



A note on Λ - g^* -closed sets in interval-valued topological spaces

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Abstract: This paper introduces the notions of Λ - g^* -closed sets and Λ - g^* -open sets in interval-valued topological spaces. Basic properties and several characterizations of these sets are discussed. The connections between interval-valued Λ - g^* -closed sets and other types of interval-valued generalized closed sets are also examined. In addition, a few important results concerning their behavior and structural properties are obtained. These newly defined sets extend the study of generalized closedness in interval-valued topology and provide further scope for investigation in this area.

Key words: interval-valued Λ - g^* -open, interval-valued λ -closed, interval-valued Λ -set and interval-valued Λ_g -closed

1. Introduction

The concept of interval sets was first introduced by Yao [8] as an efficient tool for representing uncertain and imprecise information. Subsequently, Kim *et al.* [5] developed this idea under the name interval-valued sets, considering them as a generalization of classical sets and a particular form of interval-valued fuzzy sets. They studied several kinds of interval-valued neighborhoods and classified them into distinct categories, along with investigating interval-valued closure and interior operators. More recently, Cheong *et al.* [1] introduced interval-valued relations and examined their properties from a categorical perspective, which further extended the applicability of interval-valued structures in both theoretical mathematics and computational sciences.

In 2026, I. Rajasekaran *et al.* introduced the concepts of Λ -sets and Λ_g -sets in interval-valued topological spaces [6, 7]. Later, the notions of λ -closed sets and λ -open sets were developed as generalized forms in interval-valued topology. The study of generalized closed sets has become an important area of research due to its significant role in extending classical topological concepts and investigating weaker forms of continuity and separation axioms.

Motivated by these developments, in this paper we introduce the concepts of Λ - g^* -closed sets and Λ - g^* -open sets in interval-valued topological spaces. Several basic properties and characterizations of these newly defined sets are obtained. Moreover, the relationships between interval-valued Λ - g^* -closed sets and other existing classes of interval-valued generalized closed sets are discussed. The results presented in this paper contribute to the further development of generalized structures in interval-valued topology and may provide a foundation for future investigations in this field.

2. Preliminaries

Throughout this paper (X, τ) (or X) represent interval-valued topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset $[\mathcal{M}^-, \mathcal{M}^+]$ of a space (X, τ) , $\mathcal{IVC}([\mathcal{M}^-, \mathcal{M}^+])$ and $\mathcal{IVI}([\mathcal{M}^-, \mathcal{M}^+])$ denote the interval-valued closure of $[\mathcal{M}^-, \mathcal{M}^+]$ and the interval-valued interior of $[\mathcal{M}^-, \mathcal{M}^+]$ respectively. We recall the following definitions which are useful in the sequel.

Definition 2.1. [8] Let X be a non-empty set. Then, the form $[\mathcal{M}^-, \mathcal{M}^+] = \{[S \subset X : \mathcal{M}^- \subset S \subset \mathcal{M}^+]\}$ is called an interval-valued set (briefly, $\mathcal{IVS}$) in X , if $\mathcal{M}^-, \mathcal{M}^+ \subset X$ and $\mathcal{M}^- \subset \mathcal{M}^+$.

In this case, $\mathcal{M}^-$ [resp. $\mathcal{M}^+$] represent the set of minimum [resp. maximum] memberships of elements of X to $[\mathcal{M}^-, \mathcal{M}^+]$.

Here, $[\emptyset, \emptyset]$ [resp. $[X, X]$] is called the interval-valued empty [resp. whole] set in X and denoted by $\tilde{\emptyset}$ [resp. $\tilde{X}$]. The set of all $\mathcal{IVS}$ s in X is denoted by $\mathcal{IVS}(X)$.

Definition 2.2. [5] A non-empty set X with $[\mathcal{M}^-, \mathcal{M}^+]$ and $[\mathcal{S}^-, \mathcal{S}^+]$ are $\mathcal{IVS}$ respectively. Then

1. $[\mathcal{M}^-, \mathcal{M}^+] \subset [\mathcal{S}^-, \mathcal{S}^+] \iff \mathcal{M}^- \subset \mathcal{S}^- \text{ and } \mathcal{M}^+ \subset \mathcal{S}^+$
2. $[\mathcal{M}^-, \mathcal{M}^+] = [\mathcal{S}^-, \mathcal{S}^+] \iff [\mathcal{M}^-, \mathcal{M}^+] \subset [\mathcal{S}^-, \mathcal{S}^+] \text{ and } [\mathcal{S}^-, \mathcal{S}^+] \subset [\mathcal{M}^-, \mathcal{M}^+]$
3. $([\mathcal{M}^-, \mathcal{M}^+])^c = [(\mathcal{M}^+)^c, (\mathcal{M}^-)^c]$
4. $[\mathcal{M}^-, \mathcal{M}^+] \cup [\mathcal{S}^-, \mathcal{S}^+] = [\mathcal{M}^- \cup \mathcal{S}^-, \mathcal{M}^+ \cup \mathcal{S}^+]$
5. $[\mathcal{M}^-, \mathcal{M}^+] \cap [\mathcal{S}^-, \mathcal{S}^+] = [\mathcal{M}^- \cap \mathcal{S}^-, \mathcal{M}^+ \cap \mathcal{S}^+]$

Definition 2.3. [5] Let X be a non-empty set and let τ be a non-empty family of $\mathcal{IVS}$ s on X . Then τ is called an interval-valued topology (briefly, $\mathcal{IVT}$) on X if it satisfies the following axioms:

1. $\tilde{\emptyset}, \tilde{X} \in \tau$,
2. $[\mathcal{M}^-, \mathcal{M}^+] \cap [\mathcal{S}^-, \mathcal{S}^+] \in \tau$ for any $[\mathcal{M}^-, \mathcal{M}^+], [\mathcal{S}^-, \mathcal{S}^+] \in \tau$.
3. $\bigcup_{j \in J} \mathcal{S}_j \in \tau$ for any family $(\mathcal{S}_j)_{j \in J}$ of members of τ .

In this case, the $\mathcal{T}^X$ is called an interval-valued topological space (briefly, $\mathcal{IVTS}$) and each member of τ is called an interval-valued open set (briefly, $\mathcal{IVOS}$) in X . A $\mathcal{IVS}$ of $[\mathcal{M}^-, \mathcal{M}^+]$ is called an interval-valued closed set (briefly, $\mathcal{IVCS}$) in X .

Interval valued topological spaces by is denoted by $\mathcal{IVTS}$ or $\mathcal{T}^X$.

Definition 2.4. [5] If an interval-valued topological space $\mathcal{T}^X$ with respect to X and if $[\mathcal{M}^-, \mathcal{M}^+] \subseteq \tilde{X}$, then

1. the interval-valued interior of $[\mathcal{M}^-, \mathcal{M}^+]$ is defined as the union of all an interval-valued open subsets of $[\mathcal{M}^-, \mathcal{M}^+]$ and it is denoted by $\mathcal{IVI}([\mathcal{M}^-, \mathcal{M}^+])$.

That is, $\mathcal{IVI}([\mathcal{M}^-, \mathcal{M}^+])$ is the largest an interval-valued open subset of $[\mathcal{M}^-, \mathcal{M}^+]$.

2. the interval-valued closure of $[\mathcal{M}^-, \mathcal{M}^+]$ is defined as the intersection of all an interval-valued closed sets containing $[\mathcal{M}^-, \mathcal{M}^+]$ and it is denoted by $\mathcal{IVC}([\mathcal{M}^-, \mathcal{M}^+])$.

That is, $\mathcal{IVC}([\mathcal{M}^-, \mathcal{M}^+])$ is the smallest an interval-valued closed set containing $[\mathcal{M}^-, \mathcal{M}^+]$.

Definition 2.5. A subset $[\mathcal{M}^-, \mathcal{M}^+]$ of an interval-valued topological space $\mathcal{T}^X$ is called an

1. interval-valued pre-open set (briefly, $\mathcal{IV}p$ -open) [4] if

$$[\mathcal{M}^-, \mathcal{M}^+] \subseteq \mathcal{IVI}(\mathcal{IVC}([\mathcal{M}^-, \mathcal{M}^+])).$$

2. interval-valued semi-open set (briefly, $\mathcal{IV}s$ -open) [3] if

$$[\mathcal{M}^-, \mathcal{M}^+] \subseteq \mathcal{IVC}(\mathcal{IVI}([\mathcal{M}^-, \mathcal{M}^+])).$$

Definition 2.6. A subset $[\mathcal{M}^-, \mathcal{M}^+]$ of an interval-valued topological space $\mathcal{T}^X$ is called an

1. interval-valued generalized closed (briefly, $\mathcal{IV}g$ -closed) [2] if

$$\mathcal{IVC}([\mathcal{M}^-, \mathcal{M}^+]) \subseteq [\mathcal{K}^-, \mathcal{K}^+], \text{ whenever } [\mathcal{M}^-, \mathcal{M}^+] \subseteq [\mathcal{K}^-, \mathcal{K}^+] \text{ and } [\mathcal{K}^-, \mathcal{K}^+] \text{ is } \mathcal{IV}\text{-open.}$$

2. interval-valued generalized semi closed (briefly, $\mathcal{IV}gs$ -closed) [3] if

$$\mathcal{IVsC}([\mathcal{M}^-, \mathcal{M}^+]) \subseteq [\mathcal{K}^-, \mathcal{K}^+], \text{ whenever } [\mathcal{M}^-, \mathcal{M}^+] \subseteq [\mathcal{K}^-, \mathcal{K}^+] \text{ and } [\mathcal{K}^-, \mathcal{K}^+] \text{ is } \mathcal{IV}\text{-open.}$$

3. interval-valued generalized pre closed (briefly, $\mathcal{IVgp}$ -closed) [3] if

$$\mathcal{IVpC}([\mathcal{M}^-, \mathcal{M}^+]) \subseteq [\mathcal{K}^-, \mathcal{K}^+], \text{ whenever } [\mathcal{M}^-, \mathcal{M}^+] \subseteq [\mathcal{K}^-, \mathcal{K}^+] \text{ and } [\mathcal{K}^-, \mathcal{K}^+] \text{ is } \mathcal{IV}\text{-open.}$$

Definition 2.7. [6] A subset $[\mathcal{M}^-, \mathcal{M}^+]$ of an interval-valued topological space $\mathcal{T}^X$ is called an

1. interval-valued- $\ker([\mathcal{M}^-, \mathcal{M}^+]) = \bigcap \{[\mathcal{S}^-, \mathcal{S}^+] : [\mathcal{M}^-, \mathcal{M}^+] \subseteq [\mathcal{S}^-, \mathcal{S}^+], [\mathcal{S}^-, \mathcal{S}^+] \in \tau\}$ is called the interval-valued kernel of $[\mathcal{M}^-, \mathcal{M}^+]$ and is denoted by $\mathcal{IV}\text{-ker}([\mathcal{M}^-, \mathcal{M}^+])$.
2. interval-valued Λ -set (briefly, $\mathcal{IV}\Lambda$ -set) if $[\mathcal{M}^-, \mathcal{M}^+] = \mathcal{IV}\text{-ker}([\mathcal{M}^-, \mathcal{M}^+])$.
3. interval-valued λ -closed (briefly, $\mathcal{IV}\lambda$ -closed) if $[\mathcal{M}^-, \mathcal{M}^+] = [\mathcal{S}^-, \mathcal{S}^+] \cap [\mathcal{R}^-, \mathcal{R}^+]$ where $[\mathcal{S}^-, \mathcal{S}^+]$ is $\mathcal{IV}\Lambda$ -set and $[\mathcal{R}^-, \mathcal{R}^+]$ is $\mathcal{IV}$ -closed.

Definition 2.8. [7] A subset $[\mathcal{M}^-, \mathcal{M}^+]$ of an interval-valued topological space $\mathcal{T}^X$ is called an interval-valued Λ_g -closed (briefly, $\mathcal{IV}\Lambda_g$ -closed) if $\mathcal{IVC}([\mathcal{M}^-, \mathcal{M}^+]) \subseteq [\mathcal{K}^-, \mathcal{K}^+]$, whenever $[\mathcal{M}^-, \mathcal{M}^+] \subseteq [\mathcal{K}^-, \mathcal{K}^+]$ and $[\mathcal{K}^-, \mathcal{K}^+]$ is $\mathcal{IV}\lambda$ -open.

The complement of $\mathcal{IV}\Lambda_g$ -open if $[\mathcal{K}^-, \mathcal{K}^+]^c = [X^-, X^+] - [\mathcal{K}^-, \mathcal{K}^+]$ is $\mathcal{IV}\Lambda_g$ -closed.

Lemma 2.1. [6] For a subset $[\mathcal{M}^-, \mathcal{M}^+]$ of an interval-valued topological space $\mathcal{T}^X$, the following conditions are equivalent:

1. $[\mathcal{M}^-, \mathcal{M}^+]$ is $\mathcal{IV}\lambda$ -closed set.
2. $[\mathcal{M}^-, \mathcal{M}^+] = [\mathcal{S}^-, \mathcal{S}^+] \cap \mathcal{IVC}([\mathcal{M}^-, \mathcal{M}^+])$, where $[\mathcal{S}^-, \mathcal{S}^+]$ is $\mathcal{IV}\Lambda$ -set.
3. $[\mathcal{M}^-, \mathcal{M}^+] = \mathcal{IV}\text{-ker}([\mathcal{M}^-, \mathcal{M}^+]) \cap \mathcal{IVC}([\mathcal{M}^-, \mathcal{M}^+])$.

Lemma 2.2. [6] Let $\mathcal{T}^X$ be an interval-valued topological spaces,

1. Every $\mathcal{IV}\Lambda$ -set is $\mathcal{IV}\lambda$ -closed.
2. Every $\mathcal{IV}$ -open is $\mathcal{IV}\lambda$ -closed.
3. Every $\mathcal{IV}$ -closed is $\mathcal{IV}\lambda$ -closed.

3. An interval-valued Λ -generalized closed sets

Definition 3.1. A subset $[\mathcal{M}^-, \mathcal{M}^+]$ of an interval-valued topological space $\mathcal{T}^X$ is called an

1. interval-valued g - Λ -closed (briefly, g - $\mathcal{IV}\Lambda$ -closed) if $\mathcal{IV}\lambda C([\mathcal{M}^-, \mathcal{M}^+]) \subseteq [\mathcal{K}^-, \mathcal{K}^+]$ whenever $[\mathcal{M}^-, \mathcal{M}^+] \subseteq [\mathcal{K}^-, \mathcal{K}^+]$ and $[\mathcal{K}^-, \mathcal{K}^+]$ is $\mathcal{IV}$ -open.
2. interval-valued Λ - g -closed (briefly, $\mathcal{IV}\Lambda$ - g -closed) if $\mathcal{IV}\lambda C([\mathcal{M}^-, \mathcal{M}^+]) \subseteq [\mathcal{K}^-, \mathcal{K}^+]$, whenever $[\mathcal{M}^-, \mathcal{M}^+] \subseteq [\mathcal{K}^-, \mathcal{K}^+]$ and $[\mathcal{K}^-, \mathcal{K}^+]$ is $\mathcal{IV}\lambda$ -open.
3. interval-valued Λ - g^* -closed (briefly, $\mathcal{IV}\Lambda$ - g^* -closed) if $\mathcal{IV}\lambda C([\mathcal{M}^-, \mathcal{M}^+]) \subseteq [\mathcal{K}^-, \mathcal{K}^+]$, whenever $[\mathcal{M}^-, \mathcal{M}^+] \subseteq [\mathcal{K}^-, \mathcal{K}^+]$ and $[\mathcal{K}^-, \mathcal{K}^+]$ is $\mathcal{IV}g$ -open.

Theorem 3.1. In an interval-valued topological space $\mathcal{T}^X$, every $\mathcal{IV}\lambda$ -closed set is $\mathcal{IV}\Lambda$ - g^* -closed.

Proof.

Let $[\mathcal{M}^-, \mathcal{M}^+] \subseteq [\mathcal{K}^-, \mathcal{K}^+]$, where $[\mathcal{K}^-, \mathcal{K}^+]$ is $\mathcal{IV}g$ -open. Since $[\mathcal{M}^-, \mathcal{M}^+]$ is $\mathcal{IV}\lambda$ -closed, we have $\mathcal{IV}\lambda C([\mathcal{M}^-, \mathcal{M}^+]) = [\mathcal{M}^-, \mathcal{M}^+] \subseteq [\mathcal{K}^-, \mathcal{K}^+]$. Hence, $\mathcal{IV}\lambda C([\mathcal{M}^-, \mathcal{M}^+]) \subseteq [\mathcal{K}^-, \mathcal{K}^+]$. Therefore, $[\mathcal{M}^-, \mathcal{M}^+]$ is $\mathcal{IV}\Lambda$ - g^* -closed. $\square$

Remark 3.1. The converse of the statement in Theorem 3.1 is not necessarily true, as seen from the following example.

Example 3.1. Let $X = \{a, b, c\}$ and let

$$\mathcal{T}^X = \{\tilde{\emptyset}, [\{a, b\}, \{a, b\}], \tilde{X}\}$$

be an interval-valued topology.

Consider the set $[\mathcal{M}^-, \mathcal{M}^+] = [\{a\}, \{a, c\}]$.

The $\mathcal{IV}$ -closed sets in $\mathcal{T}^X$ are $\tilde{\emptyset}$, $[\{c\}, \{c\}]$, and $\tilde{X}$. The $\mathcal{IV}g$ -closed sets include $[\{c\}, \{c\}]$, whose complement $[\{a, b\}, \{a, b\}]$ is $\mathcal{IV}g$ -open; and the $\mathcal{IV}$ -closed sets $\tilde{\emptyset}$ and $\tilde{X}$. One can verify that the only $\mathcal{IV}g$ -open set containing $[\mathcal{M}^-, \mathcal{M}^+] = [\{a\}, \{a, c\}]$ is $\tilde{X}$, since $[\{a, b\}, \{a, b\}] \not\supseteq [\{a\}, \{a, c\}]$ (as $\{a, b\} \not\supseteq \{a, c\}$).

Hence $\mathcal{IV}\lambda C([\mathcal{M}^-, \mathcal{M}^+]) \subseteq \tilde{X}$. Therefore $[\mathcal{M}^-, \mathcal{M}^+]$ satisfies the definition of an $\mathcal{IV}\Lambda$ - g^* -closed set. Now, $\mathcal{IV}\ker([\mathcal{M}^-, \mathcal{M}^+]) = \tilde{X}$ and $\mathcal{IV}C([\mathcal{M}^-, \mathcal{M}^+]) = \tilde{X}$. Therefore, $\mathcal{IV}\ker([\mathcal{M}^-, \mathcal{M}^+]) \cap \mathcal{IV}C([\mathcal{M}^-, \mathcal{M}^+]) = \tilde{X} \neq [\mathcal{M}^-, \mathcal{M}^+]$. By Lemma 2.1, $[\mathcal{M}^-, \mathcal{M}^+]$ is not $\mathcal{IV}\lambda$ -closed. Thus $[\{a\}, \{a, c\}]$ is $\mathcal{IV}\Lambda$ - g^* -closed but not $\mathcal{IV}\lambda$ -closed.

Theorem 3.2. In an interval-valued topological space $\mathcal{T}^X$, every $\mathcal{IV}$ -closed set is $\mathcal{IV}\Lambda$ - g^* -closed.

Proof.

By Lemma 2.2, every $\mathcal{IV}$ -closed set is $\mathcal{IV}\lambda$ -closed. Further, by Theorem 3.1, every $\mathcal{IV}\lambda$ -closed set is $\mathcal{IV}\Lambda$ - g^* -closed. Therefore, every $\mathcal{IV}$ -closed set is $\mathcal{IV}\Lambda$ - g^* -closed. $\square$

Remark 3.2. The converse of the statement in Theorem 3.2 is not necessarily true, as seen from the following example.

Example 3.2. In Example 3.1, the set $[\{a, b\}, \{a, b\}]$ is $\mathcal{IV}\Lambda$ - g^* -closed but not $\mathcal{IV}$ -closed.

The only $\mathcal{IV}g$ -open set containing $[\{a, b\}, \{a, b\}]$ is $\tilde{X}$, since $[\{a, b\}, \{a, b\}]$ is the only non-trivial $\mathcal{IV}g$ -open set in $\mathcal{T}^X$, and $[\{a, b\}, \{a, b\}] \not\subseteq [\{a, b\}, \{a, b\}]$ is vacuously satisfied but we verify: $[\{a, b\}, \{a, b\}] \subseteq [\{a, b\}, \{a, b\}]$ with $[\{a, b\}, \{a, b\}]$ $\mathcal{IV}g$ -open. We must check $\mathcal{IV}\lambda C([\{a, b\}, \{a, b\}]) \subseteq [\{a, b\}, \{a, b\}]$.

Since $[\{a, b\}, \{a, b\}]$ is $\mathcal{IV}$ -open,

by Lemma 2.2(2) it is $\mathcal{IV}\lambda$ -closed, so $\mathcal{IV}\lambda C([\{a, b\}, \{a, b\}]) = [\{a, b\}, \{a, b\}] \subseteq [\{a, b\}, \{a, b\}]$. Hence $[\{a, b\}, \{a, b\}]$ is $\mathcal{IV}\Lambda$ - g^* -closed. Further, the complement $\tilde{X} - [\{a, b\}, \{a, b\}] = [\{c\}, \{c\}]$ is not $\mathcal{IV}$ -open in $\mathcal{T}^X$. Therefore, $[\{a, b\}, \{a, b\}]$ is not $\mathcal{IV}$ -closed.

Theorem 3.3. In an interval-valued topological space $\mathcal{T}^X$, every $\mathcal{IV}$ -open set is $\mathcal{IV}\Lambda$ - g^* -closed.

Proof.

Let $[\mathcal{M}^-, \mathcal{M}^+]$ be a $\mathcal{IV}$ -open set. Since every $\mathcal{IV}$ -open set is $\mathcal{IV}\lambda$ -closed, by Theorem 3.1, $[\mathcal{M}^-, \mathcal{M}^+]$ is $\mathcal{IV}\Lambda$ - g^* -closed. $\square$

Remark 3.3. The converse of the statement in Theorem 3.3 is not necessarily true, as seen from the following example.

Example 3.3. In Example 3.1, the set $[\{a\}, \{a, c\}]$ is $\mathcal{IV}\Lambda$ - g^* -closed but not $\mathcal{IV}$ -open. The only $\mathcal{IV}g$ -open set containing $[\{a\}, \{a, c\}]$ is $\tilde{X}$ (as verified in Example 3.1), so $\mathcal{IV}\lambda C([\{a\}, \{a, c\}]) \subseteq \tilde{X}$. Hence $[\{a\}, \{a, c\}]$ is $\mathcal{IV}\Lambda$ - g^* -closed.

Further, $[\{a\}, \{a, c\}] \notin \mathcal{T}^X$. Therefore, $[\{a\}, \{a, c\}]$ is not $\mathcal{IV}$ -open.

Theorem 3.4. Let $[\mathcal{M}^-, \mathcal{M}^+]$ be an $\mathcal{IV}g$ -open set in an interval-valued topological space $\mathcal{T}^X$. If $[\mathcal{M}^-, \mathcal{M}^+]$ is $\mathcal{IV}\Lambda$ - g^* -closed, then $[\mathcal{M}^-, \mathcal{M}^+]$ is $\mathcal{IV}\lambda$ -closed.

Proof.

Suppose that $[\mathcal{M}^-, \mathcal{M}^+]$ is both $\mathcal{IV}g$ -open and $\mathcal{IV}\Lambda$ - g^* -closed. Since $[\mathcal{M}^-, \mathcal{M}^+] \subseteq [\mathcal{M}^-, \mathcal{M}^+]$ and $[\mathcal{M}^-, \mathcal{M}^+]$ is $\mathcal{IV}g$ -open, by the definition of $\mathcal{IV}\Lambda$ - g^* -closed sets, we obtain $\mathcal{IV}\lambda C([\mathcal{M}^-, \mathcal{M}^+]) \subseteq [\mathcal{M}^-, \mathcal{M}^+]$.

But always $[\mathcal{M}^-, \mathcal{M}^+] \subseteq \mathcal{IV}\lambda C([\mathcal{M}^-, \mathcal{M}^+])$. Therefore, $\mathcal{IV}\lambda C([\mathcal{M}^-, \mathcal{M}^+]) = [\mathcal{M}^-, \mathcal{M}^+]$. Hence $[\mathcal{M}^-, \mathcal{M}^+]$ is $\mathcal{IV}\lambda$ -closed. $\square$

Theorem 3.5. In a space $\mathcal{T}^X$, every $\mathcal{IV}\Lambda$ - g^* -closed set is g - $\mathcal{IV}\Lambda$ -closed.

Proof.

Let $[\mathcal{M}^-, \mathcal{M}^+]$ be an $\mathcal{IV}\Lambda$ - g^* -closed set and suppose that $[\mathcal{M}^-, \mathcal{M}^+] \subseteq [\mathcal{K}^-, \mathcal{K}^+]$, where $[\mathcal{K}^-, \mathcal{K}^+]$ is $\mathcal{IV}$ -open.

Since every $\mathcal{IV}$ -open set is $\mathcal{IV}g$ -open and $[\mathcal{M}^-, \mathcal{M}^+]$ is $\mathcal{IV}\Lambda$ - g^* -closed, we have $\mathcal{IV}\lambda C([\mathcal{M}^-, \mathcal{M}^+]) \subseteq [\mathcal{K}^-, \mathcal{K}^+]$. Hence, by definition, $[\mathcal{M}^-, \mathcal{M}^+]$ is g - $\mathcal{IV}\Lambda$ -closed. $\square$

Remark 3.4. The converse of the statement in Theorem 3.5 is not necessarily true, as seen from the following example.

Example 3.4. Let $X = \{a, b, c, d\}$ and let

$$\mathcal{T}^X = \{\tilde{\emptyset}, [\emptyset, \{d\}], [\{b\}, \{a, b\}], [\{b\}, \{a, b, d\}], \tilde{X}\}$$

be an interval-valued topology.

We claim that the set $[\{a\}, \{a\}]$ is $g\text{-}\mathcal{IV}\Lambda$ -closed but not $\mathcal{IV}\Lambda$ - g^* -closed. The $\mathcal{IV}$ -closed sets in $\mathcal{T}^X$ are: $\tilde{X}$, $[\{a, b, c\}, X]$, $[\{c, d\}, \{a, c, d\}]$, $[\{c\}, \{a, c, d\}]$, and $\tilde{\emptyset}$. For $[\{a\}, \{a\}]$, one computes: $\mathcal{IV}\text{-ker}([\{a\}, \{a\}]) = \tilde{X}$ (since no $\mathcal{IV}$ -open set other than $\tilde{X}$ contains $[\{a\}, \{a\}]$), and $\mathcal{IVC}([\{a\}, \{a\}]) = [\{a, b, c\}, X]$. Hence $\mathcal{IV}\lambda C([\{a\}, \{a\}]) = \mathcal{IV}\text{-ker}([\{a\}, \{a\}]) \cap \mathcal{IVC}([\{a\}, \{a\}]) = \tilde{X} \cap [\{a, b, c\}, X] = [\{a, b, c\}, X]$. Since the only $\mathcal{IV}$ -open set containing $[\{a\}, \{a\}]$ is $\tilde{X}$, and $\mathcal{IV}\lambda C([\{a\}, \{a\}]) = [\{a, b, c\}, X] \subseteq \tilde{X}$, we conclude that $[\{a\}, \{a\}]$ is $g\text{-}\mathcal{IV}\Lambda$ -closed.

However, one can verify that $[\{a, b\}, \{a, b\}]$ is $\mathcal{IV}g$ -open (since its complement $[\{c, d\}, \{c, d\}]$ is $\mathcal{IV}g$ -closed) and $[\{a\}, \{a\}] \subseteq [\{a, b\}, \{a, b\}]$. But $\mathcal{IV}\lambda C([\{a\}, \{a\}]) = [\{a, b, c\}, X] \not\subseteq [\{a, b\}, \{a, b\}]$ (since $\{a, b, c\} \not\subseteq \{a, b\}$).

Therefore, $[\{a\}, \{a\}]$ is not $\mathcal{IV}\Lambda$ - g^* -closed.

Theorem 3.6. In an interval-valued topological space $\mathcal{T}^X$, every $\mathcal{IV}\Lambda$ - g^* -closed set is $\mathcal{IV}\Lambda$ - g -closed.

Proof.

Let $[\mathcal{M}^-, \mathcal{M}^+]$ be a $\mathcal{IV}\Lambda$ - g^* -closed set and suppose that $[\mathcal{M}^-, \mathcal{M}^+] \subseteq [\mathcal{K}^-, \mathcal{K}^+]$, where $[\mathcal{K}^-, \mathcal{K}^+]$ is $\mathcal{IV}\lambda$ -open. Since every $\mathcal{IV}\lambda$ -open set is $\mathcal{IV}g$ -open, and $[\mathcal{M}^-, \mathcal{M}^+]$ is $\mathcal{IV}\Lambda$ - g^* -closed, we have $\mathcal{IV}\lambda C([\mathcal{M}^-, \mathcal{M}^+]) \subseteq [\mathcal{K}^-, \mathcal{K}^+]$.

Hence, by Definition, $[\mathcal{M}^-, \mathcal{M}^+]$ is $\mathcal{IV}\Lambda$ - g -closed. $\square$

Remark 3.5. The converse of the statement in Theorem 3.6 is not necessarily true, as seen from the following example.

Example 3.5. In Example 3.4, the set $[\{a\}, \{a\}]$ is $\mathcal{IV}\Lambda$ - g -closed but not $\mathcal{IV}\Lambda$ - g^* -closed.

One can check that

every $\mathcal{IV}\lambda$ -open set containing $[\{a\}, \{a\}]$ also contains $\mathcal{IV}\lambda C([\{a\}, \{a\}]) = [\{a, b, c\}, X]$, so $\mathcal{IV}\lambda C([\{a\}, \{a\}]) \subseteq [\mathcal{K}^-, \mathcal{K}^+]$ for every $\mathcal{IV}\lambda$ -open set $[\mathcal{K}^-, \mathcal{K}^+]$ containing $[\{a\}, \{a\}]$. Hence $[\{a\}, \{a\}]$ is $\mathcal{IV}\Lambda$ - g -closed.

However, as shown in Example 3.4, $[\{a, b\}, \{a, b\}]$ is $\mathcal{IV}g$ -open and $[\{a\}, \{a\}] \subseteq [\{a, b\}, \{a, b\}]$, but $\mathcal{IV}\lambda C([\{a\}, \{a\}]) = [\{a, b, c\}, X] \not\subseteq [\{a, b\}, \{a, b\}]$. Therefore, $[\{a\}, \{a\}]$ is not $\mathcal{IV}\Lambda$ - g^* -closed.

Theorem 3.7. In an interval-valued topological space $\mathcal{T}^X$, every $\mathcal{IV}g$ -closed set is $\mathcal{IV}\Lambda$ - g^* -closed.

Proof.

Let $[\mathcal{M}^-, \mathcal{M}^+]$ be $\mathcal{IV}g$ -closed and suppose $[\mathcal{M}^-, \mathcal{M}^+] \subseteq [\mathcal{K}^-, \mathcal{K}^+]$, where $[\mathcal{K}^-, \mathcal{K}^+]$ is $\mathcal{IV}g$ -open. Since every $\mathcal{IV}g$ -open set is $\mathcal{IV}$ -open and $[\mathcal{M}^-, \mathcal{M}^+]$ is $\mathcal{IV}g$ -closed, we have $\mathcal{IVC}([\mathcal{M}^-, \mathcal{M}^+]) \subseteq [\mathcal{K}^-, \mathcal{K}^+]$. Also, $\mathcal{IV}\lambda C([\mathcal{M}^-, \mathcal{M}^+]) \subseteq \mathcal{IVC}([\mathcal{M}^-, \mathcal{M}^+])$. Hence, $\mathcal{IV}\lambda C([\mathcal{M}^-, \mathcal{M}^+]) \subseteq [\mathcal{K}^-, \mathcal{K}^+]$. Therefore, $[\mathcal{M}^-, \mathcal{M}^+]$ is $\mathcal{IV}\Lambda$ - g^* -closed. $\square$

Remark 3.6. The converse of the statement in Theorem 3.7 is not necessarily true, as seen from the following example.

Example 3.6. In Example 3.1, the set $[\{a\}, \{a, b\}]$ is $\mathcal{TV}\Lambda$ - g^* -closed but not $\mathcal{TV}g$ -closed.

The $\mathcal{TV}g$ -open sets containing $[\{a\}, \{a, b\}]$ are $[\{a, b\}, \{a, b\}]$ and $\tilde{X}$. Since $[\{a, b\}, \{a, b\}]$ is $\mathcal{TV}$ -open, by Lemma 2.2(2) it is $\mathcal{TV}\lambda$ -closed, and $[\{a\}, \{a, b\}] \subseteq [\{a, b\}, \{a, b\}]$, so $\mathcal{TV}\lambda C([\{a\}, \{a, b\}]) \subseteq [\{a, b\}, \{a, b\}] \subseteq \tilde{X}$. Hence $[\{a\}, \{a, b\}]$ is $\mathcal{TV}\Lambda$ - g^* -closed. However, $\mathcal{TV}C([\{a\}, \{a, b\}]) = \tilde{X}$, and $[\{a\}, \{a, b\}] \subseteq [\{a, b\}, \{a, b\}]$, where $[\{a, b\}, \{a, b\}]$ is $\mathcal{TV}$ -open. But $\tilde{X} \not\subseteq [\{a, b\}, \{a, b\}]$. Therefore, $[\{a\}, \{a, b\}]$ is not $\mathcal{TV}g$ -closed.

Remark 3.7. In an interval-valued topological space $\mathcal{T}^X$, every $\mathcal{TV}\Lambda_g$ -closed set is $\mathcal{TV}\Lambda$ - g^* -closed.

Example 3.7. In Example 3.1, the set $[\{a\}, \{a, b\}]$ is $\mathcal{TV}\Lambda$ - g^* -closed but not $\mathcal{TV}\Lambda_g$ -closed.

The $\mathcal{TV}g$ -open sets containing $[\{a\}, \{a, b\}]$ are $[\{a, b\}, \{a, b\}]$ and $\tilde{X}$.

Since $\mathcal{TV}\lambda C([\{a\}, \{a, b\}]) \subseteq [\{a, b\}, \{a, b\}] \subseteq \tilde{X}$, the set $[\{a\}, \{a, b\}]$ is $\mathcal{TV}\Lambda$ - g^* -closed. However, $\mathcal{TV}C([\{a\}, \{a, b\}]) = \tilde{X}$, and $[\{a\}, \{a, b\}] \subseteq [\{a, b\}, \{a, b\}]$, where $[\{a, b\}, \{a, b\}]$ is $\mathcal{TV}\lambda$ -open (since it is $\mathcal{TV}$ -open, hence $\mathcal{TV}\lambda$ -closed, and its complement $[\{c\}, \{c\}]$ is $\mathcal{TV}$ -closed hence $\mathcal{TV}\lambda$ -closed, so $[\{a, b\}, \{a, b\}]$ is $\mathcal{TV}\lambda$ -open). But $\tilde{X} \not\subseteq [\{a, b\}, \{a, b\}]$. Therefore, $[\{a\}, \{a, b\}]$ is not $\mathcal{TV}\Lambda_g$ -closed.

Remark 3.8. The concepts of $\mathcal{TV}\Lambda$ - g^* -closed and being $\mathcal{TV}gs$ -closed, $\mathcal{TV}gp$ -closed are independent.

Example 3.8. 1. Let $X = \{a, b, c\}$ with the an interval-valued topology

$$\mathcal{T}^X = \{\tilde{\emptyset}, [\{a\}, \{a, c\}], \tilde{X}\}.$$

Now, $[\{a\}, \{a, c\}]$ is $\mathcal{TV}\Lambda$ - g^* -closed since the only $\mathcal{TV}g$ -open set containing $[\{a\}, \{a, c\}]$ is $\tilde{X}$, and hence $\mathcal{TV}\lambda C([\{a\}, \{a, c\}]) \subseteq \tilde{X}$. However, $\mathcal{TV}pC([\{a\}, \{a, c\}]) = \tilde{X}$ and $\tilde{X} \not\subseteq [\{a\}, \{a, c\}]$. Therefore, $[\{a\}, \{a, c\}]$ is not $\mathcal{TV}gp$ -closed.

2. In Example 3.4, the set $[\emptyset, \{a\}]$ is $\mathcal{TV}gp$ -closed but not $\mathcal{TV}\Lambda$ - g^* -closed. Since $\mathcal{TV}pC([\emptyset, \{a\}]) \subseteq [\emptyset, \{a, b\}]$, whenever $[\emptyset, \{a\}] \subseteq [\emptyset, \{a, b\}]$ and $[\emptyset, \{a, b\}]$ is $\mathcal{TV}$ -open, the set $[\emptyset, \{a\}]$ is $\mathcal{TV}gp$ -closed. But $\mathcal{TV}\lambda C([\emptyset, \{a\}]) \not\subseteq [\emptyset, \{a, b\}]$, where $[\emptyset, \{a, b\}]$ is $\mathcal{TV}g$ -open. Hence $[\emptyset, \{a\}]$ is not $\mathcal{TV}\Lambda$ - g^* -closed.

Example 3.9. 1. In Example 3.1, the set $[\{a\}, \{a, b\}]$ is $\mathcal{TV}\Lambda$ - g^* -closed but not $\mathcal{TV}gs$ -closed.

The $\mathcal{TV}g$ -open sets containing $[\{a\}, \{a, b\}]$ are $[\{a, b\}, \{a, b\}]$ and $\tilde{X}$, and $\mathcal{TV}\lambda C([\{a\}, \{a, b\}]) \subseteq [\{a, b\}, \{a, b\}] \subseteq \tilde{X}$. Hence $[\{a\}, \{a, b\}]$ is $\mathcal{TV}\Lambda$ - g^* -closed. However, $\mathcal{TV}sC([\{a\}, \{a, b\}]) = \tilde{X}$ and $\tilde{X} \not\subseteq [\{a\}, \{a, b\}]$. Therefore, $[\{a\}, \{a, b\}]$ is not $\mathcal{TV}gs$ -closed.

2. In Example 3.4, the set $[\emptyset, \{a\}]$ is $\mathcal{TV}gs$ -closed but not $\mathcal{TV}\Lambda$ - g^* -closed.

Since $\mathcal{TV}sC([\emptyset, \{a\}]) \subseteq [\emptyset, \{a, b\}]$, whenever $[\emptyset, \{a\}] \subseteq [\emptyset, \{a, b\}]$ and $[\emptyset, \{a, b\}]$ is $\mathcal{TV}$ -open, the set $[\emptyset, \{a\}]$ is $\mathcal{TV}gs$ -closed. But $\mathcal{TV}\lambda C([\emptyset, \{a\}]) \not\subseteq [\emptyset, \{a, b\}]$, where $[\emptyset, \{a, b\}]$ is $\mathcal{TV}g$ -open. Hence $[\emptyset, \{a\}]$ is not $\mathcal{TV}\Lambda$ - g^* -closed.

Remark 3.9. In an interval-valued topological space $\mathcal{T}^X$,

1. the intersection of two $\mathcal{TV}\Lambda$ - g^* -open sets need not be $\mathcal{TV}\Lambda$ - g^* -open.
2. the union of two $\mathcal{TV}\Lambda$ - g^* -closed sets need not be $\mathcal{TV}\Lambda$ - g^* -closed.

Example 3.10. 1. In Example 3.4, let $[\mathcal{M}^-, \mathcal{M}^+] = [\emptyset, \{b\}]$ and $[\mathcal{K}^-, \mathcal{K}^+] = [\{c\}, \{c\}]$. Then $[\mathcal{M}^-, \mathcal{M}^+]$ and $[\mathcal{K}^-, \mathcal{K}^+]$ are $\mathcal{TV}\Lambda$ - g^* -open sets. However, $[\mathcal{M}^-, \mathcal{M}^+] \cup [\mathcal{K}^-, \mathcal{K}^+] = [\{c\}, \{b, c\}]$ is not $\mathcal{TV}\Lambda$ - g^* -open.

2. In Example 3.1, let $[\mathcal{M}^-, \mathcal{M}^+] = [\{a\}, \{a, b\}]$ and $[\mathcal{K}^-, \mathcal{K}^+] = [\{a\}, \{a, c\}]$. Then $[\mathcal{M}^-, \mathcal{M}^+]$ and $[\mathcal{K}^-, \mathcal{K}^+]$ are $\mathcal{TV}\Lambda$ - g^* -closed sets. However, $[\mathcal{M}^-, \mathcal{M}^+] \cap [\mathcal{K}^-, \mathcal{K}^+] = [\{a\}, \{a\}]$ is not $\mathcal{TV}\Lambda$ - g^* -closed.

Theorem 3.8. If a subset $[\mathcal{M}^-, \mathcal{M}^+]$ is $\mathcal{TV}\Lambda$ - g^* -closed, then $\mathcal{TV}\lambda C([\mathcal{M}^-, \mathcal{M}^+]) - [\mathcal{M}^-, \mathcal{M}^+]$ does not contain any non-empty $\mathcal{TV}$ -closed set in $\tilde{X}$.

Proof.

Let $[\mathcal{M}^-, \mathcal{M}^+]$ be a $\mathcal{TV}\Lambda$ - g^* -closed set. Suppose that $[\mathcal{K}^-, \mathcal{K}^+]$ is a non-empty $\mathcal{TV}$ -closed set contained in $\mathcal{TV}\lambda C([\mathcal{M}^-, \mathcal{M}^+]) - [\mathcal{M}^-, \mathcal{M}^+]$. Then $[\mathcal{M}^-, \mathcal{M}^+] \subseteq [\mathcal{K}^-, \mathcal{K}^+]^c$, where $[\mathcal{K}^-, \mathcal{K}^+]^c$ is $\mathcal{TV}$ -open.

Since every $\mathcal{TV}$ -open set is $\mathcal{TV}g$ -open and $[\mathcal{M}^-, \mathcal{M}^+]$ is $\mathcal{TV}\Lambda$ - g^* -closed, we have $\mathcal{TV}\lambda C([\mathcal{M}^-, \mathcal{M}^+]) \subseteq [\mathcal{K}^-, \mathcal{K}^+]^c$. Hence $[\mathcal{K}^-, \mathcal{K}^+] \subseteq \tilde{X} - \mathcal{TV}\lambda C([\mathcal{M}^-, \mathcal{M}^+])$. Also, $[\mathcal{K}^-, \mathcal{K}^+] \subseteq \mathcal{TV}\lambda C([\mathcal{M}^-, \mathcal{M}^+])$. Therefore, $[\mathcal{K}^-, \mathcal{K}^+] \subseteq (\tilde{X} - \mathcal{TV}\lambda C([\mathcal{M}^-, \mathcal{M}^+])) \cap \mathcal{TV}\lambda C([\mathcal{M}^-, \mathcal{M}^+]) = \emptyset$.

This contradicts the assumption that $[\mathcal{K}^-, \mathcal{K}^+]$ is non-empty.

Hence $\mathcal{TV}\lambda C([\mathcal{M}^-, \mathcal{M}^+]) - [\mathcal{M}^-, \mathcal{M}^+]$ does not contain any non-empty $\mathcal{TV}$ -closed set. $\square$

Theorem 3.9. If a subset $[\mathcal{M}^-, \mathcal{M}^+]$ is $\mathcal{TV}\Lambda$ - g^* -closed, then $\mathcal{TV}\lambda C([\mathcal{M}^-, \mathcal{M}^+]) - [\mathcal{M}^-, \mathcal{M}^+]$ does not contain any non-empty $\mathcal{TV}g$ -closed set.

Proof.

Let $[\mathcal{M}^-, \mathcal{M}^+]$ be a $\mathcal{TV}\Lambda$ - g^* -closed set. Suppose that $[\mathcal{K}^-, \mathcal{K}^+]$ is a non-empty $\mathcal{TV}g$ -closed set contained in $\mathcal{TV}\lambda C([\mathcal{M}^-, \mathcal{M}^+]) - [\mathcal{M}^-, \mathcal{M}^+]$. Then $[\mathcal{M}^-, \mathcal{M}^+] \subseteq [\mathcal{K}^-, \mathcal{K}^+]^c$, where $[\mathcal{K}^-, \mathcal{K}^+]^c$ is $\mathcal{TV}g$ -open. Since $[\mathcal{M}^-, \mathcal{M}^+]$ is $\mathcal{TV}\Lambda$ - g^* -closed, we have $\mathcal{TV}\lambda C([\mathcal{M}^-, \mathcal{M}^+]) \subseteq [\mathcal{K}^-, \mathcal{K}^+]^c$. Hence $[\mathcal{K}^-, \mathcal{K}^+] \subseteq \tilde{X} - \mathcal{TV}\lambda C([\mathcal{M}^-, \mathcal{M}^+])$. Also, $[\mathcal{K}^-, \mathcal{K}^+] \subseteq \mathcal{TV}\lambda C([\mathcal{M}^-, \mathcal{M}^+])$.

Therefore, $[\mathcal{K}^-, \mathcal{K}^+] \subseteq (\tilde{X} - \mathcal{TV}\lambda C([\mathcal{M}^-, \mathcal{M}^+])) \cap \mathcal{TV}\lambda C([\mathcal{M}^-, \mathcal{M}^+]) = \emptyset$.

This contradicts the assumption that $[\mathcal{K}^-, \mathcal{K}^+]$ is non-empty.

Hence $\mathcal{TV}\lambda C([\mathcal{M}^-, \mathcal{M}^+]) - [\mathcal{M}^-, \mathcal{M}^+]$ does not contain any non-empty $\mathcal{TV}g$ -closed set. $\square$

Theorem 3.10. In an interval-valued topological space $\mathcal{T}^X$, for each $p \in X$, the singleton set $\{p\}$ is either $\mathcal{TV}g$ -closed or $\mathcal{TV}\Lambda$ - g^* -open.

Proof.

Suppose that $\{p\}$ is not $\mathcal{TV}g$ -closed. Then $X - \{p\}$ is not $\mathcal{TV}g$ -open. Hence the only $\mathcal{TV}g$ -open set containing $X - \{p\}$ is $\tilde{X}$. Therefore, $X - \{p\} \subseteq \tilde{X}$. Thus, $\mathcal{TV}\lambda C(X - \{p\}) \subseteq \tilde{X}$.

Hence, by definition, $\tilde{X} - \{p\}$ is $\mathcal{TV}\Lambda$ - g^* -closed. Therefore, its complement p is $\mathcal{TV}\Lambda$ - g^* -open. $\square$

Theorem 3.11. Let $[\mathcal{M}^-, \mathcal{M}^+]$ be a $\mathcal{TV}\Lambda$ - g^* -closed set. Then $[\mathcal{M}^-, \mathcal{M}^+]$ is $\mathcal{TV}\lambda$ -closed if and only if $\mathcal{TV}\lambda C([\mathcal{M}^-, \mathcal{M}^+]) - [\mathcal{M}^-, \mathcal{M}^+]$ is $\mathcal{TV}$ -closed.

Proof.

Necessity : Suppose that $[\mathcal{M}^-, \mathcal{M}^+]$ is both $\mathcal{IV}\Lambda$ - g^* -closed and $\mathcal{IV}\lambda$ -closed. Then $\mathcal{IV}\lambda C([\mathcal{M}^-, \mathcal{M}^+]) = [\mathcal{M}^-, \mathcal{M}^+]$. Hence $\mathcal{IV}\lambda C([\mathcal{M}^-, \mathcal{M}^+]) - [\mathcal{M}^-, \mathcal{M}^+] = \tilde{\emptyset}$, which is $\mathcal{IV}$ -closed.

Sufficiency : Suppose that $[\mathcal{M}^-, \mathcal{M}^+]$ is $\mathcal{IV}\Lambda$ - g^* -closed and $\mathcal{IV}\lambda C([\mathcal{M}^-, \mathcal{M}^+]) - [\mathcal{M}^-, \mathcal{M}^+]$ is $\mathcal{IV}$ -closed. By Theorem 3.8, $\mathcal{IV}\lambda C([\mathcal{M}^-, \mathcal{M}^+]) - [\mathcal{M}^-, \mathcal{M}^+]$ contains no non-empty $\mathcal{IV}$ -closed set. Therefore, $\mathcal{IV}\lambda C([\mathcal{M}^-, \mathcal{M}^+]) - [\mathcal{M}^-, \mathcal{M}^+] = \tilde{\emptyset}$.

Hence $\mathcal{IV}\lambda C([\mathcal{M}^-, \mathcal{M}^+]) = [\mathcal{M}^-, \mathcal{M}^+]$. Therefore, $[\mathcal{M}^-, \mathcal{M}^+]$ is $\mathcal{IV}\lambda$ -closed. $\square$

Theorem 3.12. *If every $\mathcal{IV}\Lambda$ - g^* -closed set is $\mathcal{IV}\lambda$ -closed, then for each $p \in X$, $\{p\}$ is either $\mathcal{IV}g$ -closed or $\mathcal{IV}\lambda$ -open.*

Proof.

Suppose that $\{p\}$ is not $\mathcal{IV}g$ -closed. Then $X - \{p\}$ is not $\mathcal{IV}g$ -open. Hence $\tilde{X}$ is the only $\mathcal{IV}g$ -open set containing $X - \{p\}$. Therefore, $\mathcal{IV}\lambda C(X - \{p\}) \subseteq \tilde{X}$. Thus, $X - \{p\}$ is $\mathcal{IV}\Lambda$ - g^* -closed. By hypothesis, $X - \{p\}$ is $\mathcal{IV}\lambda$ -closed. Hence its complement $\{p\}$ is $\mathcal{IV}\lambda$ -open. $\square$

Theorem 3.13. *Let $[\mathcal{M}^-, \mathcal{M}^+]$ be both $\mathcal{IV}g$ -open and $\mathcal{IV}\Lambda$ - g^* -closed. If $[\mathcal{K}^-, \mathcal{K}^+]$ is $\mathcal{IV}\lambda$ -closed, then $[\mathcal{M}^-, \mathcal{M}^+] \cap [\mathcal{K}^-, \mathcal{K}^+]$ is $\mathcal{IV}\Lambda$ - g^* -closed.*

Proof.

By Theorem 3.4, if a set is both $\mathcal{IV}g$ -open and $\mathcal{IV}\Lambda$ - g^* -closed, then it is $\mathcal{IV}\lambda$ -closed. Hence $[\mathcal{M}^-, \mathcal{M}^+]$ and $[\mathcal{K}^-, \mathcal{K}^+]$ are $\mathcal{IV}\lambda$ -closed sets. Since the intersection of two $\mathcal{IV}\lambda$ -closed sets is $\mathcal{IV}\lambda$ -closed, we obtain that $[\mathcal{M}^-, \mathcal{M}^+] \cap [\mathcal{K}^-, \mathcal{K}^+]$ is $\mathcal{IV}\lambda$ -closed.

Therefore, by Theorem 3.1, $[\mathcal{M}^-, \mathcal{M}^+] \cap [\mathcal{K}^-, \mathcal{K}^+]$ is $\mathcal{IV}\Lambda$ - g^* -closed. $\square$

Theorem 3.14. *For a subset $[\mathcal{M}^-, \mathcal{M}^+]$ of an interval-valued topological space $\mathcal{T}^X$, the following are equivalent:*

1. every $\mathcal{IV}g$ -open set is $\mathcal{IV}\lambda$ -closed;
2. every subset is $\mathcal{IV}\Lambda$ - g^* -closed.

Proof.

1. (1) $\Rightarrow$ (2): Let $[\mathcal{M}^-, \mathcal{M}^+]$ be any subset of $\mathcal{T}^X$ such that $[\mathcal{M}^-, \mathcal{M}^+] \subseteq [\mathcal{K}^-, \mathcal{K}^+]$, where $[\mathcal{K}^-, \mathcal{K}^+]$ is $\mathcal{IV}g$ -open. Then $\mathcal{IV}\lambda C([\mathcal{M}^-, \mathcal{M}^+]) \subseteq \mathcal{IV}\lambda C([\mathcal{K}^-, \mathcal{K}^+])$. By hypothesis, $[\mathcal{K}^-, \mathcal{K}^+]$ is $\mathcal{IV}\lambda$ -closed. Hence $\mathcal{IV}\lambda C([\mathcal{K}^-, \mathcal{K}^+]) = [\mathcal{K}^-, \mathcal{K}^+]$. Therefore, $\mathcal{IV}\lambda C([\mathcal{M}^-, \mathcal{M}^+]) \subseteq [\mathcal{K}^-, \mathcal{K}^+]$. Hence $[\mathcal{M}^-, \mathcal{M}^+]$ is $\mathcal{IV}\Lambda$ - g^* -closed.
2. (2) $\Rightarrow$ (1): Let $[\mathcal{M}^-, \mathcal{M}^+]$ be an $\mathcal{IV}g$ -open set. By hypothesis, $[\mathcal{M}^-, \mathcal{M}^+]$ is $\mathcal{IV}\Lambda$ - g^* -closed. Therefore, $\mathcal{IV}\lambda C([\mathcal{M}^-, \mathcal{M}^+]) \subseteq [\mathcal{M}^-, \mathcal{M}^+]$. Since always $[\mathcal{M}^-, \mathcal{M}^+] \subseteq \mathcal{IV}\lambda C([\mathcal{M}^-, \mathcal{M}^+])$, we obtain $\mathcal{IV}\lambda C([\mathcal{M}^-, \mathcal{M}^+]) = [\mathcal{M}^-, \mathcal{M}^+]$. Hence every $\mathcal{IV}g$ -open set is $\mathcal{IV}\lambda$ -closed. $\square$

Conclusion

In this paper, we have introduced and studied the notions of interval-valued Λ - g^* -closed sets and interval-valued Λ - g^* -open sets in interval-valued topological spaces. We have shown that every interval-valued λ -closed set, every interval-valued closed set, and every interval-valued open set is $\mathcal{IV}\Lambda$ - g^* -closed, and that every $\mathcal{IV}\Lambda$ - g^* -closed set is both g - $\mathcal{IV}\Lambda$ -closed and $\mathcal{IV}\Lambda$ - g -closed. We have also established that every interval-valued g -closed set and every interval-valued Λ_g -closed set is $\mathcal{IV}\Lambda$ - g^* -closed, while the concepts of $\mathcal{IV}\Lambda$ - g^* -closed sets and interval-valued gs -closed, gp -closed sets are independent of one another. Suitable examples have been provided throughout to justify that the converses of these implications do not hold in general, thereby placing the newly defined class of sets appropriately within the existing hierarchy of generalized closed sets in interval-valued topology. Additional characterizations were obtained, including conditions under which an $\mathcal{IV}\Lambda$ - g^* -closed set reduces to an interval-valued λ -closed set, and a decomposition-type result concerning singleton subsets of an interval-valued topological space. The results obtained in this work not only enrich the theory of generalized closedness in interval-valued topological spaces but also open up further avenues for research, such as the study of associated continuity notions (interval-valued Λ - g^* -continuous and irresolute mappings), separation axioms defined via these sets, and their possible applications to interval-valued fuzzy systems and decision-making frameworks.

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