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# A note on the algebra of triple sets

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**Abstract:** In this paper, we introduce the notion of triple sets. They can be considered as a bridge between double and intuitionistic sets or as their specific combination. Triple sets have some features of both these classes and thus they are able to model two aspects of uncertainty. On the one hand, they express the modal idea of necessity and possibility. On the other, they refer to the fact that while some objects may be accepted by decision maker (that is, considered as being true) and some can be rejected (thus treated as false), it is still possible that there are some neutral objects. We define some basic algebraic operations on triple sets and analyze their properties (comparing them e.g. with intuitionistic logic). We compare two approaches to the notion of complement (negation in the logical sense). We point out that some natural extensions of this framework should be studied too.

**Key words:** Triple sets, intuitionistic sets, double sets, topological spaces

## 1. Introduction

Intuitionistic sets were introduced by Coker in [2]. One could say that they are crisp version of intuitionistic fuzzy sets that had been studied earlier by Atanassov (e.g. in [1]). The idea of intuitionistic sets is simple: such set on a non-empty universe $X$ is an ordered pair of the form $A = (A_T, A_F)$ where both $A_T$ and $A_F$ are ordinary subsets of $X$ of with the assumption that their intersection is empty. This idea is reasonable and interesting because we can define various algebraic operations on intuitionistic sets (they will be presented in the next section). In particular, these operations (together with the concept of intuitionistic complement, empty set and universal set) are defined in such a way that both the law of excluded middle and the law of consistency do not hold. We assume that $A_T$ denotes truth set of $A$, while $A_F$ is its falsity component. As for the objects that are beyond the union of $A_T$ and $A_F$, these are just neutral elements.

Nowadays the whole theory of intuitionistic sets is very wide and complex. In particular, intuitionistic topological spaces (see [3]) are constantly investigated by many authors all over the world. However, double sets are studied too (e.g. in [6] their classical and soft versions have been described in the context of topology). Here the idea is that $A = (A_1, A_2)$, where $A_1 \subseteq A_2$ which means that $A_1$ contains those objects that are necessary or crucial while $A_2$ consists of all those elements that are just optionary or possible. As for the complement of $A_2$, it is the set of those elements that are not under consideration. The whole double set describes certain point of view.

In this paper we connect both these concepts, introducing triple sets. We show how to calculate their unions and intersections. We signalize the fact that it is possible to discuss triple topological spaces and other

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(weaker or just different) spaces. For example, recently anti-topological spaces have been studied by some authors (see [5] or [8]) but in the framework of classical $P(X)$.

## 2. Preliminary notions

First, let us formulate the definitions of intuitionistic set and double set in a formal way.

**Definition 2.1.** [2] Assume that $X$ is a non-empty universe, while $A_T$ and $A_F$ are its subsets such that their intersection is empty. Then we say that an ordered pair $A = (A_T, A_F)$ is an *intuitionistic set* on $X$.

**Definition 2.2.** [2] Let $A$ and $B$ be two intuitionistic sets on some universe $X$. Then we define their *union* as $A \cup B = (A \cup B, A \cap B)$ and their *intersection* as $A \cap B = (A \cap B, A \cup B)$. We define *complement* of $A$ as $A^c = (A_F, A_T)$, *empty intuitionistic set* as $\emptyset = (\emptyset, X)$ and *universal* intuitionistic set as $X = (X, \emptyset)$.

**Definition 2.3.** [6] Assume that $X$ is a non-empty universe, while $A_1$ and $A_2$ are its subsets such that $A_1$ is contained in $A_2$. Then we say that an ordered pair $A = (A_1, A_2)$ is a *double* (or *flou*) set on $X$.

**Definition 2.4.** [6] Let $A$ and $B$ be two double sets on some universe $X$. Then we define their *union* as $A \cup B = (A_1 \cup A_2, B_1 \cup B_2)$ and their *intersection* as $A \cap B = (A_1 \cap A_2, B_1 \cap B_2)$. We define *complement* of $A$ as $A^c = (A_2^c, A_1^c)$, *empty* double set as $\emptyset = (\emptyset, \emptyset)$ and *universal* double set as $X = (X, X)$.

The definitions above are classical and of course they are taken from the literature. The next definition will be new. It will be presented in the next section.

## 3. Triple sets

### 3.1. Triple sets with quasi-complement

The notion presented below has some features of double sets and some aspects of intuitionistic sets. Let us formalize this intuition.

**Definition 3.1.** Assume that $X$ is a non-empty universe, while $A_1$, $A_2$ and $A_3$ are its subsets such that $A_1$ is contained in $A_2$ and $A_3$ has empty intersection with $A_2$. Then we say that an ordered triple $A = (A_1, A_2, A_3)$ is a triple set on $X$.

The most natural interpretation (already suggested in Introduction) says that $A_1$ is a collection of necessary elements, $A_2$ gathers those elements that are not necessary but still possible or conditionally accepted, while $A_3$ is a collection of those objects that are explicitly rejected or forbidden. Thus, $X - (A_2 \cup A_3)$ gathers neutral elements that are not evaluated.

**Definition 3.2.** Let $A$ and $B$ be two triple sets on some universe $X$. Then we define their *union* as $A \cup B = (A_1 \cup B_1, A_2 \cup B_2, A_3 \cap B_3)$ and their *intersection* as $A \cap B = (A_1 \cap B_1, A_2 \cap B_2, A_3 \cup B_3)$. We define *quasi-complement* of $A$ as $A^c = (A_2^c, A_1^c, A_1)$, *empty* triple set as $\emptyset = (\emptyset, \emptyset, X)$ and *universal* triple set as $X = (X, X, \emptyset)$. We use the symbol $P_T(X)$ to denote the set of all triple sets on $X$.

One can easily check that both union and intersection are defined in a proper manner: they produce new triple sets. Note that it is possible to define ”strong intersection” as $A \wedge B = (A_1 \cap B_1, A_2 \cap B_2, A_3 \cap B_3)$. We leave the study of algebraic properties of this operation for future research.

**Remark 3.1.** *Note that quasi-complement was defined in a specific way. We see that this unary operation returns new triple set but it does not depend on* $A_3$. *For example, let* $X = \{a, b, c, d, e, f, g\}$ *and* $A = (\{a, b\}, \{a, b, c\}, \{f, g\})$, $B = (\{a, b\}, \{a, b, c\}, \{e\})$. *Clearly, these triple sets are different. However, in the light of our definition, they have identical quasi-complements. We see that* $A^c = B^c = (\{d, e, f, g\}, \{c, d, e, f, g\}, \{a, b\})$. *Hence, the well-known tautology* $(A^c)^c = A$ *does not hold. In this particular case we have* $(A^c)^c = (B^c)^c = (\{a, b\}, \{a, b, c\}, \{d, e, f, g\})$. *This set is different than* $A$ *and different than* $B$.

*Note, however, that* $((A^c)^c)^c = ((B^c)^c)^c = (\{d, e, f, g\}, \{c, d, e, f, g\}, \{a, b\}) = A^c$. *This will be generalized later.*

We think that the feature of quasi-complement that was mentioned in the last remark will be interesting in the context of hypothetical triple topological spaces (and their generalization, e.g. triple minimal structures, triple weak structures etc.). In classical topological spaces each closed set has its unique open counterpart because closed sets are defined as complements of open sets. This relationship is one-to-one. As we can see, in our case this property is broken. We hope that this will lead us (in our further investigations) to some interesting reflections on the concept of closure, interior or exterior (as for the last one, it was recently studied e.g. in [4] but in neutrosophic biminimal context).

As for the interpretation, quasi-complement is based on the idea that now the whole set of neutral and forbidden elements becomes necessary. Those elements that were possible but not necessary are still treated as possible. Former necessary elements are now rejected.

It is possible to define complement in our triple setting in a more typical way and this will be done later. Now let us think about inclusion.

**Definition 3.3.** Assume that $A$ and $B$ are two triple sets on a universe $X$. We say that $A \subseteq B$ (that is, $A$ is contained in $B$) if and only if $A_1 \subseteq B_1$, $A_2 \subseteq B_2$ and $B_3 \subseteq A_3$.

For example, if $X = \{a, b, c, d, e, f, g\}$, $A = (\{a\}, \{a, b\}, \{e, f\})$ and $B = (\{a, b\}, \{a, b, c\}, \{f\})$ then $A \subseteq B$.

The definition above is modelled after the one that is typical for intuitionistic sets (see [2]) and double sets. We see that transitivity of this operation holds: if $A \subseteq B$ and $B \subseteq C$, then one can easily prove that $A \subseteq C$. Moreover, one can easily check that any triple set is contained in universal triple set and empty triple set is contained in any triple set. Finally, if $A \subseteq B$ then $B^c \subseteq A^c$ because: if $A_1 \subseteq B_1$, then $B_1^c \subseteq A_1^c$, if $A_2 \subseteq B_2$, then $B_2^c \subseteq A_2^c$. As for the third components of quasi-complements, we have $A_1 \subseteq B_1$ just by assumption.

Besides, we do not use any special symbols for triple universal and empty set. Analogously, we do not use any special symbols of triple intersection and union. The context will always allow us to distinguish between e.g. classical and triple universe or classical and triple operations.

It is important to check which algebraic properties are satisfied in this environment and which are not.

**Theorem 3.1.** *Assume that* $A$, $B$ *and* $C$ *are three triple sets on some non-empty universe* $X$. *Then the following identities and relationships hold:*

1. $((A^c)^c)^c = A^c$.
2. $A \cup A = A$, $A \cap A = A$ *(idempotence).*

3. $A \cup B = B \cup A$, $A \cap B = B \cap A$ *(commutativity).*

4. $A \cup (A \cap B) = A$, $A \cap (A \cup B) = A$ *(absorption law).*

5. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$, $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ *(distributivity).*

6. $A \cup (B \cup C) = (A \cup B) \cup C$, $A \cap (B \cap C) = (A \cap B) \cap C$ *(associativity).*

*Proof.*

We know that $A^c = (A_2^c, A_1^c, A_1)$. Now $(A^c)^c = ((A_1^c)^c, (A_2^c)^c, A_2^c) = (A_1, A_2, A_2^c)$. Then $((A^c)^c)^c = (A_2^c, A_1^{;}A_1) = A^c$.

Obvious.

Obvious (because of commutativity of classical operations).

$A \cup (A \cap B) = A \cup (A_1 \cap B_1, A_2 \cap B_2, A_3 \cup B_3) = (A_1 \cup (A_1 \cap B_1), A_2 \cup (A_2 \cap B_2), A_3 \cap (A_3 \cup B_3)) = (A_1, A_2, A_3)$. But the last set is just our original $A$. We may prove the second identity in a similar manner. Clearly, we used absorption of classical operations.

$A \cup (B \cap C) = A \cup (B_1 \cap C_1, B_2 \cap C_2, B_3 \cup C_3) = (A_1 \cup (B_1 \cap C_1), A_2 \cup (B_2 \cap C_2), A_3 \cap (B_3 \cup C_3))$. We see that now we can use distributivity of classical union and intersection to obtain our expected result. The second part of the proof is similar.

$A \cup (B \cup C) = A \cup (B_1 \cup C_1, B_2 \cup C_2, B_3 \cap C_3) = (A_1 \cup (B_1 \cup C_1), A_2 \cup (B_2 \cup C_2), A_3 \cap (B_3 \cap C_3))$. But here we use classical associativity to transform the last expression into $((A_1 \cup B_1) \cup C_1, (A_2 \cup B_2) \cup C_2), (A_3 \cup B_3) \cup C_3) = (A_1 \cup B_1, A_2 \cup B_2, A_3 \cup B_3) \cup (C_1, C_2, C_3) = (A \cup B) \cup C$. The second part can be solved in a similar manner. □

We see that $P_T(X)$ together with the operations of (triple) union and intersection satisfies all the basic properties of distributive lattice.

**Remark 3.2.** *Consider the first property from Theorem 3.1 together with earlier Remark 3.1. As we already know, we cannot say that $(A^c)^c = A$ (in general). This fact is typical for intuitionistic logic. Moreover, the fact that triple negation reduces to single negation is also intuitionistic. Just like in intuitionism, we see that $A \subseteq (A^c)^c$. This is because $A_1 \subseteq A_1$, $A_2 \subseteq A_2$ (trivially) and $A_3 \subseteq A_2^c$. Clearly, the converse inclusion (of triple sets) may not be true, as we have already shown.*

What about the law of excluded middle and the law of contradiction? They do not necessarily hold.

**Remark 3.3.** *Let $X = \{a, b, c, d, e, f, g\}$. Take $A = (\{a, b\}, \{a, b, c\}, \{f, g\})$ and then we have $A^c = (\{d, e, f, g\}, \{c, d, e, f, g\}, \{a, b\})$. Now:*

*$A \cup A^c = (\{a, b, d, e, f, g\}, \{a, b, c, d, e, f, g\}, \emptyset) = (\{a, b, d, e, f, g\}, X, \emptyset)$. But this set is different than triple universal set.*

*$A \cap A^c = (\emptyset, \{c\}, \{a, b, f, g\})$. But this set is different than triple empty set.*

In fact, we can prove the following general lemma.

**Lemma 3.1.** *Let $X$ be a non-empty universe and assume that $A$ is a triple set on $X$. Then the following results hold:*

1. $A \cup A^c = (A_1 \cup A_2^c, X, \emptyset) = ((A_2 - A_1)^c, X, \emptyset)$.
2. $A \cap A^c = (\emptyset, A_2 \cap A_1^c, A_3 \cup A_1) = (\emptyset, A_2 - A_1, A_3 \cup A_1)$.

*Proof.* The proof can be considered as a simple generalization of Remark 3.3. □

What about the properties of empty triple and universal triple set?

**Lemma 3.2.** *Let $X$ be a non-empty universe and assume that $A$ is a triple set on $X$. Then the following results hold:*

1. $A \cup X = X$.
2. $A \cap \emptyset = \emptyset$.
3. $A \cup \emptyset = A$.
4. $A \cap X = A$.
5. $\emptyset^c = X$ *and* $X^c = \emptyset$.

*Proof.* We have:

1. $A \cup X = (A_1 \cup X, A_2 \cup X, A_3 \cap \emptyset) = (X, X, \emptyset) = X$.
2. $A \cap \emptyset = (A_1 \cap \emptyset, A_2 \cap \emptyset, A_3 \cup X) = (\emptyset, \emptyset, X) = \emptyset$.
3. $A \cup \emptyset = (A_1 \cup \emptyset, A_2 \cup \emptyset, A_3 \cap X) = (A_1, A_2, A_3) = A$.
4. $A \cap X = (A_1 \cap X, A_2 \cap X, A_3 \cup \emptyset) = (A_1, A_2, A_3) = A$.
5. $\emptyset^c = (\emptyset, \emptyset, X)^c = (\emptyset^c, \emptyset^c, \emptyset) = (X, X, \emptyset)$ and $X^c = (X, X, \emptyset)^c = (X^c, X^c, X) = (\emptyset, \emptyset, X)$.

□

Note that we can add the following definition to our framework.

**Definition 3.4.** Let $X$ be a non-empty universe. Then we say that $0 = (\emptyset, \emptyset, \emptyset)$ is a *total triple set*.

At first glance, 0 seems to be the best candidate for the role of triple empty set. Indeed, it is "absolutely empty". However, it does not satisfy all the expected properties of empty set. Nevertheless, the following simple observation is interesting.

**Lemma 3.3.** *Let $X$ be a non-empty universe. Then $0^c = X$.*

*Proof.* According to the definition of quasi-complement: $0^c = (\emptyset, \emptyset, \emptyset)^c = (\emptyset^c, \emptyset^c, \emptyset) = (X, X, \emptyset) = X$. □

On the other hand, we already know that $X^c = \emptyset = (\emptyset, \emptyset, X) \neq 0$. Moreover, we cannot say that 0 is contained in every triple set. In fact, it is contained only in triple sets of the form $(A_1, A_2, \emptyset)$. We cannot consider 0 as a bottom of our lattice $P_T(X)$.

What about the hypothetical "0- analogues" of b) and c) in Lemma 3.2? They are somewhat weaker than in case of $\emptyset$.

**Lemma 3.4.** *Let $X$ be a non-empty universe and assume that $A$ is a triple set on $X$. Then the following identities are true:*

1. $A \cup 0 = (A_1, A_2, \emptyset)$.
2. $A \cap 0 = (\emptyset, \emptyset, A_3)$.

Note that one could think that the object of the form $(X, X, X)$ is a natural counterpart of $0$. However, this intuition is misleading: this object does not even belong to $P_T(X)$.

The next question is: what about de Morgan laws? As we know, quasi-complement is weaker than classical negation, hence at first glance it may not be clear if these laws hold. However, we have the following theorem.

**Theorem 3.2.** *Let $X$ be a non-empty universe and let $A$ and $B$ be two triple sets on $X$. Then the following identities are true:*

1. $(A \cup B)^c = A^c \cap B^c$.
2. $(A \cap B)^c = A^c \cup B^c$.

*Proof.*

$(A \cup B)^c = (A_1 \cup B_1, A_2 \cup B_2, A_3 \cap B_3)^c = ((A_2 \cup B_2)^c, (A_1 \cup B_1)^c, A_1 \cup B_1) = (A_2^c \cap B_2^c, A_1^c \cap B_1^c, A_1 \cup B_1)$. But this is just $(A_2^c, A_1^c, A_1) \cap (B_2^c, B_1^c, B_1) = A^c \cap B^c$.

$(A \cap B)^c = (A_1 \cap B_1, A_2 \cap B_2, A_3 \cup B_3)^c = ((A_2 \cap B_2)^c, (A_1 \cap B_1)^c, A_1 \cap B_1) = (A_2^c \cup B_2^c, A_1^c \cup B_1^c, A_1 \cap B_1)$. But this is just $(A_2^c, A_1^c, A_1) \cup (B_2^c, B_1^c, B_1) = A^c \cup B^c$. □

We see that both de Morgan laws hold. This situation is different than in intuitionistic logic, where the following inclusion (implication in logical sense) fails: that $(A \cap B)^c \subseteq A^c \cup B^c$.

### 3.2. Triple sets with standard complement

Let us define triple negation in a new way that is (at least in some sense) more classical.

**Definition 3.5.** Let $X$ be a non-empty universe and $A$ be a triple set on $X$. Then we define *standard triple complement* of $A$ as $-A = (A_3, A_2^c, A_1)$.

Now the interpretation is: former forbidden elements are now necessary, all those elements that were forbidden or just neutral are now possible, while former necessary elements are now rejected. The last feature is shared with quasi-complement. Clearly, the set that is produced is a triple set again: $A_3 \subseteq A_2^c$ and $A_3 \cap A_1 = \emptyset$. An advantage of this approach is that this complement engages all three components of triple set.

One can prove the following theorem about standard complement and its relationships with quasi-complement:

**Theorem 3.3.** *Let $X$ be a non-empty universe and let $A$ and $B$ be two triple sets on $X$. Then the following properties hold:*

1. $-(-A) = A$.
2. $A \cup -A = (A_1 \cup A_3, X, \emptyset)$.
3. $A \cap -A = (\emptyset, \emptyset, A_3 \cup A_1)$.

4. *$-\emptyset = X$ and $-X = \emptyset$.*

5. *$-\emptyset = X$.*

6. *$-A \subseteq A^c$.*

7. *$-(A \cup B) = -A \cap -B$, $-(A \cap B) = -A \cup -B$.*

*Proof.* We have:

1. $-(-A) = -(A_3, A_2^c, A_1) = (A_1, (A_2^c)^c, A_3) = (A_1, A_2, A_3)$.

2. $A \cup -A = (A_1, A_2, A_3) \cup (A_3, A_2^c, A_1) = (A_1 \cup a_3, A_2 \cup A_2^c, A_3 \cap A_1) = (A_1 \cup A_3, X, \emptyset)$.

3. $A \cap -A = (A_1, A_2, A_3) \cap (A_3, A_2^c, A_1) = (A_1 \cap A_3, A_2 \cap A_2^c, A_3 \cup A_1) = (\emptyset, \emptyset, A_3 \cup A_1)$.

4. We see that $-\emptyset = -(\emptyset, \emptyset, X) = (X, \emptyset^c, \emptyset) = (X, X, \emptyset) = X$. Then $-X = -(X, X, \emptyset) = (\emptyset, X^c, X) = (\emptyset, \emptyset, X) = \emptyset$.

5. $-0 = -(\emptyset, \emptyset, \emptyset) = (\emptyset, X, \emptyset)$.

6. Clearly, $-A = (A_3, A_2^c, A_1)$ and $A^c = (A_2^c, A_1^c, A_1)$. We see that $A_3 \subseteq A_2^c$, $A_2^c \subseteq A_1^c$ and $A_1 \subseteq A_1$ (trivially). Thus, $-A \subseteq A^c$.

7. $-(A \cup B) = -(A_1 \cup B_1, A_2 \cup B_2, A_3 \cap B_3) = (A_3 \cap B_3, (A_2 \cup B_2)^c, A_1 \cup B_1) = (A_3 \cap B_3, A_2^c \cap B_2^c, A_1 \cup B_1)$. Now the last set is $(A_3, A_2^c, A_1) \cap (B_3, B_2^c, B_1) = -A \cap -B$. The other identity can be proved in an analogous manner.

□

This theorem tells us that standard triple complement is (speaking informally) "slightly more" classical than quasi-complement. Double negation in standard sense collapses to the original set, as we can see in a). However, as for the law of excluded middle and the law of contradiction, they are still both false. Besides, we see that standard complement is always contained in quasi-complement.

## 4. Two additional operations

Union and intersection seem to be "natural" operations on triple sets. In many ways, they behave like analogues of corresponding operations on classical sets. However, we can discuss less typical operations in our framework. Basically, we would like to investigate them in our further research. Nevertheless, we define them here together with some basic operations. They are modelled after similar operations on double sets (they were investigated in [7]).

**Definition 4.1.** Let $X$ be a non-empty universe and assume that $A$ and $B$ are triple sets on $X$. Then we define:

1. $A \odot B = (A_1 \cap B_1, A_2 \cup B_2, A_3 \cap B_3)$ (*intunion*).

2. $A \oplus B = ((A_1 \cup B_1) \cap (A_2 \cap B_2), A_2 \cap B_2, A_3 \cup B_3)$ (*unionint*).

As for the first two components: the idea of the first operation is that two decision makers agree to connect their possibility sets and to limit their new necessity set only to those elements that are accepted for certain by both of them. As for the rejection sets, they intersect them to make sure that the new set has empty intersection with the new second component.

The second operation is different. Now these two decision makers intersect their possibility sets and they sum up their necessity sets but only in the limits of their new possibility set. As for the rejections sets, they connect them. We can easily check that there is no risk of non-empty intersection with the new second component. However, one can see that it would be reasonable to work with intersection of rejection sets, that is: to define "strong unionint" operation. This will be studied later.

One specific property of these two operations can be pointed out even on this stage. It refers to the question of absorption law.

**Lemma 4.1.** *Let $X$ be a non-empty universe and assume that $A$ and $B$ are two triple sets on $X$. Then the following absorptions laws* do not *hold:*

1. $A \oplus (A \odot B) = A$.
2. $A \odot (A \oplus B) = A$.

*Proof.* In both cases the easiest way is to give counterexamples. However, let us check general expression too.
$A \oplus (A \odot B) = A \oplus (A_1 \cap B_1, A_2 \cup B_2, A_3 \cap B_3) = ((A_1 \cup (A_1 \cap B_1)) \cap (A_2 \cap (A_2 \cup B_2)), A_2 \cap (A_2 \cup B_2), A_3 \cap A_3 \cap B_3)$.
The last set is (we can use classical absorption law to prove this) just $(A_1 \cap A_2, A_2, A_3 \cap B_3)$. But this is $(A_1, A_2, A_3 \cap B_3)$. For example, assume that $X = \{a, b, c, d, e, f, g\}$, $A = (\{a\}, \{a, b\}, \{e, f\})$ and $B = (\{b, c\}, \{b, c, d\}, \{f, g\})$. According to our calculations, we have:
$A \oplus (A \odot B) = (A_1, A_2, A_3 \cap B_3) = (\{a\}, \{a, b\}, \{f\})$. Clearly, the last set is different than $A$.
Besides, we see that if we limit our attention only to the first two components, then this absorption law is true. However, the third component, the one of rejection, changes the whole thing.

2. Again, let us formulate general pattern:
$A \odot (A \oplus B) = A \odot ((A_1 \cup B_1) \cap (A_2 \cap B_2), A_2 \cap B_2, A_3 \cup B_3)$. This can be rewritten in the next step as:
$(A_1 \cap ((A_1 \cup B_1) \cap (A_2 \cap B_2)), A_2 \cup (A_2 \cap B_2), A_3 \cup (A_3 \cup B_3)) = (A_1 \cap A_2 \cap B_2, A_2, A_3 \cup B_3)$. This is $(A_1 \cap B_2, A_2, A_3 \cup B_3)$. Now take the same universe and sets that were used in part a). We obtain:
$A \oplus (A \odot B) = (\emptyset, \{a, b\}, \{e, f, g\})$. This is different than $A$.

□

We see that in this case even without third components we cannot obtain general absorption law. This is important. On the one hand, we see that new operations have some natural interpretation in terms of negotiations, necessity, possibility and rejection. On the other hand, the structure of equipped with these two operations is rather weak (if we do not have absorption laws which are considered as very elementary for algebras, e.g. for lattices).

## 5. Conclusion

In this paper we introduced triple sets. They are double sets with additional component that contains those objects of universe that are rejected or forbidden by a decision maker whose point of view is described by a given triple set. We defined basic algebraic operations and we compared two version of complement operation. The next steps that we would like to take are: to define and study triple topological (and weaker: e.g. minimal, weak) structures; to fuzzify the whole concept; to study additional algebraic operations presented in this paper; to reintroduce triple sets in soft setting. We can also define and analyze triple points (per analogiam with double and intuitionistic points).

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