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# Designing of Skip lot sampling plan (SkSP – 3) with Single Sampling Plan as reference plan under the conditions of Intervened Poisson distribution

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**Abstract**

In manufacturing process the lots are formed by the finished produces and a series of lots are submitted for inspection, in this situation the Skip lot sampling plans are very much useful to inspect the bulk of lots. In the production process there is a possibility of making some interventions in order to meet the demands of consumers without the knowledge of quality engineers. In order to inspect the lots of finished products which are formed from such production lines, acceptance sampling plans are designed under the conditions of intervened Poisson distribution. The primary objective of this article is to design Skip lot sampling plan of kind SkSP-3 with single sampling plan as reference plan under the conditions of intervened Poisson distribution. A methodology for obtaining the plan parameters which satisfies both the consumer and the producer is discussed with illustrations.

**Keywords:** Acceptance sampling plan, Skip lot sampling plan (SkSP-3), Single sampling plan, Intervention, Intervened Poisson distribution.

**Subject Classification:** Primary 62-XX; Secondary 62P30.

## Introduction

Acceptance sampling (AS) plans, one of the major tools in statistical quality control theory, is a planned and systematic procedure to sentence the quality of lot based on the sample information so that the quality requirement for a product will be fulfilled. In general, AS is a methodology to select a sample size and acceptance number for a submitted lot in such a way that it satisfies both the manufacturer and the purchaser by fulfilling their quality requirements and also to dilute the overall inspection cost. On the basis of how the sampling plans are designed AS is classified into

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two major categories. If the sampling plan design is based on number of defectives or defects then it is called attribute sampling plans and if the design is based on the sample average and standard deviation then the sampling design is called as variable sampling plans.

In lot-by-lot sampling plans many AS plans are developed such as single sampling plan, double sampling plan, multiple sampling plan, etc., All these sampling plans are applied on individual lots and thus the acceptance criteria applied for individual lots may not be affected by the results of preceding lots. Dodge (1955 a, b) introduced the cumulative results plans which incorporate the results of prior lots as a part of decision criteria. However, cumulative results plans uses lot-by-lot inspection plans for the stream of products or lots. In order to inspect the quality of stream of lots Dodge (1955 b) introduced Skip lot sampling plan by applying the principles of Continuous sampling plan (CSP-1) and designated as Skip lot sampling plan (SkSP-1). Further, Dodge and Perry (1971) developed a Skip lot sampling plan by applying reference plans to the series of lots and the plan was designated as SkSP-2. Vijayaraghavan and Soundarajan (1998) extended the work and designed SkSP-2 with double sampling plan as reference plan. Further, Soundarajan and Vijayaraghavan (1989) has developed a Skip lot sampling plan based on the principles of continuous sampling plan (CSP-2) of Dodge and Torrey (1951) with single sampling plan as reference plan and designated as SkSP-3.

Perry (1973) has proposed an idea that the lot-by-lot attributes plans such as single sampling plan, double sampling plan and multiple sampling plans can be used as reference plans. Among these reference plans the basic and most widely used lot-by-lot sampling plan is single sampling plan. The detailed works on determination of single sampling plan parameters are seen in Stephens (2001) and Schilling and Neubauer (2009). In single sampling plan when the inspection is based on counting the number of defects found in the sample then the number of defects can be modeled by Poisson distribution (Schilling and Neubauer (2009)). Suppose the production process has any intervention without the knowledge of quality engineers or production managers then the Poisson model cannot be a perfect model to study such interventions. In order to study such kind of interventions Shanmugam (1985) developed a model called intervened Poisson distribution (IPD) which is an extension of zero truncated Poisson distributions.

IPD has a wide range of applicability in the areas such as agriculture, epidemiology, public health, process control, medicine, manufacturing, queueing studies etc. More real time illustrations can be found in Shanmugam (1985, 2001) and Huang and Fung (1989). The characteristics of the IPD are also presented by Shanmugam (1985, 1992), Scollnik provided the Bayesian analysis for IPD in 1995 and introduced intervened generalized Poisson distribution in 2006 which is a generalized version of the IPD. Dhanavanthan (1998, 2000) introduced Compound Intervened Poisson distribution and studied its nature, characteristics and also estimated its parameters. Satheesh and Shibu (2011) introduced a modified version of IPD namely modified intervened Poisson distribution. Pradeepa Veerakumari and Azarudheen (2016) devised a single sampling plan for the above situations by imparting the conditions of IPD and proved that the consumer's risk and producer's risk reduces while comparing with the conventional single sampling plan proposed by Cameron (1952).

When the inspection lots have any intervention during the production process then to inspect such lots it is very much essential to design the acceptance sampling plans under the conditions of IPD. This paper focuses on designing of Skip lot sampling plan of type SkSP-3 using SSP as reference plan under the conditions of IPD also; the paper presents the unity value tables and methodology for obtaining the plan parameters of SkSP-3.

**Intervened Poisson Single sampling plan**

A SSP is a decision criterion to accept or reject a particular lot based on the results obtained from the random samples taken only once from the given lot. In general, the SSP is identified by its three parameters, namely lot size (N), sample size (n) and acceptance number (c). The procedure for performing SSP is as follows,

Step 1 Select n samples randomly from the submitted lot of size N.

Step 2 Inspect each selected samples and count the number of defectives d.

Step 3 If d does not exceed the specified acceptance number c (i.e. $d \leq c$) then accept the lot otherwise reject it.

The operating characteristic (OC) function for SSP is defined as,

$$P_a(p) = P[x \leq c] \tag{1}$$

where p is the lot proportion defectives. In practice, the values of $P_a(p)$ can be determined for different values of $p$ using different probability models. As quoted by Schilling and Neubauer (2009), when $n$ is large, $p$ is small and $n/N \leq 0.10$ such that $np<5$, the proportion defective $p$ is distributed according to the Poisson distribution with mean $np$.

When some intervention is applied to the production process with an intention to improve the quality, of the products during the observational period, then the mean of the rare event $p$ may get altered. In such situation the appropriate probability distribution for number of defectives in the sample is modeled by IPD.

The OC function for SSP under the conditions of IPD $(\theta,\rho)$ can be defined as,

$$P_a(p) = \sum_{x=1}^{c} P(X = x \mid \theta, \rho) = \sum_{x=1}^{c} \frac{[(1+\rho)^x - \rho^x]\theta^x}{e^{\rho\theta}(e^\theta - 1)x!} \tag{2}$$

where $\theta = np$ and $\rho$ is the intervention parameter which is measured in percentage. Hence, by obtaining an estimated value for $\rho$, the probability of acceptance can be calculated using equation (2) for given values of *n, c* and *p*.

**Construction of SkSP-3 using SSP as reference plan under the conditions of IPD**

According to Dodge (1955b), when a series of identical lots are submitted for inspection it is not necessary to inspect every lots instead the lots may skipped under a related skip lot plan. In SkSP-3 the lots are inspected based on lot by lot inspection plans together with the procedure for normal and fraction inspection. The operating procedure for SkSP-3 is as follows,

Step 1 Start the inspection normally by inspecting i consecutive lots using the reference plan.

Step 2 When i lots are accepted then inspect a fraction f of lots.

Step 3 If a lot is rejected in skipping inspection then inspect next consecutive k lots.

Step 4 If all the k lots are accepted continue skipping inspection i.e. inspect the fraction f lots.

Step 5 If a lot is rejected while inspecting k lots, resume normal inspection.

According to Soundarajan and Vijayaraghavan (1989) the operating characteristic function of SkSP-3 is defined as,

$$P_a(p) = \frac{fP + (1-f)P^i(2-P^i)}{f + (1-f)P^i(2-P^i)} \tag{3}$$

where p is the proportion defective and P is the probability of acceptance of the reference plan in this paper the P value is determined by the equation (2).

It is clear from the equation (3) that the plan SkSP-3 is portrayed by three parameters such as i, f and P. Any acceptance sampling plan should safeguard both the producer and the consumer against their risk levels. If $p_1$ and $p_2$ represent the quality levels of producer and consumer respectively and $\alpha$ and $\beta$ are the producer's and consumer's risk then the optimum sampling plan can be determined by the conditions $P_a(p_1) = 1-\alpha$ and $P_a(p_2) = \beta$. A plan obtained by these conditions will ensure the protection of both the producer and the consumer. Since the parameters are integers the above said conditions may not exactly satisfy. Thus, for the specified $p_1$, $p_2$, $\alpha$ and $\beta$ the optimum plan can be determined by following the conditions $P_a(p_1) \geq 1-\alpha$ and $P_a(p_2) \leq \beta$.

With the help of unity value approach, the unity values are computed and tabulated for different sets of ($\rho$, *c, f, i,* $P_a(p)$) in Table 1. The Table 1 also contains the operating ratios $R = np_2/np_1$ corresponding to $\alpha$=0.05 and $\beta$=0.10 which will help to discriminate the sampling plans.

For the suggested values of ρ and the strength ($p_1$, $\alpha$, $p_2$, $\beta$), the plan parameters can be determined by employing the following procedure:

i. Specify the intervention parameter ρ and the strength ($p_1$, $\alpha$, $p_2$, $\beta$).
ii. Compute operating ratio $R = p_2/p_1$.
iii. Determine the unity values $np_1$ and $np_2$ from the Table 1 for nearest value of R.
iv. Calculate the sample size $n = np_1/p_1$ or $np_2/p_2$ whichever is larger (Schilling and Neubauer (2009)). Also $i$ and $f$ are chosen corresponding to the operating ratio $R$ obtained in Step (ii).

**Illustration**

It is assumed that the lots are submitted in series and the intervention is predetermined as *5%*. If the strength of the required plan is given as $p_1 = 0.005$, $\alpha = 0.05$, $p_2 = 0.07$ and $\beta = 0.10$ then to obtain an optimal plan compute operating ratio $OR = p_2/p_1 = 0.07/0.005 = 14$. For the specified strength and the intervention parameter the OR nearer to the calculated OR which is chosen from Table 1 is *14.7* and corresponding parameters are $i = 10$ and $f = 1/4$ also $np_1$ and $np_2$ values are found to be 0.2303 and 3.3873 respectively therefore the sample size is determined as $np_1/p_1 = 46$ and $np_2/p_2 = 48$. Thus the optimum plan obtained for the specified strengths is $n = 48$, $c = 1$, $i = 10$ and $f = 1/4$.

**Conclusion**

The Skip lot sampling plan with a single sampling plan as reference plan is designed under the conditions of intervened Poisson distribution for the lots which are formed by a production process which has some intervention between the processes. Further, a procedure is suggested to determine the optimal plan parameters indexed by acceptable quality level and limiting quality level ensuring the protection of both the producer and the consumer. The table provide in this article is constructed for limited parametric values; it can be further extended to the requirement of the stack holders by employing the procedure presented in this paper.

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*Table 1:* **Unity values and Operating ratio for SkSP – 3 with SSP having c =1 as reference plan under the conditions of IPD for the given Producer's risk α = 5% and Consumer's risk β = 10%**

| *i* | *f* | $np_1$ | $np_2$ | *OR* | $np_1$ | $np_2$ | *OR* | $np_1$ | $np_2$ | *OR* |
|---|---|---|---|---|---|---|---|---|---|---|
| | | $\rho = 0.01$ | | | $\rho = 0.03$ | | | $\rho = 0.05$ | | |
| 4 | 1/5 | 0.1941 | 3.5667 | 18.3753 | 0.3754 | 3.4802 | 9.2713 | 0.3687 | 3.3924 | 9.2017 |
| 6 | | 0.1833 | 3.5644 | 19.4487 | 0.3265 | 3.4702 | 10.6277 | 0.3158 | 3.3824 | 10.7109 |
| 8 | | 0.1751 | 3.5645 | 20.3546 | 0.2921 | 3.4699 | 11.8811 | 0.2821 | 3.3820 | 11.9869 |
| 10 | | 0.1678 | 3.5645 | 21.2459 | 0.2651 | 3.4680 | 13.0820 | 0.2535 | 3.3796 | 13.3302 |
| 12 | | 0.1613 | 3.5635 | 22.0991 | 0.2441 | 3.4647 | 14.1951 | 0.2353 | 3.3767 | 14.3521 |
| 4 | 1/4 | 0.3356 | 3.5696 | 10.6364 | 0.3238 | 3.4773 | 10.7398 | 0.3127 | 3.3891 | 10.8378 |
| 6 | | 0.2985 | 3.5605 | 11.9296 | 0.2879 | 3.4686 | 12.0488 | 0.2807 | 3.3809 | 12.0424 |
| 8 | | 0.2716 | 3.5587 | 13.1010 | 0.2595 | 3.4660 | 13.3582 | 0.2484 | 3.3775 | 13.5972 |
| 10 | | 0.2480 | 3.5552 | 14.3332 | 0.2389 | 3.4629 | 14.4974 | 0.2303 | 3.3873 | 14.7088 |
| 12 | | 0.2299 | 3.5668 | 15.5128 | 0.2213 | 3.4748 | 15.6991 | 0.2137 | 3.3871 | 15.8499 |
| 4 | 2/3 | 0.1476 | 3.5591 | 24.1209 | 0.1421 | 3.4668 | 24.4052 | 0.1371 | 3.3788 | 24.6445 |
| 6 | | 0.1445 | 3.5631 | 24.6587 | 0.1371 | 3.4659 | 25.2802 | 0.1359 | 3.3792 | 24.8600 |
| 8 | | 0.1411 | 3.5585 | 25.2152 | 0.1371 | 3.4667 | 25.2858 | 0.1308 | 3.3782 | 25.8281 |
| 10 | | 0.1371 | 3.5574 | 25.9478 | 0.1324 | 3.4654 | 26.1825 | 0.1277 | 3.3773 | 26.4562 |
| 12 | | 0.1344 | 3.5566 | 26.4655 | 0.1293 | 3.4644 | 26.8010 | 0.1247 | 3.3764 | 27.0787 |
| | | $\rho = 0.07$ | | | $\rho = 0.10$ | | | $\rho = 0.20$ | | |
| 4 | 1/5 | 0.3515 | 3.3081 | 9.4126 | 0.3353 | 3.1887 | 9.5105 | 0.2901 | 2.8416 | 9.7958 |
| 6 | | 0.3056 | 3.2984 | 10.7921 | 0.2914 | 3.1792 | 10.9103 | 0.2482 | 2.8324 | 11.4107 |
| 8 | | 0.2752 | 3.2983 | 11.9870 | 0.2581 | 3.1787 | 12.3150 | 0.2228 | 2.8322 | 12.7100 |
| 10 | | 0.2473 | 3.2960 | 13.3254 | 0.2351 | 3.1765 | 13.5130 | 0.2023 | 2.8296 | 13.9896 |
| 12 | | 0.2270 | 3.2925 | 14.5033 | 0.2156 | 3.1730 | 14.7149 | 0.1862 | 2.8380 | 15.2389 |
| 4 | 1/4 | 0.3023 | 3.3048 | 10.9308 | 0.2915 | 3.1858 | 10.9292 | 0.2460 | 2.8367 | 11.5311 |
| 6 | | 0.2651 | 3.2962 | 12.4338 | 0.2539 | 3.1771 | 12.5152 | 0.2200 | 2.8303 | 12.8654 |
| 8 | | 0.2427 | 3.2940 | 13.5709 | 0.2308 | 3.1746 | 13.7571 | 0.1988 | 2.8276 | 14.2220 |
| 10 | | 0.2224 | 3.3033 | 14.8551 | 0.2117 | 3.1842 | 15.0394 | 0.1805 | 2.8378 | 15.7224 |
| 12 | | 0.2065 | 3.3031 | 15.9930 | 0.1966 | 3.1840 | 16.1941 | 0.1682 | 2.8376 | 16.8676 |
| 4 | 2/3 | 0.1327 | 3.2947 | 24.8285 | 0.1261 | 3.1752 | 25.1732 | 0.1083 | 2.8390 | 26.2239 |
| 6 | | 0.1311 | 3.2950 | 25.1256 | 0.1246 | 3.1756 | 25.4961 | 0.1066 | 2.8283 | 26.5277 |
| 8 | | 0.1263 | 3.2940 | 26.0805 | 0.1201 | 3.1746 | 26.4311 | 0.1047 | 2.8283 | 27.0157 |
| 10 | | 0.1233 | 3.2932 | 26.7130 | 0.1172 | 3.1738 | 27.0697 | 0.1026 | 2.8277 | 27.5552 |
| 12 | | 0.1204 | 3.2922 | 27.3396 | 0.1145 | 3.1729 | 27.7020 | 0.0985 | 2.8380 | 28.8235 |