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Study on fixed points for cccasionally weakly compatible in symmetric spaces

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Abstract: In this paper, we obtain a unique common fixed point theorem for OWC(Occasionally Weakly Compatible) four self- mappings in symmetric spaces. This result is a generalization, improving, extending, known comparable results existing in the literature.

Key words: Common fixed point theorem, OWC(Occasionally Weakly Compatible), Symmetric space.

AMS Subject Classifications: 47H10, 54H25.

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1. Introduction

We know that Banach contraction principle (prior to 1968)is a fundamental result in fixed point theory. Later on Kannan [14] obtained a fixed point result for a mapping satisfying contractive condition that need not satisfy continuity. Subsequently many authors has been extended and improved and generalizes these result in many ways (see for e.g. [1-13] and [15-18]). Hicks and Rhoades [10] obtained some common fixed point theorems in symmetric spaces and semi metric spaces. Recently, Abbas and Rhoades [6] obtained some common fixed point results for occasionally weakly compatible mappings satisfying a generalized contractive condition in symmetric spaces. In this paper we obtained a common fixed point theorem for OWC(Occasionally Weakly Compatible) four self- mappings in symmetric spaces. Our results are more general than the several well known results.

1.1. Preliminaries

The following are useful in our main results which are due to [6].

Definition 1.1. Two maps p and q are said to be weakly compatible if they commute at coincidence points.

Definition 1.2. Let X be a set , p, q self maps of X. A point x in X is called a coincidence point of p and q iff $px = qx$. We shall call $w = px = qx$ a point of coincidence of p and q.

Definition 1.3. Two self maps p, q of a set X. A point x in X are said to be OWC(Occasionally Weakly Compatible) iff there exists a point x in X which is a coincidence point of p and q at which p and q are commute.

Lemma 1.1 (11)**.** *Let X be a set , p, q are OWC(Occasionally Weakly Compatible) self maps of X. If p and q have a unique point coincidence $w = px = qx$, then w is a unique common fixed point of p and q.*

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Our results are proved in symmetric spaces, which are more general than metric spaces.

Definition 1.4. Let X be a set. A symmetric on X is a mapping $\rho(x,y) : X \times X \to [0,\infty)$ such that $\rho(x,y) = 0$ *if and only if* $x = y$, and $\rho(x,y) = \rho(y,x)$ for $x, y \in X$.
Let $A \in [0,\infty)$, $R_A^+ = [0, A)$. Let $F : R_A^+ \longrightarrow R$ satisfy
(i). $F(0) = 0$ and $F(t) > 0$ for each $t \in (0, A)$ and (ii). F is non decreasing on R_A^+.

Define, $F[0, A) = \{F : R_A^+ \longrightarrow R$: F satisfies (i) - (ii)$\}$. Let $A \in [0,\infty)$. Let $\Psi : R_A^+ \longrightarrow R$ satisfies (i). $\Psi(t) < t$ for each $t \in (0, A)$ and (ii). Ψ is non decreasing.

Define, $\Psi[0, A) = \{\Psi : R_A^+ \longrightarrow R : F$ satisfies (i) - (ii) above$\}$.

Some of the examples of mappings F: $R_A^+ \longrightarrow R : F$ satisfies (i) - (ii) ,we have refer to [17] .

Definition 1.5. A control function Φ defined by $\Phi : R^+ \longrightarrow R^+$ which satisfies $\Phi(t) = 0$ *if and only if* $t = 0$.

2. Main Results

In this section, we proved a unique common fixed point theorem for four self-mappings for OWC(Occasionally Weakly Compatible) mappings in symmetric spaces.
Now we prove our main theorem.

Theorem 2.1. *Let X be a set, ρ a symmetric on X. Let M, N, S and T be four self- mappings of X satisfying the following conditions:*

$$F(\Phi(\rho(Mu, Nv))) < \Psi(F(K_\Phi(u, v)), \tag{1}$$

where,
$K_\Phi(u,v) = Max\{\lambda_1[\Phi(\rho((Mu,Tv))+\Phi(\rho(Su,Mu))+\Phi(\rho(Nv,Tv))]\}+Max\{\lambda_2[\Phi(\rho(Mu,Tv))+\Phi(\rho(\frac{(Su,Nv)}{2}))]\}$.
Where, λ_1, $\lambda_2 > 0$ *and* $\lambda_1 + \lambda_2 < 1$.

For each $u, v \in X$, $F \in F[0, B)$ *and* $\Psi \in \Psi[0, F((B-0))$, *where* $B = D$ *if* $D = \infty$ *and* $B > D$ *if* $D < \infty$. *And*

$$(M,S) \text{ and } (N,T) \text{ are } OWC. \tag{2}$$

Then M, N, S and T are having a unique common fixed point in X.

Proof. Given by (2) (M,S) and (N,T) are OWC, then there exists two points $u, v \in X$ such that $Mu = Su$ and $Nv = Tv$. For suppose that $Mu \neq Nv$. Then from (1) we get that,

$$\begin{aligned}
K_\Phi(u,v) &= Max\{\lambda_1[\Phi(\rho((Mu,Nv)) + \Phi(\rho(Mu,Mu)) + \Phi(\rho(Tv,Tv))]\} + Max\{\lambda_2[\Phi(\rho(Mu,Nv)) + \Phi(\rho(\frac{(Mu,Nv)}{2}))]\}, \\
&= Max\{\lambda_1[\Phi(\rho((Mu,Nv)) + \Phi(\rho(0)) + \Phi(\rho(0))]\} + Max\{\lambda_2[\Phi(\rho(Mu,Nv)) + \Phi(\rho(\frac{(Mu,Nv)}{2}))]\}, \\
&= Max\{\lambda_1[\Phi(\rho((Mu,Nv))]\} + Max\{\lambda_2[\Phi(\rho(Mu,Nv))]\}, \\
&= \lambda_1[\Phi(\rho((Mu,Nv))] + \lambda_2[\Phi(\rho(Mu,Nv))], \\
&= (\lambda_1 + \lambda_2)[\Phi(\rho((Mu,Nv))], (since \lambda_1 + \lambda_2 < 1)
\end{aligned}$$

$$< \Phi(\rho(Mu, Nv)). \tag{3}$$

Then from (1) and (3) we get that ,

$$\begin{aligned} 0 < F(\Phi(\rho(Mu, Nv))) &< \Psi(F(K_\Phi(u, v)) \\ &< \Psi(F(\Phi(\rho(Mu, Nv))), \\ &< F(\Phi(\rho(Mu, Nv)), \ which\ is\ a\ contradiction. \\ Therefore, \Phi(\rho(Mu, Nv)) = 0. & \\ \implies \rho(Mu, Nv) = 0. & \\ \implies Mu = Nv. \ That\ is, Mu = Su = Nv = Tv. & \end{aligned}$$

Suppose that if there exists another point z such that $Mz = Sz$, then using (1) and (3) we get that $Mz = Sz = Nv = Tv = Mu = Su$ or $Mu = Mz$ and $w = Mu = Su$ is the unique point of M and S. By Lemma (1.1) , w is the only common fixed point of M and S. By symmetry there exists a unique point $z \in X$ such that $z = Nz = Tz$. Suppose that $w \neq z$ then by (1) and (3) we get that,

$$\begin{aligned} F(\Phi(\rho(w, z)) = F(\Phi(\rho(Mw, Nz)) &< \Psi(F(K_\Phi(w, z)) \\ &< \Psi(F(\Phi(\rho(w, z))) \\ &< F(\Phi(\rho(w, z)), \ which\ is\ a\ contradiction. \\ Therefore, \Phi(\rho(w, z)) = 0. & \\ \implies \rho(w, z) = 0. & \\ \implies w = z. & \end{aligned}$$

Therefore, $w = z$ is a common fixed point. By the preceding argument and it is clear that w is the unique. Therefore, M, N, S and T are having a unique common fixed point in X. And this completes the proof of the theorem. □

3. Conclusion

Our results are more general than the results of [6].

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