



Uni-soft ideals and uni-soft k -ideals of Γ -semirings

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Abstract: In this paper, we have studied the ideal theory of Γ -semirings using the concept of uni-soft sets. Beside some of its basis properties, we obtain some characterizations of k -regular and k -intra-regular Γ -semirings by means of uni-soft k -ideals (k -bi-ideals, k -quasi-ideals).

Key words: Γ -semiring, Soft set, Uni-Soft k -ideals (k -bi-ideals, k -quasi-ideals), k -regular, k -intra-regular.

1. Introduction

Nowadays, we have seen that the classical models sometimes fail to analyze the complexities which arise due to uncertain data in many fields, requiring mathematical evaluation. In order to overcome a certain type of difficulties, Molodtsov [18] introduced the concept of soft sets as a new mathematical tool for dealing with uncertainties. Maji et al. [16] discussed further soft set theory. Ali et al. [2] proposed some new operations on soft sets. As a continuation, several authors (for example, see [4, 5, 17]) described the application of soft set theory to decision making problems. Recently, the algebraic structures of soft sets [6, 24] have been studied increasingly, such as soft groups [20, 27], soft ideals in ordered semigroup [10], uni-soft semigroups [12, 13], soft rings [1], soft semirings [7, 15, 29, 30], soft near-rings [26, 28] and so on.

Semirings which are regarded as a generalization of rings have been found useful for dealing with problems in different areas of applied mathematics and information sciences, as the semiring structure provides an algebraic framework for modeling and investigating the key factors in these problems. We know that ideals of the semiring [8, 11] play a central role in the structure theory and useful for many purposes and they do not in general coincide with the usual ring ideals and so many results in ring theory have no analogues in semirings using only ideals. In [19], Nobusawa introduced the notion of Γ -ring as more general than ring. After this, many mathematicians, for example [9, 14, 21], worked on this subject. As a continuation of this, Rao [22] introduced the concept of Γ -semiring which is a generalization of semiring as well as Γ -ring. To make Γ -semiring more effective several authors investigated some of its related properties.

In this paper, we have investigated the ideal theory of Γ -semirings [23] based on uni-soft sets and obtain some related properties. Some basic operations are also investigated. Finally, we describe some characterizations of regular and intra-regular Γ -semirings by means of uni-soft k -ideals(k -bi-ideals, k -quasi-ideals).

2. Preliminaries

Definition 2.1. [22] Let S and Γ be two additive commutative semigroups with zero. Then S is called a Γ -semiring if there exists a mapping $S \times \Gamma \times S \rightarrow S$ ($(a, \alpha, b) \mapsto a\alpha b$) satisfying the following conditions:

- (i) $(a + b)\alpha c = a\alpha c + b\alpha c$
- (ii) $a\alpha(b + c) = a\alpha b + a\alpha c$
- (iii) $a(\alpha + \beta)b = a\alpha b + a\beta b$
- (iv) $a\alpha(b\beta c) = (a\alpha b)\beta c$
- (v) $0_S\alpha a = 0_S = a\alpha 0_S$
- (vi) $a0_\Gamma b = 0_S = b0_\Gamma a$

where $a, b, c \in S$, $\alpha, \beta \in \Gamma$, 0_S is the zero element of S , 0_Γ is the zero element of Γ . For simplification we write 0 instead of 0_S and 0_Γ .

Definition 2.2. [22] A left ideal I of Γ -semiring S is a nonempty subset of S satisfying the following conditions:

- (i) If $a, b \in I$ then $a + b \in I$
- (ii) If $a \in I$, $s \in S$ and $\gamma \in \Gamma$ then $s\gamma a \in I$
- (iii) $I \neq S$.

A right ideal of S is defined in an analogous manner and an ideal of S is a nonempty subset which is both a left ideal and a right ideal of S .

An ideal I of a Γ -semiring is called k -ideal of S , whenever $x \in I$ and $x + y \in I$ implies $y \in I$ for $y \in S$.

Definition 2.3. [18] Let U be an initial universe set and E be a set of parameters. Let $P(U)$ denotes the power set of U . Consider a nonempty set A , $A \subseteq E$. A pair (f, A) is called a soft set over U , where f is a mapping given by $f : A \rightarrow P(U)$.

It is clear to see that a soft set is a parameterized family of subsets of the set U . Note that the set of all soft sets over U will be denoted by $S(U)$.

Definition 2.4. [1] Let (f, A) be a soft set. The set $Supp(f, A) = \{x \in A | f(x) \neq \phi\}$ is called the support of the soft set (f, A) . The soft set is said to be non-empty if its support is not equal to the empty set.

Definition 2.5. [5] Let $f, g \in S(U)$, then

- (i) The union of f and g , denoted by $f \cup g$, is defined as $(f \cup g)(x) = f(x) \cup g(x)$ for all $x \in E$.
- (ii) The $\vee$ -product of f and g , denoted by $f \vee g$, is defined as $(f \vee g)(x, y) = f(x) \cup g(y)$ for all $x, y \in E$.

Definition 2.6. [13] Let $A \subseteq E$, the set of parameters and U be the universe. We denote χ_A , the uni-soft characteristic function of A and define as

$$\begin{aligned} \chi_A(x) &= \phi \text{ if } x \in A \\ &= U \text{ if } x \notin A. \end{aligned}$$

3. Uni-soft k -ideals with some operations

Throughout this paper unless otherwise mentioned S denotes the Γ -semiring, χ_S denotes its characteristic function.

Definition 3.1. Let f be a non-empty soft set of an Γ -semiring S (i.e. $f(x) \neq \phi$ for some $x \in S$) over U . Then f is called a uni-soft left ideal [resp. uni-soft right ideal] of S over U if

- (i) $f(x + y) \subseteq f(x) \cup f(y)$
- (ii) $f(x\gamma y) \subseteq f(y)$ [resp. $f(x\gamma y) \subseteq f(x)$]

for all $x, y \in S$ and $\gamma \in \Gamma$.

Note that for uni-soft k -ideal the following additional relation must hold:

For $x, a, b \in S$ with $x + a = b \Rightarrow f(x) \subseteq f(a) \cup f(b)$.

Example 3.1. Let $U = S = Z_4 = \{0, 1, 2, 3\}$ and $\Gamma = Z_2 = \{0, 1\}$. Then S forms a Γ -semiring with respect to usual addition and multiplication of integers. Define $f(1) = f(3) = \{0, 1, 2, 3\}$ and $f(0) = f(2) = \{0, 2\}$. Then f is a uni-soft ideal of S over U .

Example 3.2. Let $U = Z^+$, $S = Z_4 = \{0, 1, 2, 3\}$ and $\Gamma = Z_2 = \{0, 1\}$. Define soft set f of S over U by $f(0) = \{4n | n \in Z^+\}$, $f(1) = f(3) = \{n | n \in Z^+\}$ and $f(2) = \{2n | n \in Z^+\}$. Then we can easily check that f is a uni-soft k -ideal of S over U .

Proposition 3.1. Union of a non-empty collection of uni-soft left (resp. right) ideals is also a uni-soft left (resp. right) ideal of S over U .

Proof. Let $\{f_i : i \in I\}$ be a non-empty family of uni-soft left ideals of S over U and $x, y \in S$, $\gamma \in \Gamma$.

Then

$$\begin{aligned} (\bigcup_{i \in I} f_i)(x + y) &= \bigcup_{i \in I} \{f_i(x + y)\} \subseteq \bigcup_{i \in I} \{f_i(x) \cup f_i(y)\} \\ &= \bigcup_{i \in I} \{f_i(x), f_i(y)\} = (\bigcup_{i \in I} f_i)(x) \cup (\bigcup_{i \in I} f_i)(y). \end{aligned}$$

Again

$$(\bigcup_{i \in I} f_i)(x\gamma y) = \bigcup_{i \in I} \{f_i(x\gamma y)\} \subseteq \bigcup_{i \in I} f_i(y) = (\bigcup_{i \in I} f_i)(y).$$

Hence $\bigcup_{i \in I} f_i$ is a uni-soft left ideal of S .

Similarly we can prove the result for uni-soft right ideal also. □

Theorem 3.1. Let f and g be uni-soft left ideals of an Γ -semiring S over U . Then $f \vee g$ is a uni-soft left ideal of $S \times S$ over U .

Proof. Let $(x_1, x_2), (y_1, y_2) \in S \times S$ and $\gamma \in \Gamma$. Then

$$\begin{aligned} (f \vee g)((x_1, x_2) + (y_1, y_2)) &= (f \vee g)(x_1 + y_1, x_2 + y_2) \\ &= (f(x_1 + y_1) \cup g(x_2 + y_2)) \\ &= (f(x_1 + y_1)) \cup (g(x_2 + y_2)) \\ &\subseteq (f(x_1) \cup f(y_1)) \cup (g(x_2) \cup g(y_2)) \\ &= (f \vee g)(x_1, x_2) \cup (f \vee g)(y_1, y_2) \end{aligned}$$

and

$$\begin{aligned}
 (f \vee g)((x_1, x_2)\gamma(y_1, y_2)) &= (f \vee g)(x_1\gamma y_1, x_2\gamma y_2) \\
 &= (f(x_1\gamma y_1) \cup g(x_2\gamma y_2)) \\
 &= (f(x_1\gamma y_1)) \cup (g(x_2\gamma y_2)) \\
 &\subseteq f(y_1) \cup g(y_2) = (f \vee g)(y_1, y_2).
 \end{aligned}$$

Therefore $f \vee g$ is a uni-soft left ideal of $S \times S$ over U . $\square$

Theorem 3.2. *Let f be a soft set of an Γ -semiring S over U . Then f is a uni-soft left ideal of S if and only if $f \vee f$ is a uni-soft left ideal of $S \times S$ over U .*

Proof. Assume that f is a uni-soft left ideal of S over U . Then by Theorem 3.1, $f \vee f$ is a uni-soft left ideal of $S \times S$ over U .

Conversely, suppose that $f \vee f$ is a uni-soft left ideal of $S \times S$ over U . Let $x_1, x_2, y_1, y_2 \in S$ and $\gamma \in \Gamma$. Then

$$\begin{aligned}
 (f(x_1 + y_1) \cup f(x_2 + y_2)) &= (f \vee f)(x_1 + y_1, x_2 + y_2) \\
 &= (f \vee f)((x_1, x_2) + (y_1, y_2)) \\
 &\subseteq (f \vee f)(x_1, x_2) \cup (f \vee f)(y_1, y_2) \\
 &= (f(x_1) \cup f(x_2)) \cup (f(y_1) \cup f(y_2)).
 \end{aligned}$$

Now, putting $x_1 = x$, $x_2 = 0$, $y_1 = y$ and $y_2 = 0$, in this inequality and noting that $f(0) \subseteq f(x)$ for all $x \in S$, we obtain $f(x + y) \subseteq f(x) \cup f(y)$.

Next, we have

$$\begin{aligned}
 (f(x_1\gamma y_1) \cup f(x_2\gamma y_2)) &= (f \vee f)(x_1\gamma y_1, x_2\gamma y_2) \\
 &= (f \vee f)((x_1, x_2)\gamma(y_1, y_2)) \\
 &\subseteq (f \vee f)(y_1, y_2) \\
 &= f(y_1) \cup f(y_2).
 \end{aligned}$$

Taking $x_1 = x$, $y_1 = y$ and $y_2 = 0$, we obtain $f(x\gamma y) \subseteq f(y)$.

Hence f is a uni-soft left ideal of S over U . $\square$

Definition 3.2. Let f and g be two soft sets of an Γ -semiring S over U . We define composition of f and g as follows:

$$\begin{aligned}
 fog(x) &= \cap[\cup_i \{f(a_i), f(c_i), g(b_i), g(d_i)\}] \\
 &\quad x + \sum_{i=1}^n a_i \alpha_i b_i = \sum_{i=1}^n c_i \beta_i d_i \\
 &= 0, \text{ if } x \text{ cannot be expressed as above} \\
 &\quad \text{where } x, z, a_i, b_i, c_i, d_i \in S \text{ and } \alpha_i, \beta_i \in \Gamma.
 \end{aligned}$$

Theorem 3.3. *Let f_1, f_2 be any two uni-soft k -ideals of S over U . Then $f_1 \circ f_2$ is a uni-soft ideal of S over U .*

Proof. Let f_1, f_2 be any two uni-soft k -ideals of S over U and $x, y \in S$ and $\gamma \in \Gamma$. Then

$$\begin{aligned}
 & (f_1 o f_2)(x + y) \\
 &= \cap \{ \cup_i \{ f_1(a_i), f_2(b_i), f_1(c_i), f_2(d_i) \} \} \\
 &\subseteq \cap \{ \cup_i \{ f_1(x_{1i}), f_1(x_{3i}), f_1(y_{1i}), f_1(y_{3i}), f_2(x_{2i}), f_2(x_{4i}), f_2(y_{2i}), f_2(y_{4i}) \} \} \\
 &\subseteq \cup \{ \cap \{ \cup_i \{ f_1(x_{1i}), f_1(x_{3i}), f_2(x_{2i}), f_2(x_{4i}) \} \}, \cap \{ \cup_i \{ f_1(y_{1i}), f_1(y_{3i}), f_2(y_{2i}), f_2(y_{4i}) \} \} \} \\
 &= (f_1 o f_2)(x) \cup (f_1 o f_2)(y).
 \end{aligned}$$

Now assuming f_1, f_2 are uni-soft right ideals we have

$$\begin{aligned}
 (f_1 o f_2)(x\gamma y) &= \cap \{ \cup_i \{ f_1(a_i), f_2(b_i), f_1(c_i), f_2(d_i) \} \} \\
 &\subseteq \cap \{ \cup_i \{ f_1(x_{1i}), f_1(x_{3i}), f_2(x_{2i}\gamma y), f_2(x_{4i}\gamma y) \} \} \\
 &\subseteq \cap \{ \cup_i \{ f_1(x_{1i}), f_1(x_{3i}), f_2(x_{2i}), f_2(x_{4i}) \} \} \\
 &= (f_1 o f_2)(x).
 \end{aligned}$$

Similarly, assuming f_1, f_2 are as uni-soft left k -ideals we can show that $(f_1 o f_2)(x\gamma y) \subseteq (f_1 o f_2)(y)$.

Hence $f_1 o f_2$ is a uni-soft ideal of S over U . $\square$

4. Uni-soft ideals of regular Γ -semiring

Definition 4.1. A soft set f of a Γ -semiring S over U is called uni-soft bi-ideal if for all $x, y \in S$ and $\alpha, \beta \in \Gamma$ we have

- (i) $f(x + y) \subseteq f(x) \cup f(y)$
- (ii) $f(x\alpha y) \subseteq f(x) \cup f(y)$
- (iii) $f(x\alpha y\beta z) \subseteq f(x) \cup f(z)$

Definition 4.2. A soft set f of a Γ -semiring S over U is called uni-soft quasi-ideal if for all $x, y \in S$ we have

- (i) $f(x + y) \subseteq f(x) \cup f(y)$
- (ii) $((f o \chi_S) \cup (\chi_S o f))(x) \subseteq f(x)$

Proposition 4.1. Intersection of a non-empty collection of uni-soft bi-ideals of S over U is also a uni-soft bi-ideal of S over U .

Proof. The proof follows by routine verifications. $\square$

Proposition 4.2. Let $\{f_i : i \in I\}$ be a family of uni-soft bi-ideals of S over U such that $f_i \subseteq f_j$ or $f_j \subseteq f_i$ for $i, j \in I$. Then $\cup_{i \in I} f_i$ is a uni-soft bi-ideal of S over U .

Proof. Straightforward. $\square$

Lemma 4.1. In a Γ -semiring, every uni-soft quasi ideal is uni-soft bi-ideal.

Proof. Let f be a uni-soft quasi ideal of S over U . It is sufficient to prove that $f(x\alpha y\beta z) \subseteq f(x) \cup f(z)$ for all $x, y, z \in S$ and $\alpha, \beta \in \Gamma$. Since f is a uni-soft quasi ideal of S over U , we have

$$\begin{aligned} f(x\alpha y\beta z) &\subseteq ((fo\chi_S) \cup (\chi_S o\mu))(x\alpha y\beta z) \\ &= \{(fo\chi_S)(x\alpha y\beta z) \cup (\chi_S o\mu)(x\alpha y\beta z)\} \\ &= \cup \left\{ \cap \left(\bigcup_i (f(a_i) \cup f(c_i)) \right), \cap \left(\bigcup_i (f(b_i) \cup f(d_i)) \right) \right\} \\ &\quad x\alpha y\beta z + \sum_{i=1}^n a_i \gamma_i b_i = \sum_{i=1}^n c_i \delta_i d_i \quad x\alpha y\beta z + \sum_{i=1}^n a_i \gamma_i b_i = \sum_{i=1}^n c_i \delta_i d_i \\ &\subseteq f(0) \cup f(x) \cup f(0) \cup f(z) \text{ (since } x\alpha y\beta z + 0\gamma 0 + 0 = x\alpha y\beta z + 0) \\ &= f(x) \cup f(z). \end{aligned}$$

Similarly, we can show that $f(x\alpha y) \subseteq f(x) \cup f(y)$ for all $x, y \in S$ and $\alpha \in \Gamma$.

Example 4.1. Let $U = Z$, $S = \Gamma = \{0, a, b\}$. Then S forms a Γ -semiring with respect to addition and multiplication defined as:

$$\begin{array}{|c|c|c|c|} \hline + & 0 & a & b \\ \hline 0 & 0 & a & b \\ a & a & a & a \\ b & b & a & b \\ \hline \end{array} \quad \text{and} \quad \begin{array}{|c|c|c|c|} \hline \cdot & 0 & a & b \\ \hline 0 & 0 & 0 & 0 \\ a & 0 & a & a \\ b & 0 & a & a \\ \hline \end{array}$$

Now define soft set f on S over U as: $f(0) = 4Z$, $f(a) = 2Z$, $f(b) = Z$. Then f is a uni-soft bi-ideal of S over U which is not a uni-soft quasi-ideal.

□

Definition 4.3. A Γ -semiring S is said to be k -regular if for each $x \in S$, there exist $a, b \in S$ and $\alpha, \beta, \gamma, \delta \in \Gamma$ such that $x + x\alpha a\beta x = x\gamma b\delta x$.

Definition 4.4. A Γ -semiring S is said to be k -intra-regular if for each $x \in S$, there exist $z, a_i, a'_i, b_i, b'_i \in S$, $\alpha_{1i}, \alpha_{2i}, \alpha_{3i}, \beta_{1i}, \beta_{2i}, \beta_{3i} \in \Gamma$, $i \in \mathbb{N}$, the set of natural numbers, such that $x + \sum_{i=1}^n a_i \alpha_{1i} x \alpha_{2i} x \alpha_{3i} a'_i = \sum_{i=1}^n b_i \beta_{1i} x \beta_{2i} x \beta_{3i} b'_i$.

Theorem 4.1. Let S be a k -regular Γ -semiring and $x \in S$. Then

- (i) $f(x) \subseteq (fo\chi_S of)(x)$ for every uni-soft k -bi-ideal f of S over U .
- (ii) $f(x) \subseteq (fo\chi_S of)(x)$ for every uni-soft k -quasi-ideal f of S over U .

Proof. Let S be a k -regular Γ -semiring and x be any element of S . Suppose f be any uni-soft k -bi-ideal of S . Since S is k -regular there exist $a, b \in S$ and $\alpha, \beta, \gamma, \delta \in \Gamma$ such that $x + x\alpha a\beta x = x\gamma b\delta x$. Now

$$\begin{aligned}
 & (fo\chi_S of)(x) \\
 &= \cap(\cup_i \{(fo\chi_S)(a_i), (fo\chi_S)(c_i), f(b_i), f(d_i)\}) \\
 & \quad x + \sum_{i=1}^n a_i \alpha_i b_i = \sum_{i=1}^n c_i \beta_i d_i \\
 & \subseteq \cap \{(fo\chi_S)(x\alpha a), (fo\chi_S)(x\gamma b), f(x)\} \\
 & \quad [\text{since } x + x\alpha a\beta x = x\gamma b\delta x] \\
 &= \cap \{ \cap(\cup_i \{(f(a_i), f(c_i))\}) \quad , \quad \cap(\cup_i \{(f(a_i), f(c_i))\}) \quad , f(x) \} \\
 & \quad x\alpha a + \sum_{i=1}^n a_i \alpha_{1i} b_i = \sum_{i=1}^n c_i \beta_{1i} d_i \quad x\gamma b + \sum_{i=1}^n a_i \alpha_{2i} b_i = \sum_{i=1}^n c_i \beta_{2i} d_i \\
 & \subseteq \cap \{f(x), f(x), f(x)\} \\
 & \quad [\text{since } x\alpha a + x\alpha a\beta x\alpha a = x\gamma b\delta x\alpha a \text{ and } x\gamma b + x\alpha a\beta x\gamma b = x\gamma b\delta x\gamma b] \\
 &= f(x).
 \end{aligned}$$

This implies that $f(x) \subseteq (fo\chi_S of)(x)$.

(i) $\Rightarrow$ (ii) is straightforward from Lemma 4.1. □

Theorem 4.2. Let S be a k -regular Γ -semiring and $x \in S$. Then

- (i) $(f \cup g)(x) \subseteq (fogof)(x)$ for every uni-soft k -bi-ideal f and every uni-soft k -ideal g of S over U .
- (ii) $(f \cup g)(x) \subseteq (fogof)(x)$ for every uni-soft k -quasi-ideal f and every uni-soft k -ideal g of S over U .

Proof. Assume that S is a k -regular Γ -semiring. Let f and g be any uni-soft k -bi-ideal and uni-soft k -ideal of S over U , respectively and x be any element of S . Since S is k -regular, there exist $a, b \in S$ and $\alpha, \beta, \gamma, \delta \in \Gamma$ such that $x + x\alpha a\beta x = x\gamma b\delta x$. Then

$$\begin{aligned}
 & (fogof)(x) \\
 &= \cap(\cup_i \{(fog)(a_i), (fog)(c_i), f(b_i), f(d_i)\}) \\
 & \quad x + \sum_{i=1}^n a_i \alpha_i b_i = \sum_{i=1}^n c_i \beta_i d_i \\
 & \subseteq \cap \{(fog)(x\alpha a), (fog)(x\gamma b), f(x)\} \\
 & \quad [\text{since } x + x\alpha a\beta x = x\gamma b\delta x] \\
 &= \cap \{ \cap(\cup_i \{(f(a_i), f(c_i), g(b_i), g(d_i))\}) \quad , \quad \cap(\cup_i \{(f(a_i), f(c_i), g(b_i), g(d_i))\}) \quad , f(x) \} \\
 & \quad x\alpha a + \sum_{i=1}^n a_i \alpha_i b_i = \sum_{i=1}^n c_i \beta_i d_i \quad x\gamma b + \sum_{i=1}^n a_i \gamma_i b_i = \sum_{i=1}^n c_i \delta_i d_i \\
 & \subseteq \cup \{ \cup \{f(x), f(a\beta x\alpha a), g(b\delta x\alpha a)\} \quad , \quad \cup \{f(x), g(a\beta x\gamma b), g(b\delta x\gamma b)\} \quad , f(x) \} \\
 & \quad [\text{since } x\alpha a + x\alpha a\beta x\alpha a = x\gamma b\delta x\alpha a \text{ and } x\gamma b + x\alpha a\beta x\gamma b = x\gamma b\delta x\gamma b] \\
 & \subseteq f(x) \cup g(x) = (f \cup g)(x).
 \end{aligned}$$

(i) $\Rightarrow$ (ii) is straightforward from Lemma 4.1. □

Theorem 4.3. Let S is both k -regular and k -intra-regular Γ -semiring and $x \in S$. Then

- (i) $f(x) = (f of)(x)$ for every uni-soft k -bi-ideal f of S over U .
- (ii) $f(x) = (f of)(x)$ for every uni-soft k -quasi-ideal f of S over U .

Proof. Suppose S is both k -regular and k -intra-regular Γ -semiring. Let $x \in S$ and f be any uni-soft k -bi-ideal of S over U . Since S is both k -regular and k -intra-regular there exist $a_i, b_i, c_i, d_i \in S$, $\alpha_{1i}, \alpha_{2i}, \alpha_{3i}, \alpha_{4i}, \alpha_{5i}, \beta_{1i}, \beta_{2i}, \beta_{3i}, \beta_{4i}, \beta_{5i} \in \Gamma$, $i \in \mathbf{N}$ such that $x + \sum_{i=1}^n x\alpha_{1i}a_i\alpha_{2i}x\alpha_{3i}x\alpha_{4i}b_i\alpha_{5i}x = \sum_{i=1}^n x\beta_{1i}c_i\beta_{2i}x\beta_{3i}x\beta_{4i}d_i\beta_{5i}x$.

Therefore

$$\begin{aligned} (f \circ f)(x) &= \cap [\cup_i \{f(a_i), f(c_i), f(b_i), f(d_i)\}] \\ &\quad x + \sum_{i=1}^n a_i \alpha_i b_i = \sum_{i=1}^n c_i \beta_i d_i \\ &\subseteq \cap [\cup_i \{f(x\alpha_{1i}a_i\alpha_{2i}x), f(x\alpha_{4i}b_i\alpha_{5i}x), f(x\beta_{1i}c_i\beta_{2i}x), f(x\beta_{4i}d_i\beta_{5i}x)\}] \\ &\quad x + \sum_{i=1}^n x\alpha_{1i}a_i\alpha_{2i}x\alpha_{3i}x\alpha_{4i}b_i\alpha_{5i}x = \sum_{i=1}^n x\beta_{1i}c_i\beta_{2i}x\beta_{3i}x\beta_{4i}d_i\beta_{5i}x \\ &\subseteq f(x). \end{aligned}$$

Now $(f \circ f)(x) \subseteq (f \circ \chi_S)(x) \subseteq f(x)$. Hence $f(x) = (f \circ f)(x)$ for every uni-soft k -bi-ideal f of S .

(i) $\Rightarrow$ (ii) is straightforward from the Lemma 4.1. □

Theorem 4.4. Let S is both k -regular and k -intra-regular Γ -semiring and $x \in S$. Then

- (i) $(f \cup g)(x) \subseteq (f \circ g)(x)$ for all uni-soft k -bi-ideals f and g of S over U .
- (ii) $(f \cup g)(x) \subseteq (f \circ g)(x)$ for every uni-soft k -bi-ideals f and every uni-soft k -quasi-ideal g of S over U .
- (iii) $(f \cup g)(x) \subseteq (f \circ g)(x)$ for every uni-soft k -quasi-ideals f and every uni-soft k -bi-ideal g of S over U .
- (iv) $(f \cup g)(x) \subseteq (f \circ g)(x)$ for all uni-soft k -quasi-ideals f and g of S over U .

Proof. Assume that S is both k -regular and k -intra-regular Γ -semiring. Let $x \in S$ and f, g be any uni-soft k -bi-ideals of S over U . Since S is both k -regular and k -intra-regular there exist $a_i, b_i, c_i, d_i \in S$, $\alpha_{1i}, \alpha_{2i}, \alpha_{3i}, \alpha_{4i}, \alpha_{5i}, \beta_{1i}, \beta_{2i}, \beta_{3i}, \beta_{4i}, \beta_{5i} \in \Gamma$, $i \in \mathbf{N}$ such that $x + \sum_{i=1}^n x\alpha_{1i}a_i\alpha_{2i}x\alpha_{3i}x\alpha_{4i}b_i\alpha_{5i}x \subseteq \sum_{i=1}^n x\beta_{1i}c_i\beta_{2i}x\beta_{3i}x\beta_{4i}d_i\beta_{5i}x$. Therefore

$$\begin{aligned} (f \circ g)(x) &= \cap [\cup_i \{f(a_i), f(c_i), g(b_i), g(d_i)\}] \\ &\quad x + \sum_{i=1}^n a_i \alpha_i b_i = \sum_{i=1}^n c_i \beta_i d_i \\ &\subseteq \cap [\cup_i \{f(x\alpha_{1i}a_i\alpha_{2i}x), g(x\alpha_{4i}b_i\alpha_{5i}x), f(x\beta_{1i}c_i\beta_{2i}x), g(x\beta_{4i}d_i\beta_{5i}x)\}] \\ &\subseteq f(x) \cup g(x) = (f \cup g)(x). \end{aligned}$$

(i) $\Rightarrow$ (ii) $\Rightarrow$ (iv) and (i) $\Rightarrow$ (iii) $\Rightarrow$ (iv) are obvious from Lemma 4.1. □

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