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A STUDY ON T-FUZZYSOFT SUBHEMIRINGS OF A HEMIRING

Dr.N.Anitha[1] and M.Latha[2]

[1] Department of Mathematics, Periyar University , PG Extension Centre, Dharmapuri-05.

[2] Departments of Mathematics, Karpagam University, Coimbatore-21.

Abstract

In this paper, we made an attempt to study the algebraic nature of a T-fuzzy soft subhemirings of a hemiring and some of the its properities.

Keywords: Fuzzy soft set, T-fuzzy soft subhemiring, T-fuzzy soft normal subhemiring and pseudo Fuzzy soft coset.

AMS Subject classification: 03E72, 06D72, 08A72.

INTRODUCTION

There are many concepts of universal algebras generalizing an associative ring (R; +; .). Some of them in particular, nearrings and several kinds of semirings have been proven very useful. Semirings (called also halfrings) are algebras (R; +; .) share the same properties as a ring except that (R ; +) is assumed to be a semigroup rather than a commutative group. Semirings appear in a natural manner in some applications to the theory of automata and formal languages. An algebra (R ; +, .) is said to be a semiring if (R ; +) and (R ; .) are semigroups satisfying a. (b + c) = a. b + a. c and (b + c) .a = b. a + c. a for all a, b and c in R. A semiring R is said to be additively commutative if a + b = b + a for all a, b and c in R. A semiring R may have an identity 1, defined by 1. a = a = a. 1 and a zero 0, defined by 0 + a = a = a + 0 and a. 0 = 0 = 0. a for all a in R. A semiring R is said to be a hemiring if it is an additively commutative with zero. After the introduction of fuzzy sets by L.A. Zadeh [17], several researchers explored on the generalization of the concept of fuzzy sets. M. Borah, T. J. Neog and D. K. Sut,[5] were developed some operations of fuzzy soft sets, On operations of soft sets was developed by A.Sezgin and A. O. Atagun,[13] and Kumud Borgohain and Chittaranjan Gohain,[7] was developed some New operations on Fuzzy Soft Sets, In this paper, we introduce some Theorems in T-Fuzzy soft subhemiring of a hemiring.

1. PRELIMINARIES

Definition1.1. A pair (F,E) is called a soft set (over U) if and only if F is a mapping of E into the set of all subsets of the set U.

* Corresponding author: lathasenthil18@gmail.com
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In other words, the soft set is a parameterized family of subsets of the set U. Every set F(ε)($\varepsilon \epsilon E$) from this family may be considered as the set of ε -elements of the soft sets (F, E) or as the set of ε - approximate elements of the soft set.

Definition 1.2. Let (U,E) be a soft universe and A ⊆ E. Let $\mathcal{F}$(U) be the set of all fuzzy subsets in U. A pair ($\widetilde{F}$,A) is called a fuzzy soft set over U, where $\widetilde{F}$, is a mapping given by $\tilde{F}$: A → $\mathcal{F}$(U).

Definition 1.3: A T-norm is a binary operation T: $[0,1] \mathrm{X}\, [0,1] \rightarrow [0,1]$ satisfying the following requirements:

(i) 1 T x = x (boundary conditions)

(ii) x T y = y Tx (commutativity)

(iii) x T (y T z)= (x T y) T z (associativity)

(iv) If x £ y and w £ z, then x T w £ y T z (monotonicity).

Definition 1.4 Let (R, +,.) be a hemiring. A fuzzy subset of R is said to be an T-fuzzy subhemiring(fuzzy subhemiring with respect to T-norm) of R if it satisfies the following conditions:

(i) $\mu_{(\mathrm{F,A})}(\mathrm{x}+\mathrm{y}) \leq T\left((\mu_{(\mathrm{F,A})}(\mathrm{x}), \mu_{(\mathrm{F,A})}(\mathrm{y})\right)$

(ii) $\mu_{(\mathrm{F,A})}(\mathrm{xy}) \leq\ T\left((\mu_{(\mathrm{F,A})}(\mathrm{x}), \mu_{(\mathrm{F,A})}(\mathrm{y})\right)$, for all x and y in R.

Definition 1.5 Let (R, +,.) be a hemiring. A T- Fuzzysoft subhemiring (F,A) of R is said to be an Fuzzy soft normal subhemiring (TFSNSHR) of R if it satisfies the following conditions:
(i) $\mu_{(\mathrm{F,A})}$ (xy) = $\mu_{(\mathrm{F,A})}$ (yx), for all x and y in R.

Definition 1.6 If (R, +, **.**) and (R^{I}, +, **.**) are any two hemirings, then the function f: R → R^{I} is called a **homomorphism** if f(x+y) = f(x)+f(y) and f(xy)=f(x)f(y), for all x and y in R.

Definition 1.7 If (R, +, **.**) and (R^{I}, +, **.**) are any two hemirings, then the function f: R → R^{I} is called an **anti-homomorphism** if f(x+y) = f(y)+f(x) and f(xy)=f(y)f(x), for all x and y in R.

Definition 1.8 Let (R, +,.) and (R^{I}, +, .) be any two hemirings. Then the function f: R → R^{I} be a hemiring homomorphism. If f is one-to-one and onto, then f is called a **hemiring isomorphism**.

Definition 1.9 Let (R, +,.) and (R^{I}, +, .) be any two hemirings. Then the function f: R → R^{I} be a hemiring anti-homomorphism. If f is one-to-one and onto, then f is called a **hemiring anti-isomorphism**.

Definition 1.10 Let R and R^{l} be any two hemirings. Let f: R → R^{l} be any function and let A be a T- Fuzzy soft subhemiring in R, V be an T-Fuzzy soft subhemiring in f(R) = R^{l}, defined by $\mu_{V(y)} = \sup_{x \in f^{-1}(y)} (\mu_{(F,A)}(x)$ for all x in R and y in R^{l}. Then A is called a preimage of V under f and is denoted by $f^{-1}(V)$.

Definition 1.11 Let (F,A) be an T-Fuzzy soft subhemiring of a hemiring (R, +, ·) and a in R. Then the pseudo T-Fuzzy soft coset $(a\,(F,A))^{p}$ is defined by $((a\mu_{(F,A)})^{p})(x) = p(a)\ \mu_{(F,A)}(x)$, for every x in R and for some p in P.

2. T-FUZZY SOFT SUBHEMIRINGS OF A HEMIRING

Theorem: 2.1 If (F, A) is an T-Fuzzy soft subhemiring of a hemiring (R, +,.), then (F, □A) is an T-Fuzzy soft subhemiring of R.

Proof: Let (F, A) be an T-fuzzy soft subhemiring of a hemiring R.Consider (F,A) = {⟨ x, $\mu_{(F,A)}$ (x),⟩}, for all x in R, we take (F, A) = (F,B) ={⟨ x, $\mu_{(F,B)}$ (x), ⟩}, where$\mu_{(F,B)}$ (x) = $\mu_{(F,A)}$ (x),. Clearly, $\mu_{(F,B)}(x+y) \leq T\{\mu_{(F,B)}(x), \mu_{(F,B)}(y)\}$, for all x and y in R and $\mu_{(F,B)}(xy) \leq T\{\mu_{(F,B)}(x), \mu_{(F,B)}(y)\}$, for all x and y in R. Since A is an T-fuzzy soft subhemiring of R, we have $\mu_{(F,A)}(x+y) \leq T\{\mu_{(F,A)}(x), \mu_{(F,A)}(y)\}$, for all x and y in R, And $\mu_{(F,A)}(xy) \leq$
$T\{\mu_{(F,A)}(x), \mu_{(F,A)}(y)\}$, for all x and y in R. Hence (F,B) = (F, □A) is an T-fuzzy soft subhemiring of a hemiring R.

Theorem: 2.2 If (F,A) is an T-fuzzy soft subhemiring of a hemiring (R, +, .), then (F, ◊A) is an T-fuzzy soft subhemiring of R.

Proof: Let (F,A) be an T-fuzzy soft subhemiring of a hemiring R. That is
(F,A) = {⟨ x, $\mu_{(F,A)}$ (x) ⟩}, for all x in R. Let (F,◊A) = (F,B)= ⟨x, $\mu_{(F,B)}$ (x)⟩}, for all x and y in R. Since (F,A) is an T-fuzzy soft subhemiring of R, which implies that $1-\mu_{(F,B)}(xy) \geq T\{(1-\mu_{(F,B)}(x)), (1-\mu_{(F,B)}(y))\}$,which implies that $\mu_{(F,B)}(xy) \leq 1- T\{(1-\mu_{(F,B)}(x)), (1-\mu_{(F,B)}(y))\}= T\{\mu_{(F,B)}(x), \mu_{(F,B)}(y)\}$. Therefore, $\mu_{(F,B)}(xy) \leq T\{\mu_{(F,B)}(x), \mu_{(F,B)}(y)\}$, for all x and y in R. Hence (F,B) = (F,◊A) is an T-fuzzy soft subhemiring of a hemiring R.

Theorem: **2.3** Let (R, +, .) and (R′, +, .) be any two hemi rings. The homomorphic image of an T-fuzzy soft subhemiring of R is an T-fuzzy soft subhemiring of R′.
Proof: Let f : R → R' be a homomorphism. Then, $f(x+y) = f(x) + f(y)$ and $f(xy) = f(x)f(y)$, for all x and y in R.Let $(G, V) = f((F, A)$, where $(F, A)$is an $T -$ fuzzy soft subhemiring of R. Now, for $f(x), f(y)$in R',
$\mu_{(G,V)}\big(f(x)\big) + \big(f(y)\big) \leq \mu_{(F,A)}(x+y) \leq T(\mu_{(F,A)}(x), \mu_{(F,A)}(y)$, which implies that
$\mu_{(G,V)}(\big(f(x)\big) + \big(f(y)\big)) \leq T(\mu_{(G,V)}\Big(\big(f(x)\big), \mu_{(G,V)}\big(f(y)\big)\Big)$.Again,

$\mu_{(G,V)}((f(x))(f(y))) \leq \mu_{(F,A)}(xy) \leq T(\mu_{(F,A)}(x), \mu_{(F,A)}(y)$, which implies that $\mu_{(G,V)}((f(x))(f(y))) \leq T(\mu_{(F,A)}(x), \mu_{(F,A)}(y)$.Hence (G, V) is an T-fuzzy soft subhemiring of hemiring R'.

Theorem: **2.4** Let (R, +, .) and (R', +, .) be any two hemirings. The homomorphic preimage of a T-fuzzy soft subhemiring of R' is a T-fuzzy soft subhemiring of R.
Proof: Let f : R→ R' be a homomorphism. Then,$f(x+y) = f(x) + f(y)$ and $f(xy) = f(x)f(y)$, for all x and y in R. let $(G, V) = f(F, A)$, where $(G, V)$is a $T -$ fuzzy soft subhemiring of R'. Now, for x, y in R, $\mu_{(F,A)}((x+y)) = \mu_{(G,V)}((f(x)) + f(y) \leq T(\mu_{(G,V)}(f(x)), \mu_{(G,V)}(f(y)) = T(\mu_{(F,A)}(x), \mu_{(F,A)}(y))$ which implies that$\mu_{(F,A)}((x) + (y)) \leq T(\mu_{(F,A)}(x), \mu_{(F,A)}(y)$. Again,$\mu_{(F,A)}((xy)) = \mu_{(G,V)}((f(x))(f(y)) \leq T(\mu_{(G,V)}(f(x)), \mu_{(G,V)}(f(y)) = T(\mu_{(F,A)}(x), \mu_{(F,A)}(y)$,which implies that $\mu_{(F,A)}((xy)) \leq T(\mu_{(F,A)}(x), \mu_{(F,A)}(y)$. Hence (F,A) is a T-fuzzy soft subhemiring of hemiring R.

Theorem: **2.5** Let (R, +, .) and ($\mathbf{R}'$, +, .) be any two hemirings. The anti-homomorphic image of a T-fuzzy soft subhemiring of R is a T-fuzzy soft subhemiring of $\mathbf{R}'$.
Proof: Let f : R → R' be a homomorphism. Then, $f(x + y) = (x) + f(y)$ and $f(xy) = f(x)f(y)$, for all x and y in R.Let $(G, V) = f((F, A))$, where $(F, A)$is a $T -$ fuzzy soft subhemiring of R. $\mu_{(G,V)}\left((f(x)) + (f(y))\right) \leq (G, V)(y + x) \leq T(\mu_{(F,A)}(y), \mu_{(F,A)}(x, q) = T(\mu_{(F,A)}(x), \mu_{(F,A)}(y)$, which implies that $\mu_{(G,V)}((f(x)) + (f(y))) \leq T(\mu_{(G,V)}\left((f(x)), \mu_{(G,V)}(f(y))\right)$. Again, $\mu_{(G,V)}((f(x))(f(y))) \leq \mu_{(F,A)}(yx) \leq T(\mu_{(F,A)}(y), \mu_{(F,A)}(x) = T(\mu_{(F,A)}(x), \mu_{(F,A)}(y)$ which implies that $\mu_{(G,V)}((f(x))(f(y))) \leq T(\mu_{(G,V)}\left((f(x)), \mu_{(G,V)}(f(y))\right)$. Hence (G,V) is a T-fuzzy soft subhemiring of hemiring $\mathbf{R}'$.

Theorem: **2.6** Let (R, +, .) and ($\mathbf{R}'$, +, .) be any two hemirings. The anti-homomorphic preimage of an T-fuzzy soft subhemiring of $\mathbf{R}'$ is an T-fuzzy soft subhemiring of R.
Proof: Let $(G, V) = f((F, A))$ where $(G, V)$is a $T -$ fuzzy soft subhemiring of R'. Let x and y in R. Then $\mu_{(F,A)}((x+y)) = \mu_{(G,V)}((f(x)) + (f(y)) \leq T(\mu_{(G,V)}(f(y)), \mu_{(G,V)}(f(x)) = T(\mu_{(F,A)}(x), \mu_{(F,A)}(y)$,which implies that $\mu_{(F,A)}((x+y)) \leq T(\mu_{(F,A)}(x), \mu_{(F,A)}(y)$. Again$\mu_{(F,A)}((xy)) = \mu_{(G,V)}((f(x))(f(y)) \leq T(\mu_{(G,V)}(f(y)), \mu_{(G,V)}(f(x)) = T(\mu_{(F,A)}(x), \mu_{(F,A)}(y)$,which implies that $\mu_{(F,A)}((xy)) \leq T(\mu_{(F,A)}(x), \mu_{(F,A)}(y)$. Hence (F,A) is a T-fuzzy soft subhemiring of hemiring R.

In the following Theorem ◦ is the composition operation of functions

Theorem: 2.7 Let (F,A) be an T- fuzzy soft subhemiring of a hemiring H and f is an isomorphism from a hemiring R onto H. Then (F,A)◦f is a T- fuzzy soft subhemiring of R.

Proof: Let x and y in R and (F,A) be an T-fuzzy soft subhemiring of a hemiring H. Then we have, $(\mu_{(F,A)} \circ f)(x+y) = \mu_{(F,A)}(f(x+y)) = \mu_{(F,A)}(f(x)+f(y))$, as f is an isomorphism $\leq T\{\mu_{(F,A)}(f(x)), \mu_{(F,A)}(f(y))\} = T\{(\mu_{(F,A)} \circ f)(x), (\mu_{(F,A)} \circ f)(y)\}$, which implies that $(\mu_{(F,A)} \circ f)(x+y) \leq T\{(\mu_{(F,A)} \circ f)(x), (\mu_{(F,A)} \circ f)(y)\}$. And, $(\mu_{(F,A)} \circ f)(xy) = \mu_{(F,A)}(f(xy)) = \mu_{(F,A)}(f(x)f(y))$, as f is an isomorphism $\leq T\{\mu_{(F,A)}(f(x)), \mu_{(F,A)}(f(y))\} = T\{(\mu_{(F,A)} \circ f)(x), (\mu_{(F,A)} \circ f)(y)\}$, which implies that $(\mu_{(F,A)} \circ f)(xy) \leq T\{(\mu_{(F,A)} \circ f)(x), (\mu_{(F,A)} \circ f)(y)\}$. Therefore (F,A)∘f is a T- Fuzzy soft subhemiring of a hemiring R.

Theorem: 2.8 Let (F,A) be an T-fuzzy soft subhemiring of a hemiring h and f is an anti-isomorphism from a hemiring r onto h. then (F,A)∘f is a T-fuzzy soft subhemiring of R.
Proof: Let x and y in R and (F,A) be an T- fuzzy soft subhemiring of a hemiring H. Then we have, $(\mu_{(F,A)} \circ f)(x+y) = \mu_{(F,A)}(f(x+y)) = \mu_{(F,A)}(f(y)+f(x))$, as f is an anti-isomorphism $\leq T\{\mu_{(F,A)}(f(x)), \mu_{(F,A)}(f(y))\} = T\{(\mu_{(F,A)} \circ f)(x), (\mu_{(F,A)} \circ f)(y)\}$, which implies that $(\mu_{(F,A)} \circ f)(x+y) \leq T\{(\mu_{(F,A)} \circ f)(x), (\mu_{(F,A)} \circ f)(y)\}$. And, $(\mu_{(F,A)} \circ f)(xy) = \mu_{(F,A)}(f(xy)) = \mu_{(F,A)}(f(y)f(x))$, as f is an anti-isomorphism $\leq T\{\mu_{(F,A)}(f(x)), \mu_{(F,A)}(f(y))\} = T\{(\mu_{(F,A)} \circ f)(x), (\mu_{(F,A)} \circ f)(y)\}$, which implies that $(\mu_{(F,A)} \circ f)(xy) \leq T\{(\mu_{(F,A)} \circ f)(x), (\mu_{(F,A)} \circ f)(y)\}$. Therefore (F,A)∘f is an T-fuzzy soft subhemiring of the hemiring R.

Theorem: 2.9 Let (F,A) be an T-fuzzy soft subhemiring of a hemiring (R, +, .), then the pseudo fuzzy soft coset $(a(F,A))^p$ is an T- fuzzy soft subhemiring of a hemiring R, for every a in R.
Proof: Let (F,A) be an T-fuzzy soft subhemiring of a hemiring R. For every x and y in R, we have, $((a\mu_{(F,A)})^p)(x + y) = p(a)\,\mu_{(F,A)}(x+y) \leq p(a)\,T\{\mu_{(F,A)}(x), \mu_{(F,A)}(y)\} = T\{p(a)\,\mu_{(F,A)}(x), p(a)\,\mu_{(F,A)}(y)\} = T\{((a\mu_{(F,A)})^p)(x), ((a\mu_{(F,A)})^p)(y)\}$. Therefore, $((a\mu_{(F,A)})^p)(x + y) \leq T\{((a\mu_{(F,A)})^p)(x), ((a\mu_{(F,A)})^p)(y)\}$. Now, $((a\mu_{(F,A)})^p)(xy) = p(a)\,\mu_{(F,A)}(xy) \leq p(a)T\{\mu_{(F,A)}(x), \mu_{(F,A)}(y)\} = T\{p(a)\,\mu_{(F,A)}(x), p(a)\,\mu_{(F,A)}(y)\} = T\{((a\mu_{(F,A)})^p)(x), ((a\mu_{(F,A)})^p)(y)\}$. Therefore, $((a\mu_{(F,A)})^p)(xy) \leq T\{((a\mu_{(F,A)})^p)(x), ((a\mu_{(F,A)})^p)(y)\}$. Hence $(a(F,A))^p$ is an T-fuzzy soft subhemiring of a hemiring R.

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