



## On $\Lambda$ -generalized closed sets in interval-valued topological spaces

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**Abstract:** In this paper, introduce a new class of sets called  $\Lambda_g$ -closed sets in interval-valued topological spaces. The concept is defined as a generalization of closed sets and is shown to be a weaker form of certain existing closed sets in interval-valued topology. Several fundamental properties of  $\Lambda_g$ -closed sets are established and their behavior with respect to basic set operations is investigated. We also introduce the notion of  $\Lambda_g$ -open sets as the complements of  $\Lambda_g$ -closed sets and study their basic properties. Furthermore, the relationships between  $\Lambda_g$ -closed sets and other well-known classes of sets such as closed sets,  $g$ -closed sets, and semi-closed sets are discussed. Various characterizations and results are obtained to highlight the structural behavior of these sets in interval-valued topological spaces. Suitable examples are provided to illustrate the introduced concepts and to show that the implications between different classes of sets are proper. The results obtained in this work contribute to the further development of generalized closed sets in an interval-valued topology.

**Key words:**  $\mathcal{IV}$ -closed,  $\mathcal{IV}\Lambda$ -set,  $\mathcal{IV}\lambda$ -closed and  $\mathcal{IV}\Lambda_g$ -closed set

### 1. Introduction

The idea of interval sets was originally proposed by Yao [6] as a useful approach for representing and approximating uncertain or complicated information. Subsequently, Kim et al. [4] described this concept using the term interval-valued sets and viewed it both as an extension of classical set theory and as a particular form of interval-valued fuzzy sets. In their study, they analyzed different types of interval-valued neighborhoods and grouped them into two broad classes. They also investigated the associated interval-valued closure and interior operators. More recently, Cheong et al. [1] developed the concept of interval-valued relations and explored their properties using a category-theoretic framework, which further broadened the applications of interval-valued structures in both theoretical and computational contexts.

In 2026, Rajasekaran et al. [5] introduced the notion of  $\Lambda$ -sets in interval-valued topological spaces. A  $\Lambda$ -set is defined as a set  $[\mathcal{M}^-, \mathcal{M}^+]$  which is equal to its interval-valued kernel. This concept plays an important role in the study of generalized closed sets in interval-valued topology. They also introduced the notions of interval-valued  $\lambda$ -closed sets and interval-valued  $\lambda$ -open sets and investigated several of their fundamental properties. These concepts have contributed to the further development of interval-valued topological structures and have motivated the study of new generalized types of closed sets in interval-valued and related topological

spaces.

In this paper, we introduce a new class of sets called  $\Lambda_g$ -closed sets in interval-valued topological spaces. This concept is defined as a generalization of closed sets and is shown to be a weaker form of certain existing closed sets. We also introduce  $\Lambda_g$ -open sets as the complements of  $\Lambda_g$ -closed sets and study their basic properties. Furthermore, relationships between  $\Lambda_g$ -closed sets and other types of closed sets such as closed,  $g$ -closed, and semi-closed sets are investigated. Several results and examples are provided to illustrate these concepts.

## 2. Preliminaries

**Definition 2.1.** [6] Let  $X$  be a non-empty set. Then, the form  $[\mathcal{M}^-, \mathcal{M}^+] = \{[S \subset X : \mathcal{M}^- \subset S \subset \mathcal{M}^+]\}$  is called an interval-valued set (briefly,  $\mathcal{IVS}$ ) in  $X$ , if  $\mathcal{M}^-, \mathcal{M}^+ \subset X$  and  $\mathcal{M}^- \subset \mathcal{M}^+$ .

In this case,  $\mathcal{M}^-$  [resp.  $\mathcal{M}^+$ ] represent the set of minimum [resp. maximum] memberships of elements of  $X$  to  $[\mathcal{M}^-, \mathcal{M}^+]$ .

Here,  $[\emptyset, \emptyset]$  [resp.  $[X, X]$ ] is called the interval-valued empty [resp. whole] set in  $X$  and denoted by  $\tilde{\emptyset}$  [resp.  $\tilde{X}$ ]. The set of all  $\mathcal{IVS}$ s in  $X$  is denoted by  $\mathcal{IVS}(X)$ . Interval valued topological spaces by is denoted by  $\mathcal{IVTS}$  or  $\mathcal{T}^X$ .

**Definition 2.2.** [4] A non-empty set  $X$  with  $[\mathcal{M}^-, \mathcal{M}^+]$  and  $[\mathcal{S}^-, \mathcal{S}^+]$  are  $\mathcal{IVS}$  respectively. Then

1.  $[\mathcal{M}^-, \mathcal{M}^+] \subset [\mathcal{S}^-, \mathcal{S}^+] \iff \mathcal{M}^- \subset \mathcal{S}^-$  and  $\mathcal{M}^+ \subset \mathcal{S}^+$
2.  $[\mathcal{M}^-, \mathcal{M}^+] = [\mathcal{S}^-, \mathcal{S}^+] \iff [\mathcal{M}^-, \mathcal{M}^+] \subset [\mathcal{S}^-, \mathcal{S}^+]$  and  $[\mathcal{S}^-, \mathcal{S}^+] \subset [\mathcal{M}^-, \mathcal{M}^+]$
3.  $([\mathcal{M}^-, \mathcal{M}^+])^c = [(\mathcal{M}^+)^c, (\mathcal{M}^-)^c]$
4.  $[\mathcal{M}^-, \mathcal{M}^+] \cup [\mathcal{S}^-, \mathcal{S}^+] = [\mathcal{M}^- \cup \mathcal{S}^-, \mathcal{M}^+ \cup \mathcal{S}^+]$
5.  $[\mathcal{M}^-, \mathcal{M}^+] \cap [\mathcal{S}^-, \mathcal{S}^+] = [\mathcal{M}^- \cap \mathcal{S}^-, \mathcal{M}^+ \cap \mathcal{S}^+]$

**Definition 2.3.** [4] Let  $X$  be a non-empty set and let  $\tau$  be a non-empty family of  $\mathcal{IVS}$ s on  $X$ . Then  $\tau$  is called an interval-valued topology (briefly,  $\mathcal{IVT}$ ) on  $X$  if it satisfies the following axioms:

1.  $\tilde{\emptyset}, \tilde{X} \in \tau$ ,
2.  $[\mathcal{M}^-, \mathcal{M}^+] \cap [\mathcal{S}^-, \mathcal{S}^+] \in \tau$  for any  $[\mathcal{M}^-, \mathcal{M}^+], [\mathcal{S}^-, \mathcal{S}^+] \in \tau$ .
3.  $\bigcup_{j \in J} \mathcal{S}_j \in \tau$  for any family  $(\mathcal{S}_j)_{j \in J}$  of members of  $\tau$ .

In this case, the  $\mathcal{T}^X$  is called an interval-valued topological space (briefly,  $\mathcal{IVTS}$ ) and each member of  $\tau$  is called an interval-valued open set (briefly,  $\mathcal{IVOS}$ ) in  $X$ . A  $\mathcal{IVS}$  of  $[\mathcal{M}^-, \mathcal{M}^+]$  is called an interval-valued closed set (briefly,  $\mathcal{IVCS}$ ) in  $X$ .

**Definition 2.4.** [4] If the space  $\mathcal{T}^X$  with respect to  $X$  and if  $[\mathcal{M}^-, \mathcal{M}^+] \subseteq \tilde{X}$ , then

1. the an interval-valued interior of  $[\mathcal{M}^-, \mathcal{M}^+]$  is defined as the union of all an interval-valued open subsets of  $[\mathcal{M}^-, \mathcal{M}^+]$  and it is denoted by  $\mathcal{IVI}([\mathcal{M}^-, \mathcal{M}^+])$ .

That is,  $\mathcal{IVI}([\mathcal{M}^-, \mathcal{M}^+])$  is the largest an interval-valued open subset of  $[\mathcal{M}^-, \mathcal{M}^+]$ .

2. the an interval-valued closure of  $[\mathcal{M}^-, \mathcal{M}^+]$  is defined as the intersection of all an interval-valued closed sets containing  $[\mathcal{M}^-, \mathcal{M}^+]$  and it is denoted by  $\mathcal{I}\mathcal{V}C([\mathcal{M}^-, \mathcal{M}^+])$ .

That is,  $\mathcal{I}\mathcal{V}C([\mathcal{M}^-, \mathcal{M}^+])$  is the smallest an interval-valued closed set containing  $[\mathcal{M}^-, \mathcal{M}^+]$ .

**Definition 2.5.** [5] A subset  $[\mathcal{M}^-, \mathcal{M}^+]$  of a space  $\mathcal{T}^X$  is called a

1. an interval-valued- $\ker([\mathcal{M}^-, \mathcal{M}^+]) = \bigcap \{[\mathcal{S}^-, \mathcal{S}^+] : [\mathcal{M}^-, \mathcal{M}^+] \subseteq [\mathcal{S}^-, \mathcal{S}^+], [\mathcal{S}^-, \mathcal{S}^+] \in \tau\}$  is called the interval-valued kernal of  $[\mathcal{M}^-, \mathcal{M}^+]$  and is denoted by  $\mathcal{I}\mathcal{V}\text{-ker}([\mathcal{M}^-, \mathcal{M}^+])$ .
2. an interval-valued  $\Lambda$ -set (briefly,  $\mathcal{I}\mathcal{V}\Lambda$ -set) if  $[\mathcal{M}^-, \mathcal{M}^+] = \mathcal{I}\mathcal{V}\text{-ker}([\mathcal{M}^-, \mathcal{M}^+])$ .
3. an interval-valued  $\lambda$ -closed (briefly,  $\mathcal{I}\mathcal{V}\lambda$ -closed) if  $[\mathcal{M}^-, \mathcal{M}^+] = [\mathcal{S}^-, \mathcal{S}^+] \cap [\mathcal{R}^-, \mathcal{R}^+]$  where  $[\mathcal{S}^-, \mathcal{S}^+]$  is an  $\mathcal{I}\mathcal{V}\Lambda$ -set and  $[\mathcal{R}^-, \mathcal{R}^+]$  is an  $\mathcal{I}\mathcal{V}$ -closed.

**Definition 2.6.** [2]

A subset  $[\mathcal{M}^-, \mathcal{M}^+]$  from  $\mathcal{I}\mathcal{V}\mathcal{S}(X)$  is an interval-valued generalized closed (briefly,  $\mathcal{I}\mathcal{V}g$ -closed) if  $\mathcal{I}\mathcal{V}C([\mathcal{M}^-, \mathcal{M}^+]) \subseteq [\mathcal{K}^-, \mathcal{K}^+]$ , whenever  $[\mathcal{M}^-, \mathcal{M}^+] \subseteq [\mathcal{K}^-, \mathcal{K}^+]$  and  $[\mathcal{K}^-, \mathcal{K}^+]$  is an  $\mathcal{I}\mathcal{V}$ -open.

**Remark 2.1.** [5] *Within an interval-valued topological space, the class of  $\mathcal{I}\mathcal{V}g$ -closed sets and the class of  $\mathcal{I}\mathcal{V}\lambda$ -closed sets do not coincide. In general, an  $\mathcal{I}\mathcal{V}g$ -closed set need not be  $\mathcal{I}\mathcal{V}\lambda$ -closed and vice versa.*

### 3. On $\Lambda_g$ -closed sets in interval-valued topological spaces

**Definition 3.1.** A subset  $[\mathcal{M}^-, \mathcal{M}^+]$  of a space  $\mathcal{T}^X$  is called an interval-valued  $\Lambda_g$ -closed (briefly,  $\mathcal{I}\mathcal{V}\Lambda_g$ -closed) if  $\mathcal{I}\mathcal{V}C([\mathcal{M}^-, \mathcal{M}^+]) \subseteq [\mathcal{K}^-, \mathcal{K}^+]$ , whenever  $[\mathcal{M}^-, \mathcal{M}^+] \subseteq [\mathcal{K}^-, \mathcal{K}^+]$  and  $[\mathcal{K}^-, \mathcal{K}^+]$  is an  $\mathcal{I}\mathcal{V}\lambda$ -open.

The complement of  $\mathcal{I}\mathcal{V}\Lambda_g$ -open if  $[\mathcal{K}^-, \mathcal{K}^+]^c = [X^-, X^+] - [\mathcal{K}^-, \mathcal{K}^+]$  is an  $\mathcal{I}\mathcal{V}\Lambda_g$ -closed.

**Example 3.1.** Consider the space  $\mathcal{T}^X$ , where

$X = \{a, b, c, d\}$  and

$\tau = \{\emptyset, \tilde{X}, [\{a\}, \{a\}], [\{b\}, \{b, d\}], [\{a, b\}, \{a, b, d\}]\}$  is interval-valued open sets and

$\tau^c = \{\tilde{\emptyset}, \tilde{X}, [\{b, c, d\}, \{b, c, d\}], [\{a, c\}, \{a, c, d\}], [\{c\}, \{c, d\}]\}$  interval-valued closed set.

Hence  $[\mathcal{M}^-, \mathcal{M}^+] = [\{a\}, \{a, c\}]$  is  $\mathcal{I}\mathcal{V}\Lambda_g$ -closed set.

**Lemma 3.1.** In a space  $\mathcal{T}^X$ , every  $\mathcal{I}\mathcal{V}$ -open set is necessarily  $\mathcal{I}\mathcal{V}\Lambda_g$ -open but  $\mathcal{I}\mathcal{V}\Lambda_g$ -openness does not mandate that a set be  $\mathcal{I}\mathcal{V}$ -open.

*Proof.*

Let  $[\mathcal{M}^-, \mathcal{M}^+]$  be an  $\mathcal{I}\mathcal{V}$ -open set in  $\mathcal{T}^X$ . By the Definition of  $\mathcal{I}\mathcal{V}\Lambda_g$ -open sets is said to be an  $\mathcal{I}\mathcal{V}\Lambda_g$ -open if its complement is an  $\mathcal{I}\mathcal{V}\Lambda_g$ -closed.

Since  $[\mathcal{M}^-, \mathcal{M}^+]$  is an  $\mathcal{I}\mathcal{V}$ -open, its complement  $[\mathcal{M}^-, \mathcal{M}^+]^c$  is an  $\mathcal{I}\mathcal{V}$ -closed.

Every  $\mathcal{I}\mathcal{V}$ -closed set is an  $\mathcal{I}\mathcal{V}\Lambda_g$ -closed. Hence  $[\mathcal{M}^-, \mathcal{M}^+]^c$  is an  $\mathcal{I}\mathcal{V}\Lambda_g$ -closed. Therefore,  $[\mathcal{M}^-, \mathcal{M}^+]$  is  $\mathcal{I}\mathcal{V}\Lambda_g$ -open. Thus every  $\mathcal{I}\mathcal{V}$ -open set is  $\mathcal{I}\mathcal{V}\Lambda_g$ -open.

However, the converse does not necessarily hold. There exists a set which is  $\mathcal{I}\mathcal{V}\Lambda_g$ -open but not  $\mathcal{I}\mathcal{V}$ -open. □

**Remark 3.1.** *The reverse implication of Lemma 3.1 does not generally hold, as illustrated by the example given below.*

**Example 3.2.** *Referring to Example 3.1,  $[\emptyset, \{b\}]$  is an  $\mathcal{IV}\Lambda_g$ -open but  $[\emptyset, \{b\}]$  is not  $\mathcal{IV}$ -open.*

**Remark 3.2.** *The example presented below indicates that the classes of  $\mathcal{IV}\Lambda_g$ -closed sets and  $\mathcal{IV}\lambda$ -closed sets are mutually independent; in general, one does not imply the other.*

**Example 3.3.** *Referring to Example 3.1,*

1.  $[\emptyset, \{b, c\}]$  is an  $\mathcal{IV}\Lambda_g$ -closed but  $[\emptyset, \{b, c\}]$  is not  $\mathcal{IV}\lambda$ -closed.
2.  $[\emptyset, \{a\}]$  is an  $\mathcal{IV}\lambda$ -closed but  $[\emptyset, \{a\}]$  is not  $\mathcal{IV}\Lambda_g$ -closed.

**Theorem 3.1.** *In an space  $\mathcal{T}^X$ , the union of two  $\mathcal{IV}\Lambda_g$ -closed sets is an  $\mathcal{IV}\Lambda_g$ -closed.*

*Proof.*

Let  $[\mathcal{M}^-, \mathcal{M}^+] \cup [\mathcal{S}^-, \mathcal{S}^+] \subseteq [\mathcal{K}^-, \mathcal{K}^+]$ , then  $[\mathcal{M}^-, \mathcal{M}^+] \subseteq [\mathcal{K}^-, \mathcal{K}^+]$  and  $[\mathcal{S}^-, \mathcal{S}^+] \subseteq [\mathcal{K}^-, \mathcal{K}^+]$  where  $[\mathcal{K}^-, \mathcal{K}^+]$  is an  $\mathcal{IV}\lambda$ -open. As  $[\mathcal{M}^-, \mathcal{M}^+]$  and  $[\mathcal{S}^-, \mathcal{S}^+]$  are an  $\mathcal{IV}\Lambda_g$ -closed,  $\mathcal{IVC}([\mathcal{M}^-, \mathcal{M}^+]) \subseteq [\mathcal{K}^-, \mathcal{K}^+]$  and  $\mathcal{IVC}([\mathcal{S}^-, \mathcal{S}^+]) \subseteq [\mathcal{K}^-, \mathcal{K}^+]$ .

$$\text{Hence } \mathcal{IVC}([\mathcal{M}^-, \mathcal{M}^+] \cup [\mathcal{S}^-, \mathcal{S}^+]) = \mathcal{IVC}([\mathcal{M}^-, \mathcal{M}^+]) \cup \mathcal{IVC}([\mathcal{S}^-, \mathcal{S}^+]) \subseteq [\mathcal{K}^-, \mathcal{K}^+].$$

□

**Example 3.4.** *Referring to Example 3.1, consider the sets*

$$[\mathcal{M}^-, \mathcal{M}^+] = [\{a\}, \{a, c\}] \quad \text{and} \quad [\mathcal{S}^-, \mathcal{S}^+] = [\{b\}, \{b, c\}].$$

*From Example 3.1,  $[\mathcal{M}^-, \mathcal{M}^+]$  and  $[\mathcal{S}^-, \mathcal{S}^+]$  is  $\mathcal{IV}\Lambda_g$ -closed.*

*Now consider their union:*

$$[\mathcal{M}^-, \mathcal{M}^+] \cup [\mathcal{S}^-, \mathcal{S}^+] = [\{a, b\}, \{a, b, c\}].$$

*Hence the set is  $\mathcal{IV}\Lambda_g$ -closed.*

*Therefore, the union of two  $\mathcal{IV}\Lambda_g$ -closed sets is again an  $\mathcal{IV}\Lambda_g$ -closed set, illustrating Theorem 3.1.*

**Theorem 3.2.** *In a space  $\mathcal{T}^X$ , the intersection of two  $\mathcal{IV}\Lambda_g$ -open sets is an  $\mathcal{IV}\Lambda_g$ -open.*

*Proof.*

Let  $[\mathcal{A}^-, \mathcal{A}^+]$  and  $[\mathcal{B}^-, \mathcal{B}^+]$  be two  $\mathcal{IV}\Lambda_g$ -open. Then their complements  $[\mathcal{A}^-, \mathcal{A}^+]^c$  and  $[\mathcal{B}^-, \mathcal{B}^+]^c$  are  $\mathcal{IV}\Lambda_g$ -closed sets. Now,  $([\mathcal{A}^-, \mathcal{A}^+] \cap [\mathcal{B}^-, \mathcal{B}^+])^c = [\mathcal{A}^-, \mathcal{A}^+]^c \cup [\mathcal{B}^-, \mathcal{B}^+]^c$ . Since the union of two  $\mathcal{IV}\Lambda_g$ -closed sets is an  $\mathcal{IV}\Lambda_g$ -closed (by Theorem 3.8), it follows that  $([\mathcal{A}^-, \mathcal{A}^+] \cap [\mathcal{B}^-, \mathcal{B}^+])^c$  is an  $\mathcal{IV}\Lambda_g$ -closed.

Therefore,  $[\mathcal{A}^-, \mathcal{A}^+] \cap [\mathcal{B}^-, \mathcal{B}^+]$  is an  $\mathcal{IV}\Lambda_g$ -open. Hence the intersection of two  $\mathcal{IV}\Lambda_g$ -open sets is an  $\mathcal{IV}\Lambda_g$ -open. □

**Example 3.5.** *Consider the space  $\mathcal{T}^X$ , where*

$$X = \{a, b, c, d\} \quad \text{and}$$

$$\tau = \{\tilde{\emptyset}, \tilde{X}, [\{b\}, \{b\}], [\{c\}, \{c, d\}], [\{b, c\}, \{b, c, d\}]\} \text{ is interval-valued open sets and}$$

$\tau^c = \{\tilde{\emptyset}, \tilde{X}, [\{a, c, d\}, \{a, c, d\}], [\{b, c\}, \{a, b, d\}], [\{d\}, \{a, d\}]\}$  interval-valued closed set. consider the two interval-valued sets

$$[\mathcal{A}^-, \mathcal{A}^+] = [\{b\}, \{b, c\}] \quad \text{and} \quad [\mathcal{B}^-, \mathcal{B}^+] = [\emptyset, \{b, d\}].$$

Both of these sets are  $\mathcal{IV}\Lambda g$ -open sets in the space  $\mathcal{T}^X$ .

Now consider their intersection:

$$[\mathcal{A}^-, \mathcal{A}^+] \cap [\mathcal{B}^-, \mathcal{B}^+] = [\emptyset, \{b\}].$$

The resulting set is also an  $\mathcal{IV}\Lambda g$ -open set in  $\mathcal{T}^X$ .

Thus, this example illustrates Theorem 3.2, showing that the intersection of two  $\mathcal{IV}\Lambda g$ -open sets is again an  $\mathcal{IV}\Lambda g$ -open set.

**Remark 3.3.** In a space  $\mathcal{T}^X$ , the intersection of two  $\mathcal{IV}\Lambda g$ -closed sets but not  $\mathcal{IV}\Lambda g$ -closed.

**Example 3.6.** Consider the space  $\mathcal{T}^X$ , where

$$X = \{a, b, c\} \quad \text{and}$$

$$\tau = \{\tilde{\emptyset}, \tilde{X}, [\emptyset, \{a\}]\} \quad \text{is interval-valued open sets and}$$

$$\tau^c = \{\tilde{\emptyset}, \tilde{X}, [\{b, c\}, X]\} \quad \text{interval-valued closed set. consider the two interval-valued sets}$$

$$[\mathcal{A}^-, \mathcal{A}^+] = [\{a\}, \{a, b\}] \quad \text{and} \quad [\mathcal{B}^-, \mathcal{B}^+] = [\emptyset, \{a, c\}].$$

Both of these sets are  $\mathcal{IV}\Lambda g$ -closed sets in the space  $\mathcal{T}^X$ .

Now consider their intersection:

$$[\mathcal{A}^-, \mathcal{A}^+] \cap [\mathcal{B}^-, \mathcal{B}^+] = [\emptyset, \{a\}].$$

The resulting set is also an  $\mathcal{IV}\Lambda g$ -closed set in  $\mathcal{T}^X$ .

**Theorem 3.3.** Let a space  $\mathcal{T}^X$  and suppose that  $[\mathcal{M}^-, \mathcal{M}^+]$  is an  $\mathcal{IV}\Lambda g$ -closed set in  $[X^-, X^+]$ . Then the set  $\mathcal{IVC}([\mathcal{M}^-, \mathcal{M}^+]) - [\mathcal{M}^-, \mathcal{M}^+]$  does not contain any nonempty  $\mathcal{IV}$ -closed.

*Proof.*

Let  $[\mathcal{K}^-, \mathcal{K}^+]$  be an  $\mathcal{IV}$ -closed subset contains in  $\mathcal{IVC}([\mathcal{M}^-, \mathcal{M}^+]) - [\mathcal{M}^-, \mathcal{M}^+]$ . Clearly  $[\mathcal{M}^-, \mathcal{M}^+] \subseteq [\mathcal{K}^-, \mathcal{K}^+]^c$  where  $[\mathcal{M}^-, \mathcal{M}^+]$  is an  $\mathcal{IV}\Lambda g$ -closed and  $[\mathcal{K}^-, \mathcal{K}^+]^c$  is an  $\mathcal{IV}$ -open set of  $[X, X]$ . Thus  $\mathcal{IVC}([\mathcal{M}^-, \mathcal{M}^+]) \subseteq [\mathcal{K}^-, \mathcal{K}^+]^c$  (or)  $[\mathcal{K}^-, \mathcal{K}^+] \subseteq (\mathcal{IVC}([\mathcal{M}^-, \mathcal{M}^+]))^c$ . Then  $[\mathcal{K}^-, \mathcal{K}^+] \subseteq (\mathcal{IVC}([\mathcal{M}^-, \mathcal{M}^+]))^c \cap (\mathcal{IVC}([\mathcal{M}^-, \mathcal{M}^+]) - [\mathcal{M}^-, \mathcal{M}^+]) \subseteq (\mathcal{IVC}([\mathcal{M}^-, \mathcal{M}^+]))^c \cap \mathcal{IVC}([\mathcal{M}^-, \mathcal{M}^+]) = \tilde{\emptyset}$ . This is show that  $[\mathcal{K}^-, \mathcal{K}^+] = \tilde{\emptyset}$ .  $\square$

**Theorem 3.4.** A subset  $[\mathcal{M}^-, \mathcal{M}^+]$  of a space  $\mathcal{T}^X$  is an  $\mathcal{IV}\Lambda g$ -closed set  $\iff \mathcal{IVC}([\mathcal{M}^-, \mathcal{M}^+]) - [\mathcal{M}^-, \mathcal{M}^+]$  contains no nonempty  $\mathcal{IV}\lambda$ -closed.

*Proof.*

**Necessity.** Assume that  $[\mathcal{M}^-, \mathcal{M}^+]$  is an  $\mathcal{IV}\Lambda g$ -closed set. Let  $[\mathcal{K}^-, \mathcal{K}^+]$  be an  $\mathcal{IV}\lambda$ -closed subset of  $\mathcal{IVC}([\mathcal{M}^-, \mathcal{M}^+]) - [\mathcal{M}^-, \mathcal{M}^+]$ . Then  $[\mathcal{K}^-, \mathcal{K}^+] \subseteq \mathcal{IVC}([\mathcal{M}^-, \mathcal{M}^+])$  and  $[\mathcal{K}^-, \mathcal{K}^+] \cap [\mathcal{M}^-, \mathcal{M}^+] = \tilde{\emptyset}$ . Hence  $[\mathcal{M}^-, \mathcal{M}^+] \subseteq ([\mathcal{K}^-, \mathcal{K}^+])^c$ . Since  $[\mathcal{K}^-, \mathcal{K}^+]$  is  $\mathcal{IV}\lambda$ -closed, its complement  $([\mathcal{K}^-, \mathcal{K}^+])^c$  is an  $\mathcal{IV}\lambda$ -open. Because  $[\mathcal{M}^-, \mathcal{M}^+]$  is an  $\mathcal{IV}\Lambda g$ -closed, it follows that  $\mathcal{IVC}([\mathcal{M}^-, \mathcal{M}^+]) \subseteq ([\mathcal{K}^-, \mathcal{K}^+])^c$ .

Therefore  $[\mathcal{K}^-, \mathcal{K}^+] \subseteq (\mathcal{I}\mathcal{V}\mathcal{C}([\mathcal{M}^-, \mathcal{M}^+]))^c$ . But this contradicts the fact that  $[\mathcal{K}^-, \mathcal{K}^+] \subseteq \mathcal{I}\mathcal{V}\mathcal{C}([\mathcal{M}^-, \mathcal{M}^+])$  unless  $[\mathcal{K}^-, \mathcal{K}^+] = \tilde{\emptyset}$ . Hence  $\mathcal{I}\mathcal{V}\mathcal{C}([\mathcal{M}^-, \mathcal{M}^+]) - [\mathcal{M}^-, \mathcal{M}^+]$  contains no nonempty  $\mathcal{I}\mathcal{V}\lambda$ -closed set.

**Sufficiency.** Conversely, suppose that  $\mathcal{I}\mathcal{V}\mathcal{C}([\mathcal{M}^-, \mathcal{M}^+]) - [\mathcal{M}^-, \mathcal{M}^+]$  contains no nonempty  $\mathcal{I}\mathcal{V}\lambda$ -closed subset. Let  $[\mathcal{M}^-, \mathcal{M}^+] \subseteq [\mathcal{S}^-, \mathcal{S}^+]$ , where  $[\mathcal{S}^-, \mathcal{S}^+]$  is an  $\mathcal{I}\mathcal{V}\lambda$ -open.

If  $\mathcal{I}\mathcal{V}\mathcal{C}([\mathcal{M}^-, \mathcal{M}^+]) \not\subseteq [\mathcal{S}^-, \mathcal{S}^+]$ , then  $\mathcal{I}\mathcal{V}\mathcal{C}([\mathcal{M}^-, \mathcal{M}^+]) \cap ([\mathcal{S}^-, \mathcal{S}^+])^c$  is a nonempty  $\mathcal{I}\mathcal{V}\lambda$ -closed subset of  $\mathcal{I}\mathcal{V}\mathcal{C}([\mathcal{M}^-, \mathcal{M}^+]) - [\mathcal{M}^-, \mathcal{M}^+]$ , which contradicts the assumption. Therefore  $\mathcal{I}\mathcal{V}\mathcal{C}([\mathcal{M}^-, \mathcal{M}^+]) \subseteq [\mathcal{S}^-, \mathcal{S}^+]$ . Hence  $[\mathcal{M}^-, \mathcal{M}^+]$  is an  $\mathcal{I}\mathcal{V}\Lambda_g$ -closed set.  $\square$

**Theorem 3.5.** *Let  $\mathcal{T}^X$  be a space. If  $[\mathcal{M}^-, \mathcal{M}^+]$  is an  $\mathcal{I}\mathcal{V}\Lambda_g$ -closed and  $[\mathcal{M}^-, \mathcal{M}^+] \subseteq [\mathcal{S}^-, \mathcal{S}^+] \subseteq \mathcal{I}\mathcal{V}\mathcal{C}([\mathcal{M}^-, \mathcal{M}^+])$ , then  $[\mathcal{S}^-, \mathcal{S}^+]$  is an  $\mathcal{I}\mathcal{V}\Lambda_g$ -closed.*

*Proof.*

Suppose that  $[\mathcal{M}^-, \mathcal{M}^+]$  is an  $\mathcal{I}\mathcal{V}\Lambda_g$ -closed and  $[\mathcal{M}^-, \mathcal{M}^+] \subseteq [\mathcal{S}^-, \mathcal{S}^+] \subseteq \mathcal{I}\mathcal{V}\mathcal{C}([\mathcal{M}^-, \mathcal{M}^+])$ .

Since  $[\mathcal{S}^-, \mathcal{S}^+] \subseteq \mathcal{I}\mathcal{V}\mathcal{C}([\mathcal{M}^-, \mathcal{M}^+])$ , we obtain  $\mathcal{I}\mathcal{V}\mathcal{C}([\mathcal{S}^-, \mathcal{S}^+]) \subseteq \mathcal{I}\mathcal{V}\mathcal{C}([\mathcal{M}^-, \mathcal{M}^+])$ . Thus  $\mathcal{I}\mathcal{V}\mathcal{C}([\mathcal{S}^-, \mathcal{S}^+]) - [\mathcal{S}^-, \mathcal{S}^+] \subseteq \mathcal{I}\mathcal{V}\mathcal{C}([\mathcal{M}^-, \mathcal{M}^+]) - [\mathcal{M}^-, \mathcal{M}^+]$ .

But since  $[\mathcal{M}^-, \mathcal{M}^+]$  is an  $\mathcal{I}\mathcal{V}\Lambda_g$ -closed, the set  $\mathcal{I}\mathcal{V}\mathcal{C}([\mathcal{M}^-, \mathcal{M}^+]) - [\mathcal{M}^-, \mathcal{M}^+]$  contains no nonempty  $\mathcal{I}\mathcal{V}\lambda$ -closed.

Hence  $\mathcal{I}\mathcal{V}\mathcal{C}([\mathcal{S}^-, \mathcal{S}^+]) - [\mathcal{S}^-, \mathcal{S}^+]$  also contains no nonempty  $\mathcal{I}\mathcal{V}\lambda$ -closed subset. Therefore  $[\mathcal{S}^-, \mathcal{S}^+]$  is an  $\mathcal{I}\mathcal{V}\Lambda_g$ -closed.  $\square$

**Theorem 3.6.** *In a space  $\mathcal{T}^X$ , if  $[\mathcal{M}^-, \mathcal{M}^+]$  is an  $\mathcal{I}\mathcal{V}\lambda$ -open and  $\mathcal{I}\mathcal{V}\Lambda_g$ -closed, then  $[\mathcal{M}^-, \mathcal{M}^+]$  is an  $\mathcal{I}\mathcal{V}$ -closed.*

*Proof.*

Let  $[\mathcal{M}^-, \mathcal{M}^+]$  be both  $\mathcal{I}\mathcal{V}\lambda$ -open and  $\mathcal{I}\mathcal{V}\Lambda_g$ -closed. Since  $[\mathcal{M}^-, \mathcal{M}^+]$  is an  $\mathcal{I}\mathcal{V}\lambda$ -open, its complement  $([\mathcal{M}^-, \mathcal{M}^+])^c$  is an  $\mathcal{I}\mathcal{V}\lambda$ -closed.

Because  $[\mathcal{M}^-, \mathcal{M}^+]$  is an  $\mathcal{I}\mathcal{V}\Lambda_g$ -closed, we have  $\mathcal{I}\mathcal{V}\mathcal{C}([\mathcal{M}^-, \mathcal{M}^+]) \subseteq [\mathcal{M}^-, \mathcal{M}^+]$  whenever  $[\mathcal{M}^-, \mathcal{M}^+] \subseteq [\mathcal{K}^-, \mathcal{K}^+]$  and  $[\mathcal{K}^-, \mathcal{K}^+]$  is an  $\mathcal{I}\mathcal{V}\lambda$ -open.

Taking  $[\mathcal{K}^-, \mathcal{K}^+] = [\mathcal{M}^-, \mathcal{M}^+]$ , which is an  $\mathcal{I}\mathcal{V}\lambda$ -open, we obtain  $\mathcal{I}\mathcal{V}\mathcal{C}([\mathcal{M}^-, \mathcal{M}^+]) \subseteq [\mathcal{M}^-, \mathcal{M}^+]$ .

Since  $[\mathcal{M}^-, \mathcal{M}^+] \subseteq \mathcal{I}\mathcal{V}\mathcal{C}([\mathcal{M}^-, \mathcal{M}^+])$  always holds, it follows that  $\mathcal{I}\mathcal{V}\mathcal{C}([\mathcal{M}^-, \mathcal{M}^+]) = [\mathcal{M}^-, \mathcal{M}^+]$ .

Hence  $[\mathcal{M}^-, \mathcal{M}^+]$  is an  $\mathcal{I}\mathcal{V}$ -closed set.  $\square$

**Theorem 3.7.** *For each  $(\{p\}, \{p\}) \in [X, X]$ , either  $(\{p\}, \{p\})$  is an  $\mathcal{I}\mathcal{V}\lambda$ -closed or  $(\{p\}, \{p\})^c$  is an  $\mathcal{I}\mathcal{V}\Lambda_g$ -closed.*

*Proof.*

Assume that  $(\{p\}, \{p\})$  is not an  $\mathcal{I}\mathcal{V}\lambda$ -closed. Then its complement  $(\{p\}, \{p\})^c$  is not  $\mathcal{I}\mathcal{V}\lambda$ -open. Hence the only  $\mathcal{I}\mathcal{V}\lambda$ -open set containing  $(\{p\}, \{p\})^c$  is the whole space  $\tilde{X}$ .

Therefore  $\mathcal{I}\mathcal{V}\mathcal{C}((\{p\}, \{p\})^c) \subseteq \tilde{X}$ . This shows that  $(\{p\}, \{p\})^c$  satisfies the definition of an  $\mathcal{I}\mathcal{V}\Lambda_g$ -closed set.

Hence either  $(\{p\}, \{p\})$  is an  $\mathcal{I}\mathcal{V}\lambda$ -closed or its complement  $(\{p\}, \{p\})^c$  is an  $\mathcal{I}\mathcal{V}\Lambda_g$ -closed.  $\square$

**Theorem 3.8.** *In a space  $\mathcal{T}^X$ ,  $[\mathcal{M}^-, \mathcal{M}^+]$  is an  $\mathcal{I}\mathcal{V}\Lambda_g$ -open  $\iff [\mathcal{K}^-, \mathcal{K}^+] \subseteq \mathcal{I}\mathcal{V}I([\mathcal{M}^-, \mathcal{M}^+])$  whenever  $[\mathcal{K}^-, \mathcal{K}^+]$  is an  $\mathcal{I}\mathcal{V}\lambda$ -closed and  $[\mathcal{K}^-, \mathcal{K}^+] \subseteq [\mathcal{M}^-, \mathcal{M}^+]$ .*

*Proof.*

Assume that  $[\mathcal{K}^-, \mathcal{K}^+] \subseteq \mathcal{I}\mathcal{V}I([\mathcal{M}^-, \mathcal{M}^+])$  whenever  $[\mathcal{K}^-, \mathcal{K}^+]$  is an  $\mathcal{I}\mathcal{V}\lambda$ -closed and  $[\mathcal{K}^-, \mathcal{K}^+] \subseteq [\mathcal{M}^-, \mathcal{M}^+]$ . Let  $[\mathcal{M}^-, \mathcal{M}^+]^c \subseteq [\mathcal{S}^-, \mathcal{S}^+]$ , where  $[\mathcal{S}^-, \mathcal{S}^+]$  is an  $\mathcal{I}\mathcal{V}\lambda$ -open. Hence  $[\mathcal{S}^-, \mathcal{S}^+]^c \subseteq [\mathcal{M}^-, \mathcal{M}^+]$ .

By assumption  $[\mathcal{S}^-, \mathcal{S}^+]^c \subseteq \mathcal{I}\mathcal{V}I([\mathcal{M}^-, \mathcal{M}^+])$  which implies that  $(\mathcal{I}\mathcal{V}I([\mathcal{M}^-, \mathcal{M}^+]))^c \subseteq [\mathcal{S}^-, \mathcal{S}^+]$ , so  $\mathcal{I}\mathcal{V}C([\mathcal{M}^-, \mathcal{M}^+]^c) \subseteq [\mathcal{S}^-, \mathcal{S}^+]$ . Hence  $[\mathcal{M}^-, \mathcal{M}^+]^c$  is an  $\mathcal{I}\mathcal{V}\Lambda_g$ -closed that is,  $[\mathcal{M}^-, \mathcal{M}^+]$  is an  $\mathcal{I}\mathcal{V}\Lambda_g$ -open.

Conversely, let  $[\mathcal{M}^-, \mathcal{M}^+]$  be an  $\mathcal{I}\mathcal{V}\Lambda_g$ -open. Then  $[\mathcal{M}^-, \mathcal{M}^+]^c$  is  $\mathcal{I}\mathcal{V}\Lambda_g$ -closed. Also let  $[\mathcal{K}^-, \mathcal{K}^+]$  be an  $\mathcal{I}\mathcal{V}\lambda$ -closed set contained in  $[\mathcal{M}^-, \mathcal{M}^+]$ . Then  $[\mathcal{K}^-, \mathcal{K}^+]^c$  is an  $\mathcal{I}\mathcal{V}\lambda$ -open. Therefore whenever  $[\mathcal{M}^-, \mathcal{M}^+]^c \subseteq [\mathcal{K}^-, \mathcal{K}^+]^c$ ,  $\mathcal{I}\mathcal{V}C([\mathcal{M}^-, \mathcal{M}^+]^c) \subseteq [\mathcal{K}^-, \mathcal{K}^+]^c$ . This implies that  $[\mathcal{K}^-, \mathcal{K}^+] \subseteq (\mathcal{I}\mathcal{V}C([\mathcal{M}^-, \mathcal{M}^+]^c))^c = \mathcal{I}\mathcal{V}I([\mathcal{M}^-, \mathcal{M}^+])$ . Thus  $[\mathcal{M}^-, \mathcal{M}^+] \subseteq \mathcal{I}\mathcal{V}I([\mathcal{M}^-, \mathcal{M}^+])$ . □

**Theorem 3.9.** *In a space  $\mathcal{T}^X$ ,  $[\mathcal{M}^-, \mathcal{M}^+]$  is an  $\mathcal{I}\mathcal{V}\Lambda_g$ -open  $\iff [\mathcal{S}^-, \mathcal{S}^+] = [X, X]$  whenever  $[\mathcal{S}^-, \mathcal{S}^+]$  is an  $\mathcal{I}\mathcal{V}\lambda$ -open and  $\mathcal{I}\mathcal{V}I([\mathcal{M}^-, \mathcal{M}^+]) \cup [\mathcal{M}^-, \mathcal{M}^+]^c \subseteq [\mathcal{S}^-, \mathcal{S}^+]$ .*

*Proof.*

Let  $[\mathcal{M}^-, \mathcal{M}^+]$  be an  $\mathcal{I}\mathcal{V}\Lambda_g$ -open,  $[\mathcal{S}^-, \mathcal{S}^+]$  be an  $\mathcal{I}\mathcal{V}\lambda$ -open and  $\mathcal{I}\mathcal{V}I([\mathcal{M}^-, \mathcal{M}^+]) \cup [\mathcal{M}^-, \mathcal{M}^+]^c \subseteq [\mathcal{S}^-, \mathcal{S}^+]$ . Then  $[\mathcal{S}^-, \mathcal{S}^+]^c \subseteq (\mathcal{I}\mathcal{V}I([\mathcal{M}^-, \mathcal{M}^+]))^c \cap ([\mathcal{M}^-, \mathcal{M}^+]^c)^c = (\mathcal{I}\mathcal{V}I([\mathcal{M}^-, \mathcal{M}^+]))^c - [\mathcal{M}^-, \mathcal{M}^+]^c = \mathcal{I}\mathcal{V}C([\mathcal{M}^-, \mathcal{M}^+]^c) - [\mathcal{M}^-, \mathcal{M}^+]^c$ . Since  $[\mathcal{M}^-, \mathcal{M}^+]^c$  is an  $\mathcal{I}\mathcal{V}\Lambda_g$ -closed and  $[\mathcal{S}^-, \mathcal{S}^+]^c$  is an  $\mathcal{I}\mathcal{V}\lambda$ -closed, by Theorem 3.4 it follows that  $[\mathcal{S}^-, \mathcal{S}^+]^c = \emptyset$ . Therefore  $\tilde{X} = [\mathcal{S}^-, \mathcal{S}^+]$ .

Conversely, suppose that  $[\mathcal{K}^-, \mathcal{K}^+]$  is an  $\mathcal{I}\mathcal{V}\lambda$ -closed and  $[\mathcal{K}^-, \mathcal{K}^+] \subseteq [\mathcal{M}^-, \mathcal{M}^+]$ . Then  $\mathcal{I}\mathcal{V}I([\mathcal{M}^-, \mathcal{M}^+]) \cup [\mathcal{M}^-, \mathcal{M}^+]^c \subseteq \mathcal{I}\mathcal{V}I([\mathcal{M}^-, \mathcal{M}^+]) \cup [\mathcal{K}^-, \mathcal{K}^+]^c$ . It follows that  $\mathcal{I}\mathcal{V}I([\mathcal{M}^-, \mathcal{M}^+]) \cup [\mathcal{K}^-, \mathcal{K}^+]^c = [X, X]$  and hence  $[\mathcal{K}^-, \mathcal{K}^+] \subseteq \mathcal{I}\mathcal{V}I([\mathcal{M}^-, \mathcal{M}^+])$ . Therefore  $[\mathcal{M}^-, \mathcal{M}^+]$  is an  $\mathcal{I}\mathcal{V}\Lambda_g$ -open. □

**Theorem 3.10.** *In a space  $\mathcal{T}^X$ , if  $\mathcal{I}\mathcal{V}I([\mathcal{M}^-, \mathcal{M}^+]) \subseteq [\mathcal{S}^-, \mathcal{S}^+] \subseteq [\mathcal{M}^-, \mathcal{M}^+]$  and  $[\mathcal{M}^-, \mathcal{M}^+]$  is an  $\mathcal{I}\mathcal{V}\Lambda_g$ -open set, then  $[\mathcal{S}^-, \mathcal{S}^+]$  is also an  $\mathcal{I}\mathcal{V}\Lambda_g$ -open set.*

*Proof.*

Assume that  $\mathcal{I}\mathcal{V}I([\mathcal{M}^-, \mathcal{M}^+]) \subseteq [\mathcal{S}^-, \mathcal{S}^+] \subseteq [\mathcal{M}^-, \mathcal{M}^+]$  and that  $[\mathcal{M}^-, \mathcal{M}^+]$  is  $\mathcal{I}\mathcal{V}\Lambda_g$ -open. Then its complement  $([\mathcal{M}^-, \mathcal{M}^+]^c)$  is an  $\mathcal{I}\mathcal{V}\Lambda_g$ -closed.

$\mathcal{I}\mathcal{V}C([\mathcal{M}^-, \mathcal{M}^+]^c)$ . Thus we obtain  $([\mathcal{M}^-, \mathcal{M}^+]^c) \subseteq ([\mathcal{S}^-, \mathcal{S}^+]^c) \subseteq \mathcal{I}\mathcal{V}C([\mathcal{M}^-, \mathcal{M}^+]^c)$ .

Since  $([\mathcal{M}^-, \mathcal{M}^+]^c)$  is an  $\mathcal{I}\mathcal{V}\Lambda_g$ -closed, by Theorem 3.5 it follows that  $([\mathcal{S}^-, \mathcal{S}^+]^c)$  is also  $\mathcal{I}\mathcal{V}\Lambda_g$ -closed. Hence  $[\mathcal{S}^-, \mathcal{S}^+]$  is an  $\mathcal{I}\mathcal{V}\Lambda_g$ -open. □

**Theorem 3.11.** *In a space  $\mathcal{T}^X$ ,  $[\mathcal{M}^-, \mathcal{M}^+]$  is an  $\mathcal{I}\mathcal{V}\Lambda_g$ -closed  $\iff \mathcal{I}\mathcal{V}C([\mathcal{M}^-, \mathcal{M}^+]) - [\mathcal{M}^-, \mathcal{M}^+]$  is an  $\mathcal{I}\mathcal{V}\Lambda_g$ -open.*

*Proof.*

**Necessity:** Assume that  $[\mathcal{M}^-, \mathcal{M}^+]$  is an  $\mathcal{I}\mathcal{V}\Lambda_g$ -closed.

Let  $[\mathcal{K}^-, \mathcal{K}^+] \subseteq \mathcal{I}\mathcal{V}\mathcal{C}([\mathcal{M}^-, \mathcal{M}^+]) - [\mathcal{M}^-, \mathcal{M}^+]$ , where  $[\mathcal{K}^-, \mathcal{K}^+]$  is  $\mathcal{I}\mathcal{V}\lambda$ -closed. Then  $[\mathcal{K}^-, \mathcal{K}^+] \subseteq \mathcal{I}\mathcal{V}\mathcal{C}([\mathcal{M}^-, \mathcal{M}^+]) - [\mathcal{M}^-, \mathcal{M}^+]$ . Since  $[\mathcal{M}^-, \mathcal{M}^+]$  is  $\mathcal{I}\mathcal{V}\Lambda_g$ -closed, by Theorem 3.4 the set  $\mathcal{I}\mathcal{V}\mathcal{C}([\mathcal{M}^-, \mathcal{M}^+]) - [\mathcal{M}^-, \mathcal{M}^+]$  contains no nonempty  $\mathcal{I}\mathcal{V}\lambda$ -closed set. Hence  $[\mathcal{K}^-, \mathcal{K}^+] = \tilde{\emptyset}$ . Therefore,  $[\mathcal{K}^-, \mathcal{K}^+] \subseteq \mathcal{I}\mathcal{V}\mathcal{I}(\mathcal{I}\mathcal{V}\mathcal{C}([\mathcal{M}^-, \mathcal{M}^+]) - [\mathcal{M}^-, \mathcal{M}^+])$ . Thus, by Theorem 3.8,  $\mathcal{I}\mathcal{V}\mathcal{C}([\mathcal{M}^-, \mathcal{M}^+]) - [\mathcal{M}^-, \mathcal{M}^+]$  is an  $\mathcal{I}\mathcal{V}\Lambda_g$ -open set.

**Sufficiency.** Suppose that  $\mathcal{I}\mathcal{V}\mathcal{C}([\mathcal{M}^-, \mathcal{M}^+]) - [\mathcal{M}^-, \mathcal{M}^+]$  is an  $\mathcal{I}\mathcal{V}\Lambda_g$ -open.

$\mathcal{I}\mathcal{V}\mathcal{C}([\mathcal{M}^-, \mathcal{M}^+]) - [\mathcal{M}^-, \mathcal{M}^+]$ . Since  $\mathcal{I}\mathcal{V}\mathcal{C}([\mathcal{M}^-, \mathcal{M}^+]) \cap [\mathcal{S}^-, \mathcal{S}^+]^c$  is an  $\mathcal{I}\mathcal{V}\lambda$ -closed and  $\mathcal{I}\mathcal{V}\mathcal{C}([\mathcal{M}^-, \mathcal{M}^+]) - [\mathcal{M}^-, \mathcal{M}^+]$  is an  $\mathcal{I}\mathcal{V}\Lambda_g$ -open,

by Theorem 3.8 we obtain  $\mathcal{I}\mathcal{V}\mathcal{C}([\mathcal{M}^-, \mathcal{M}^+]) \cap [\mathcal{S}^-, \mathcal{S}^+]^c \subseteq \mathcal{I}\mathcal{V}\mathcal{I}(\mathcal{I}\mathcal{V}\mathcal{C}([\mathcal{M}^-, \mathcal{M}^+]) - [\mathcal{M}^-, \mathcal{M}^+]) = \tilde{\emptyset}$ .

Hence  $\mathcal{I}\mathcal{V}\mathcal{C}([\mathcal{M}^-, \mathcal{M}^+]) \cap [\mathcal{S}^-, \mathcal{S}^+]^c = \tilde{\emptyset}$ , which implies  $\mathcal{I}\mathcal{V}\mathcal{C}([\mathcal{M}^-, \mathcal{M}^+]) \subseteq [\mathcal{S}^-, \mathcal{S}^+]$ . Therefore  $[\mathcal{M}^-, \mathcal{M}^+]$  is an  $\mathcal{I}\mathcal{V}\Lambda_g$ -closed.  $\square$

**Theorem 3.12.** In a space  $\mathcal{T}^X$ , the following statements are equivalent:

1.  $[\mathcal{M}^-, \mathcal{M}^+]$  is an  $\mathcal{I}\mathcal{V}\Lambda_g$ -closed set.
2.  $\mathcal{I}\mathcal{V}\mathcal{C}([\mathcal{M}^-, \mathcal{M}^+]) - [\mathcal{M}^-, \mathcal{M}^+]$  contains no nonempty  $\mathcal{I}\mathcal{V}\lambda$ -closed set.
3.  $\mathcal{I}\mathcal{V}\mathcal{C}([\mathcal{M}^-, \mathcal{M}^+]) - [\mathcal{M}^-, \mathcal{M}^+]$  is an  $\mathcal{I}\mathcal{V}\Lambda_g$ -open set.

*Proof.*

The equivalence between (1) and (2) follows directly from Theorem 3.4. Further, by Theorem 3.11, a set is  $\mathcal{I}\mathcal{V}\Lambda_g$ -open if and only if its complement is  $\mathcal{I}\mathcal{V}\Lambda_g$ -closed. Applying this result to the set  $\mathcal{I}\mathcal{V}\mathcal{C}([\mathcal{M}^-, \mathcal{M}^+]) - [\mathcal{M}^-, \mathcal{M}^+]$ , we obtain the equivalence of (2) and (3). Hence the three statements are equivalent.  $\square$

**Definition 3.2.** Let  $\mathcal{T}^X$  be a space and let  $[\mathcal{M}^-, \mathcal{M}^+]$  be a subset of  $X$ . Then

1. The set  $[\mathcal{M}^-, \mathcal{M}^+]$  is said to be an *interval-valued  $_g\Lambda$ -closed* (briefly,  $\mathcal{I}\mathcal{V}_g\Lambda$ -closed) if  $\mathcal{I}\mathcal{V}\lambda\mathcal{C}([\mathcal{M}^-, \mathcal{M}^+]) \subseteq [\mathcal{K}^-, \mathcal{K}^+]$  whenever  $[\mathcal{M}^-, \mathcal{M}^+] \subseteq [\mathcal{K}^-, \mathcal{K}^+]$  and  $[\mathcal{K}^-, \mathcal{K}^+]$  is an  $\mathcal{I}\mathcal{V}$ -open set.
2. The set  $[\mathcal{M}^-, \mathcal{M}^+]$  is called an *interval-valued  $\Lambda$ - $g$ -closed* (briefly,  $\mathcal{I}\mathcal{V}\Lambda$ - $g$ -closed) if  $\mathcal{I}\mathcal{V}\lambda\mathcal{C}([\mathcal{M}^-, \mathcal{M}^+]) \subseteq [\mathcal{K}^-, \mathcal{K}^+]$  whenever  $[\mathcal{M}^-, \mathcal{M}^+] \subseteq [\mathcal{K}^-, \mathcal{K}^+]$  and  $[\mathcal{K}^-, \mathcal{K}^+]$  is an  $\mathcal{I}\mathcal{V}\lambda$ -open set.

The complements of these sets are called the corresponding  $_g\Lambda$ -open sets and  $\Lambda$ - $g$ -open sets, respectively.

**Example 3.7.** Referring to Example 3.1, Consider the interval-valued set

$$[\mathcal{M}^-, \mathcal{M}^+] = [\{a\}, \{a, b, c\}]$$

$$[\mathcal{S}^-, \mathcal{S}^+] = [\{b\}, \{b, c\}]$$

Clearly  $[\mathcal{S}^-, \mathcal{S}^+]$  is  $\mathcal{I}\mathcal{V}\Lambda$ - $g$ -closed.

Further,  $[\mathcal{M}^-, \mathcal{M}^+]$  is also an  $\mathcal{I}\mathcal{V}_g\Lambda$ -closed.

Thus this example shows that a set can simultaneously be  $\mathcal{I}\mathcal{V}\Lambda$ - $g$ -closed and  $\mathcal{I}\mathcal{V}_g\Lambda$ -closed in the space  $\mathcal{T}^X$ .

**Remark 3.4.** For a subset of a space  $\mathcal{T}^X$ , we have the following implications:

$$\begin{array}{ccc}
 \mathcal{IV}\text{-closed} & \rightarrow & \mathcal{IV}\lambda\text{-closed} \\
 \downarrow & & \downarrow \\
 \mathcal{IV}\Lambda_g\text{-closed} & \rightarrow & \mathcal{IV}\Lambda\text{-}g\text{-closed} \\
 \downarrow & & \downarrow \\
 \mathcal{IV}g\text{-closed} & \rightarrow & \mathcal{IV}_g\Lambda\text{-closed}
 \end{array}$$

None of the above implications is reversible.

**Theorem 3.13.** *In a space  $\mathcal{T}^X$ ,  $[\mathcal{M}^-, \mathcal{M}^+]$  is an  $\mathcal{IV}\Lambda_g$ -closed  $\iff \mathcal{IV}\lambda C(\{p\}, \{p\}) \cap [\mathcal{M}^-, \mathcal{M}^+] \neq \tilde{\emptyset}$  for every  $(\{p\}, \{p\}) \in \mathcal{IV}C([\mathcal{M}^-, \mathcal{M}^+])$ .*

*Proof.*

Necessity. Suppose that  $\mathcal{IV}\lambda C(\{p\}, \{p\}) \cap [\mathcal{M}^-, \mathcal{M}^+] = \tilde{\emptyset}$  for some  $(\{p\}, \{p\}) \in \mathcal{IV}C([\mathcal{M}^-, \mathcal{M}^+])$ . Then  $\tilde{X} - \mathcal{IV}\lambda C(\{p\}, \{p\})$  is an  $\mathcal{IV}\lambda$ -open set containing  $[\mathcal{M}^-, \mathcal{M}^+]$ .

Furthermore,  $(\{p\}, \{p\}) \in \mathcal{IV}C([\mathcal{M}^-, \mathcal{M}^+]) - (\tilde{X} - \mathcal{IV}\lambda C(\{p\}, \{p\}))$  and hence  $\mathcal{IV}C([\mathcal{M}^-, \mathcal{M}^+]) \not\subseteq \tilde{X} - \mathcal{IV}\lambda C(\{p\}, \{p\})$ . This shows that  $[\mathcal{M}^-, \mathcal{M}^+]$  is not  $\mathcal{IV}\Lambda_g$ -closed.

Sufficiency. Suppose that  $[\mathcal{M}^-, \mathcal{M}^+]$  is not  $\mathcal{IV}\Lambda_g$ -closed. There exist a  $\mathcal{IV}\lambda$ -open set  $[\mathcal{K}^-, \mathcal{K}^+]$  containing  $[\mathcal{M}^-, \mathcal{M}^+]$  such that  $\mathcal{IV}C([\mathcal{M}^-, \mathcal{M}^+]) - [\mathcal{K}^-, \mathcal{K}^+] \neq \tilde{\emptyset}$ . There exist  $(\{p\}, \{p\}) \in \mathcal{IV}C([\mathcal{M}^-, \mathcal{M}^+])$  such that  $(\{p\}, \{p\}) \notin [\mathcal{K}^-, \mathcal{K}^+]$ , hence  $\mathcal{IV}\lambda C(\{p\}, \{p\}) \cap [\mathcal{K}^-, \mathcal{K}^+] = \tilde{\emptyset}$ . Therefore,  $\mathcal{IV}\lambda C(\{p\}, \{p\}) \cap [\mathcal{M}^-, \mathcal{M}^+] = \tilde{\emptyset}$  for some  $(\{p\}, \{p\}) \in \mathcal{IV}C([\mathcal{M}^-, \mathcal{M}^+])$ .  $\square$

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