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# Locally closed sets and $g$-locally closed sets in binary topological spaces

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**Abstract:** In this paper, we focused on locally closed sets in binary topological spaces and certain properties of these investigated. Also, we studied the binary generalized locally closed sets, binary $g$-$lc^{\star}$, binary $g$-$lc^{\star\star}$ and established their various characteristic properties.

**Key words:** binary locally closed sets, binary $g$-lc, binary $g$-$lc^{\star}$, binary $g$-$lc^{\star\star}$

## 1. Introduction and Preliminaries

In 2011, S.Nithyanantha Jothi and P.Thangavelu [8] introduced topology between two sets and also studied some of their properties. Topology between two sets is the binary structure from $X$ to $Y$ which is defined to be the ordered pairs $(A, B)$ where $A \subseteq X$ and $B \subseteq Y$. Recently some researcher developed some new kind of modern topologies for example ([12], [13], and [1]). In this paper, we focused on locally closed sets in binary topological spaces and certain properties of these investigated. Also, we studied the binary generalized locally closed sets, binary $g$-$lc^{\star}$, binary $g$-$lc^{\star\star}$ and established their various characteristic properties.

We denote the boundary of $(A, B)$ by $bd(A, B)$.

Let $X$ and $Y$ be any two nonempty sets. A binary topology [8] from $X$ to $Y$ is a binary structure $\mathcal{M} \subseteq \mathbb{P}(X) \times \mathbb{P}(Y)$ that satisfies the axioms namely

1. $(\phi, \phi)$ and $(X, Y) \in \mathcal{M}$,
2. $(A_1 \cap A_2, B_1 \cap B_2) \in \mathcal{M}$ whenever $(A_1, B_1) \in \mathcal{M}$ and $(A_2, B_2) \in \mathcal{M}$, and
3. If $\{(A_\alpha, B_\alpha) : \alpha \in \delta\}$ is a family of members of $\mathcal{M}$, then $(\bigcup_{\alpha\in\delta} A_\alpha, \bigcup_{\alpha\in\delta} B_\alpha) \in \mathcal{M}$.

If $\mathcal{M}$ is a binary topology from $X$ to $Y$ then the triplet $(X, Y, \mathcal{M})$ is called a binary topological space and the members of $\mathcal{M}$ are called the binary open subsets of the binary topological space $(X, Y, \mathcal{M})$. The elements of $X \times Y$ are called the binary points of the binary topological space $(X, Y, \mathcal{M})$. If $Y = X$ then $\mathcal{M}$ is called a binary topology on $X$ in which case we write $(X, \mathcal{M})$ as a binary topological space.

**Definition 1.1.** [8] Let $X$ and $Y$ be any two nonempty sets and let $(A, B)$ and $(C, D) \in \mathbb{P}(X) \times \mathbb{P}(Y)$. We say that $(A, B) \subseteq (C, D)$ if $A \subseteq C$ and $B \subseteq D$.

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**Definition 1.2.** [8] Let $(X, Y, \mathcal{M})$ be a binary topological space and $A \subseteq X$, $B \subseteq Y$. Then $(A, B)$ is called binary closed in $(X, Y, \mathcal{M})$ if $(X \backslash A, Y \backslash B) \in \mathcal{M}$.

**Proposition 1.1.** *[8] Let $(X, Y, \mathcal{M})$ be a binary topological space and $(A, B) \subseteq (X, Y)$. Let $(A, B)^{1*} = \cap\{A_\alpha : (A_\alpha, B_\alpha)$ is binary closed and $(A, B) \subseteq (A_\alpha, B_\alpha)\}$ and $(A, B)^{2*} = \cap\{B_\alpha : (A_\alpha, B_\alpha)$ is binary closed and $(A, B) \subseteq (A_\alpha, B_\alpha)\}$. Then $((A, B)^{1*}, (A, B)^{2*})$ is binary closed and $(A, B) \subseteq ((A, B)^{1*}, (A, B)^{2*})$.*

**Proposition 1.2.** *[8] Let $(X, Y, \mathcal{M})$ be a binary topological space and $(A, B) \subseteq (X, Y)$. Let $(A, B)^{1*} = \cup\{A_\alpha : (A_\alpha, B_\alpha)$ is binary open and $(A_\alpha, B_\alpha) \subseteq (A, B)\}$ and $(A, B)^{2*} = \cup\{B_\alpha : (A_\alpha, B_\alpha)$ is binary open and $(A_\alpha, B_\alpha) \subseteq (A, B)\}$.*

**Definition 1.3.** [8] The ordered pair $((A, B)^{1*}, (A, B)^{2*})$ is called the binary closure of $(A, B)$, denoted by $b\text{-}cl(A, B)$ in the binary space $(X, Y, \mathcal{M})$ where $(A, B) \subseteq (X, Y)$.

**Definition 1.4.** [8] The ordered pair $((A, B)^{1*}, (A, B)^{2*})$ defined in proposition 1.2 is called the binary interior of of $(A, B)$, denoted by $b\text{-}int(A, B)$. Here $((A, B)^{1*}, (A, B)^{2*})$ is binary open and $((A, B)^{1*}, (A, B)^{2*}) \subseteq (A, B)$.

**Definition 1.5.** [10] A subset $(A, B)$ of a binary topological space $(X, Y, \mathcal{M})$ is called a binary semi open set if $(A, B) \subseteq b\text{-}cl(b\text{-}int(A, B))$.

**Definition 1.6.** [9] A subset $(A, B)$ of a binary topological space $(X, Y, \mathcal{M})$ is called a binary $g$-closed set if $b\text{-}cl(A, B) \subseteq (U, V)$ whenever $(A, B) \subseteq (U, V)$ and $(U, V)$ is binary open.

## 2. Binary Locally Closed Sets

**Definition 2.1.** A subset $(A, B)$ of a binary topological space $(X, Y, \mathcal{M})$ is called binary locally closed if $(A, B) = (E, F) \cap (G, H)$ where $(E, F)$ is binary open and $(G, H)$ is binary closed in $(X, Y, \mathcal{M})$.

The set of all binary locally closed sets of $(X, Y, \mathcal{M})$ is denoted by $BLC(X, Y)$.

**Proposition 2.1.**
1. *Let $(X, Y, \mathcal{M})$ be a binary $T_1$-space and let $(A, B)$ be a discrete subset of $(X, Y, \mathcal{M})$. Then $(A, B)$ is binary locally closed.*
2. *Let $(X, Y, \mathcal{M})$ be binary dense-in-itself and $(A, B)$ be a discrete subset. Then $(X, Y) - (A, B)$ is binary locally closed if and only if $(A, B)$ is binary closed.*

*Proof.* Let $(A, B)$ be a discrete subset of the binary $T_1$-space $(X, Y, \mathcal{M})$, i.e. for each $(i, j) \in (A, B)$ there is an binary open set $(E, F)_{(X,Y)}$ such that $(E, F)_{(X,Y)} \cap (A, B) = \{(i, j)\}$. If $(E, F) = \{(E, F)_{(X,Y)} | (i, j) \in (A, B)\}$ then it is easily verified that $(A, B) = (E, F) \cap b\text{-}cl(A, B)$. This proves (1).

In order to prove (2), observe that in a binary dense-in-itself space any discrete subset has empty interior.

**Proposition 2.2.** *Let $(X, Y, \mathcal{M})$ be a space and let $(U, V) \in BLC(X, Y)$. If $(A, B) \in (U, V)$ and $(A, B) \in BLC((U, V), \mathcal{M}/(U, V))$ then $(A, B) \in BLC(X, Y)$.*

**Proposition 2.3.** *Let $(A, B)$ and $(C, D)$ be binary locally closed subsets of a space $(X, Y, \mathcal{M})$. If $(A, B)$ and $(C, D)$ are separated, i.e. if $(A, B) \cap b\text{-}cl(C, D) = b\text{-}cl((A, B) \cap (C, D)) = (\phi, \phi)$, then $(A, B) \cup (C, D) \in BLC(X, Y)$.*

*Proof.* Suppose there are binary open sets $(P,Q)$ and $(S,T)$ such that $(A,B) = (P,Q) \cap b\text{-}cl(A,B)$ and $(C,D) = (S,T) \cap b\text{-}cl(C,D)$. Since $(A,B)$ and $(C,D)$ are separated we may assume that $(P,Q) \cap b\text{-}cl(C,D) = (S,T) \cap b\text{-}cl(A,B) = (\phi,\phi)$. Consequently $(A,B) \cup (C,D) = ((P,Q) \cup (S,T)) \cap b\text{-}cl((A,B) \cup (C,D))$ showing that $(A,B) \cup (C,D) \in BLC(X,Y)$.

**Theorem 2.1.** *Let $\{(U,V)_i | i \in I\}$ be either an binary open cover or a binary locally finite closed cover of a space $(X,Y,\mathcal{M})$ and let $(A,B) \subseteq (X,Y)$. If $(A,B) \cap (U,V)_i \in BLC((U,V)_i, \mathcal{M}/(U,V)_i)$ for each $i \in I$ then $(A,B) \in BLC(X,Y)$.*

*Proof.* First suppose that $\{(U,V)_i | i \in I\}$ is an binary open cover of $(X,Y,\mathcal{M})$. For each $i \in I$, since $(A,B) \cap (U,V)_i \in BLC((U,V)_i, \mathcal{M}/(U,V)_i)$ we may assume that $(A,B) \cap (U,V)_i = (S,T)_i \cap b\text{-}cl((A,B) \cap (U,V)_i)$ where $(S,T)_i \in \mathcal{M}$ and $(S,T)_i \subseteq (U,V)_i$. Now $(S,T)_i \cap b\text{-}cl(A,B) = (S,T)_i \cap (U,V)_i \cap b\text{-}cl(A,B) \subseteq (S,T)_i \cap b\text{-}cl((A,B) \cap (U,V)_i) = (A,B) \cap (U,V)_i$. Hence if $(S,T) = \bigcup\{(S,T)_i | i \in I\}$ we have $(S,T) \cap b\text{-}cl(A,B) = (A,B)$.

Now suppose that $\{(U,V)_i | i \in I\}$ is a binary locally finite closed cover of $(X,Y,\mathcal{M})$. For each $i \in I$, since $(A,B) \cap (U,V)_i \in BLC((U,V)_i, \mathcal{M}/(U,V)_i)$ we have $(A,B) \cap (U,V)_i = (S,T)_i \cap b\text{-}cl((A,B) \cap (U,V)_i)$ where $(S,T)_i \in \mathcal{M}$. Let $(x,y) \in (A,B)$. Since $\{(U,V)_i | i \in I\}$ is a binary locally finite closed cover, hence a point-finite and closure-preserving cover, there is a finite subset $I_{(X,Y)} \subseteq I$ such that $(x,y) \in (U,V)_i$ if $I \in I_{(X,Y)}$ and $(x,y) \notin \bigcup\{(U,V)_i | i \in I - I_{(X,Y)}\}$. Moreover, there is an binary open set $(P,Q)_{(X,Y)}$ containing $(x,y)$ such that $(P,Q)_{(X,Y)} \subseteq \bigcap\{(S,T)_i | i \in I_{(X,Y)}\}$ and $(P,Q)_{(X,Y)} \cap ((P,Q)\{(U,V)_i | i \in I - I_{(X,Y)}\}) = (\phi,\phi)$.

If $(P,Q) = \bigcup\{(P,Q)_{(x,y)} | (x,y) \in (A,B)\}$ then clearly $(A,B) \subseteq (P,Q) \cap b\text{-}cl(A,B)$. Let $(k,l) \in (P,Q) \cap b\text{-}cl(A,B)$. Then $(k,l) \in (P,Q)_{(X,Y)}$ for some $(x,y) \in (A,B)$. Since $(k,l) \in b\text{-}cl(A,B) = \bigcup\{b\text{-}cl((A,B) \cap (U,V)_i) | i \in I\}$ we have $(k,l) \in b\text{-}cl((A,B) \cap (U,V)_j)$ for some $j \in I$. Hence $j \in I_{(X,Y)}$ and $(P,Q)_{(X,Y)} \subseteq (S,T)_j$. Thus $(k,l) \in (S,T)_j \cap b\text{-}cl((A,B) \cap (U,V)_j) = (A,B) \cap (U,V)_j \subseteq (A,B)$. It follows that $(A,B) = (P,Q) \cap b\text{-}cl(A,B)$.

**Theorem 2.2.** *For a binary $T_1$-space $(X,Y,\mathcal{M})$ the following are equivalent:*

1. *$(A,B) \in BLC(X,Y)$ if and only if $(X,Y) - (A,B) \in BLC(X,Y)$.*
2. *$BLC(X,Y)$ is binary closed under finite unions.*
3. *The boundary of each binary open set is a discrete subset.*
4. *The boundary of each binary semi-open set is a discrete subset.*
5. *Every binary semi-open set is binary locally closed.*

*Proof.* $(1) \Leftrightarrow (2)$ is obvious.

$(2) \Rightarrow (3)$ : Let $(P,Q)$ be binary open and let $(x,y) \in bd(P,Q) \cap b\text{-}cl(P,Q) \cap ((X,Y) - (P,Q))$. By assumption, if $(A,B) = (P,Q) \cup \{(x,y)\}$ then $(A,B) \in BLC(X,Y)$. Let $(A,B) = (S,T) \cap b\text{-}cl(A,B)$ for some binary open set $(S,T)$. One easily verifies that $(S,T) \cap bd(P,Q) = \{(x,y)\}$.

$(3) \Rightarrow (4)$ : Let $(A,B)$ be binary semi-open in $(X,Y,\mathcal{M})$ and let $(P,Q) = b\text{-}int(A,B)$. Then $bd(A,B) \subseteq bd(P,Q)$ and hence $bd(A,B)$ is a discrete subset.

(4) $\Rightarrow$ (5) : Let $(A,B)$ be binary semi-open in $(X,Y,\mathcal{M})$. For each $(x,y) \in (A,B) \cap bd(A,B)$ there is an binary open set $(P,Q)$ such that $(P,Q)_{(X,Y)} \cap bd(A,B) = \{(x,y)\}$. If $(P,Q) = b\text{-}int(A,B) \cup [\bigcup\{(P,Q)_{(X,Y)} | (x,y) \in (A,B) \cap bd(A,B)\}]$ it is easily verified that $(A,B) = (P,Q) \cap b\text{-}cl(A,B)$.

(5) $\Rightarrow$ (1) : We will show that any union of an binary open set and a binary closed set is binary locally closed. Let $(C,D) = (E,F) \cup (G,H)$ where $(E,F)$ is binary open and $(G,H)$ is binary closed. We may assume that $(E,F) \cap (G,H) = (\phi,\phi)$. If $(A,B) = (E,F) \cup b\text{-}(cl(E,F) \cap (G,H))$ then $(A,B)$ is binary semi-open and hence $(A,B) = (S,T) \cap b\text{-}cl(A,B) = (S,T) \cap b\text{-}cl(A,B) = (S,T) \cap cl(E,F)$ for some binary open set $(S,T)$. If $(W,Z) = (S,T) \cup ((X,Y) - b\text{-}cl(E,F))$ then clearly $(C,D) = (W,Z) \cap b\text{-}cl(C,D)$. Thus $(C,D) \in BLC(X,Y)$.

## 3. Binary $g$-Locally Closed Sets

**Definition 3.1.** A subset $(A,B)$ of a binary topological space $(X,Y,\mathcal{M})$, is called

1. binary Generalized locally closed set (briefly *bglc*) if $(A,B) = (E,F) \cap (G,H)$, where $(E,F)$ is binary $g$-open and $(G,H)$ is binary $g$-closed in $(X,Y,\mathcal{M})$.
2. $bg\text{-}lc^*$-set if $(A,B) = (E,F) \cap (G,H)$, where $(E,F)$ is binary $g$-open set and $(G,H)$ is binary closed set in $(X,Y,\mathcal{M})$.
3. $bg\text{-}lc^{**}$-set if $(A,B) = (E,F) \cap (G,H)$, where $(E,F)$ is binary open set and $(G,H)$ is binary $g$-closed set in $(X,Y,\mathcal{M})$.

The collection of all binary Generalized locally closed sets (resp. $bglc^*$-sets and $bglc^{**}$-sets) of $(X,Y,\mathcal{M})$ will be denoted by $BGLC(X,Y)$ (resp. $BGLC^*(X,Y)$ and $BGLC^{**}(X,Y)$).

**Remark 3.1.** *Every binary $g$-closed set (resp. binary $g$-open set) is bglc.*

**Theorem 3.1.** *For a subset $(A,B)$ of $(X,Y,\mathcal{M})$, the following are equivalent.*

1. $(A,B) \in BGLC^*(X,Y)$.
2. $(A,B) = (E,F) \cap b\text{-}cl(A,B)$ *for some binary $g$-open set* $(E,F)$.
3. $b\text{-}cl(A,B) - (A,B)$ *is binary $g$-closed.*
4. $(A,B) \cup ((P,Q) - b\text{-}cl(A,B))$ *is binary $g$-open.*

*Proof.* (1) $\Rightarrow$ (2) Let $(A,B) \in BGLC^*(X,Y)$. Then $(A,B) = (E,F) \cap (G,H)$ where $(E,F)$ is binary $g$-open and $(G,H)$ is binary closed. Since $(A,B) \subseteq (E,F)$ and $(A,B) \subseteq b\text{-}cl(A,B)$, $(A,B) \subseteq (E,F) \cap b\text{-}cl(A,B)$. Conversely, since $(A,B) \subseteq (G,H)$, $b\text{-}cl(A,B) \subseteq (G,H)$, we have $(A,B) = (E,F) \cap (G,H)$ contains $(E,F) \cap b\text{-}cl(A,B)$. That is $(E,F) \cap b\text{-}cl(A,B) \subseteq (A,B)$. Therefore we have $(A,B) = (E,F) \cap b\text{-}cl(A,B)$.

(2) $\Rightarrow$ (1) Since $(E,F)$ is binary $g$-open and $b\text{-}cl(A,B)$ is binary closed, $(E,F) \cap b\text{-}cl(A,B) \in BGLC^*(X,Y)$ by Definition of $BGLC^*(X,Y)$.

(2) $\Rightarrow$ (3) $(A,B) = (E,F) \cap b\text{-}cl(A,B)$ implies that $b\text{-}cl(A,B) - (A,B) = b\text{-}cl(A,B) \cap (E,F)^c$ which is binary $g$-closed, Since $(E,F)^c$ is binary $g$-closed.

(3) $\Rightarrow$ (2) Let $(E,F) = (b\text{-}cl(A,B) - (A,B))^c$. Then by assumption, $(E,F)$ is binary $g$-open in $(X,Y,\mathcal{M})$ and $(A,B) = (E,F) \cap b\text{-}cl(A,B)$.

(3) $\Rightarrow$ (4) $(A,B)\cup((P,Q)-b\text{-}cl(A,B)) = (A,B)\cup(b\text{-}cl(A,B))^c = (b\text{-}cl(A,B)-(A,B))^c$ and by assumption $(b\text{-}cl(A,B)-(A,B))^c$ is binary $g$-open and $(A,B)\cup((P,Q)-b\text{-}cl(A,B))$ is binary $g$-open.

(4) $\Rightarrow$ (3) Let $(E,F) = (A,B)\cup(b\text{-}cl(A,B))^c$. Then $(E,F)^c$ is binary $g$-closed and $(E,F)^c = b\text{-}cl(A,B)-(A,B)$ and therefore $b\text{-}cl(A,B)-(A,B)$ is binary $g$-closed.

**Theorem 3.2.** *For a subset $(A,B)$ of $(X,Y,\mathcal{M})$, the following statements are equivalent.*

1. $(A,B)\in BGLC(X,Y)$.
2. $(A,B)=(E,F)\cap b\text{-}gcl(A,B)$ *for some binary $g$-open set $(E,F)$.*
3. $b\text{-}gcl(A,B)-(A,B)$ *is binary $g$-closed.*
4. $(A,B)\cup(b\text{-}gcl(A,B))^c$ *is binary $g$-open.*
5. $(A,B)\subseteq b\text{-}gint((A,B)\cup(b\text{-}gcl(A,B))^c)$

*Proof.* (1) $\Rightarrow$ (2) Let $(A,B)\in BGLC(X,Y)$. Then $(A,B)=(E,F)\cap(G,H)$, where $(E,F)$ is binary $g$-open and $(G,H)$ is binary $g$-closed. Since $(A,B)\subseteq(G,H)$, $b\text{-}gcl(A,B)\subseteq(G,H)$ and therefore $(E,F)\cap b\text{-}gcl(A,B)\subseteq(A,B)$. Also $(A,B)\subseteq(E,F)$ and $(A,B)\subseteq b\text{-}gcl(A,B)$ implies $(A,B)\subseteq(E,F)\cap b\text{-}gcl(A,B)$ and therefore $(A,B)=(E,F)\cap b\text{-}gcl(A,B)$.

(2) $\Rightarrow$ (3) $(A,B)=(E,F)\cap b\text{-}gcl(A,B)$ implies $b\text{-}gcl(A,B)-(A,B)=b\text{-}gcl(A,B)\cap(E,F)^c$ which is binary $g$-closed since $(E,F)^c$ is binary $g$-closed.

(3) $\Rightarrow$ (4) $A\cup(Ngcl(A))^c=(Ngcl(A)-A)^c$ and by assumption $(Ngcl(A)-A)^c$ is $Ng$-open and so is $A\cup(Ngcl(A))^c$.

(4) $\Rightarrow$ (5) By assumption, $(A,B)\cup(b\text{-}gcl(A,B))^c=b\text{-}gint((A,B)\cup(b\text{-}gcl(A,B))^c)$ and hence $(A,B)\subseteq b\text{-}gint((A,B)\cup(b\text{-}gcl(A,B))^c)$.

(5) $\Rightarrow$ (1) By assumption and since $(A,B)\subseteq b\text{-}gcl(A,B)$, $(A,B)=b\text{-}gint((A,B)\cup(b\text{-}gcl(A,B))^c)\cap b\text{-}gcl(A,B)\in BGLC(X,Y)$.

**Theorem 3.3.** *Let $(A,B)$ be a subset of $(X,Y,\mathcal{M})$. Then $(A,B)\in BGLC^{**}(X,Y)$ if and only if $(A,B)=(E,F)\cap b\text{-}gcl(A,B)$ for some binary open set $(E,F)$.*

*Proof.* Let $(A,B)\in BGLC^{**}(X,Y)$. Then $(P,Q)=(E,F)\cap(G,H)$ where $(E,F)$ is binary open and $(G,H)$ is binary $g$-closed. Since $(P,Q)\subseteq(G,H)$, $b\text{-}gcl(A,B)\subseteq(G,H)$. Now $(A,B)=(A,B)\cap b\text{-}gcl(A,B)=(E,F)\cap(G,H)\cap b\text{-}gcl(A,B)=(E,F)\cap b\text{-}gcl(A,B)$. Here the converse part is trivial.

**Corollary 3.1.** *Let $(A,B)$ be a subset of $(X,Y,\mathcal{M})$. If $(A,B)\in BGLC^{**}(X,Y)$, then $b\text{-}gcl(A,B)-(A,B)$ is binary $g$-closed and $(A,B)\cup(b\text{-}gcl(A,B))^c$ is binary $g$-open.*

*Proof.* Let $(A,B)\in BGLC^{**}(X,Y)$. Then by Theorem 3.3, $(A,B)=(E,F)\cap b\text{-}gcl(A,B)$ for some binary open set $(E,F)$ and $b\text{-}gcl(A,B)-(A,B)=b\text{-}gcl(A,B)\cap(E,F)^c$ is binary $g$-closed in $(X,Y,\mathcal{M})$. If $(G,H)=b\text{-}gcl(A,B)-(A,B)$, then $(G,H)^c=(A,B)\cup(b\text{-}gcl(A,B))^c$ and $(G,H)^c$ is binary $g$-open and therefore $(A,B)\cup(b\text{-}gcl(A,B))^c$ is binary $g$-open.

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