. $(\varrho_1, \varrho_2) \subseteq (G_1, G_2) \subseteq \overline{N_B}({N_B}^\circ(\varrho_1, \varrho_2)) \subseteq (\varrho_1, \varrho_2) \Rightarrow (\varrho_1, \varrho_2) \subseteq (G_1, G_2) \subseteq (\varrho_1, \varrho_2) \Rightarrow (\varrho_1, \varrho_2) = (\mathcal{G}_1, \mathcal{G}_2)$. *Therefore,* $(\mathcal{G}_1, \mathcal{G}_2)$ *is mildly* N_B *g-closed.*

Corollary 3.1. *In a* N_B *topological space* $(U_1, U_2, \tau_R(X_1, X_2))$, *if* (ϱ_1, ϱ_2) *is mildly* N_B *g-closed and an* N_B *open set, then* $\overline{N_B}(\varrho_1, \varrho_2)$ *is mildly* N_B *g-closed.*

Proof: *Since* (ϱ_1, ϱ_2) *is* N_B *open in* (U_1, U_2), $(\varrho_1, \varrho_2) \subseteq \overline{N_B}(\varrho_1, \varrho_2) = \overline{N_B}({N_B}^\circ (\varrho_1, \varrho_2))$. *By the above theorem,* $\overline{N_B}(\varrho_1, \varrho_2)$ *is mildly* N_B *g-closed.*

Theorem 3.15. *In a N_B topological space $(U_1, U_2, \tau_R(X_1, X_2))$, a N_B nowhere dense subset is mildly N_B g-closed.*

Proof: *If (ϱ_1, ϱ_2) is N_B nowhere dense subset in (U_1, U_2), then ${N_B}^\circ(\overline{N_B}(\varrho_1, \varrho_2)) = (\phi, \phi)$. Since ${N_B}^\circ(\varrho_1, \varrho_2) \subseteq {N_B}^\circ(\overline{N_B}(\varrho_1, \varrho_2)) = (\phi, \phi)$. Therefore, ${N_B}^\circ(\varrho_1, \varrho_2) = (\phi, \phi)$. Hence $\overline{N_B}({N_B}^\circ(\varrho_1, \varrho_2)) = \overline{N_B}(\phi, \phi) = (\phi, \phi) \subseteq (\varrho_1, \varrho_2)$. Therefore, (ϱ_1, ϱ_2) is mildly N_B g-closed.*

Remark 3.4. *The reverse of the prior theorem is not true as shown in the following example.*

Example 3.4. *In example 2.11, $(\{q\}, \{7\})$ is mildly N_B g-closed but not N_B nowhere dense. Because ${N_B}^\circ(\overline{N_B}(\{q\}, \{7\}) = {N_B}^\circ(\{o, q\}, \{7\}) = (\{o, q\}, \{7\}) \neq (\phi, \phi)$.*

4. Conclusion:

Some new nano binary generalized closed sets were introduced and their properties were discussed. In future we will discuss N_B kernel and N_B separation axioms in N_B topological spaces.

References

[1] Lellis Thivagar. M and Carmel Richard, On Nano forms of weakly open sets, International Journal of Mathematics and Statistics Invention, 1(1) 2013, 31-37.

[2] Nithyanantha Jothi. S and P. Thangavelu, On Binary Topological Spaces, Pacific ? Asian Journal of Mathematics.

[3] Hari Siva Annam. G and J. Jasmine Elizabeth, Cognition of Nano Binary Topological Spaces, Global Journal of Pure and Applied Mathematics, 6(2019), 1045-1054.

[4] J. Jasmine Elizabeth and G. Hari Siva Annam, Some notions on nano binary continuous, Malaya Journal of Matematik, 2021.

[5] J. Jasmine Elizabeth and G. Hari Siva Annam, Nano binary contra continuous functions in nano binary topological spaces, J. Math. Comput. Sci. 11 (2021), No. 4, 4994-5011.

[6] C. Janaki and A. Jeyalakshmi, A new form of generalized closed sets via regular local function in ideal topological spaces, Malaya Journal of Matematik, S(1) (2015), 1-9.

[7] O. Njastad, On some classes of nearly open sets, Pacific J. Math.,15 (1965), 961-970.

[8] E. F. Lashin and T. Medhat, Topological reduction of information systems, Chaos, Solitions and Fractals, 25 (2015), 277-286.

[9] C. Richard, Studies on nano topological spaces, Ph.D. Thesis, Madurai Kamaraj University, India (2013).