

# Nano $\Delta$ generalized-locally closed sets in nano topological spaces

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Abstract: The purpose of this paper is to introduce a new class of sets called generalized nano locally closed set in nanotopological spaces, also introduce generalized nanolocally continuous map, generalized nanolocally closed irresolute map and studied some of its properties.

**Key words:**  $n\Delta$ -closed,  $n\Delta$  g-closed sets,  $n\Delta$ -lc-set,  $n\Delta$  g-lc-set

### 1. Introduction

Several notions of open-like and closed-like sets in nano topological spaces were introduced and studied. The beginning was with M. Lellis Thivagar and Carmel Richard who initiated the notion of nano forms of weakly open sets and nano continuity, [7, 8]. We introduced and studied the notion of nano $\Delta$ -open sets in nano topological spaces,[10]. The concept of nano continuity in nano topological spaces was extended to generalized nano $\Delta$ -continuity, [3].

A set in a topological space is called  $\Delta$ -open if it is the symmetric difference of two open sets. The notion of  $\Delta$ -open sets appeared in [9] and in [1]. However, it was pointed out in [9] and in [1] that the notion of  $\Delta$ -open sets is due to a preprint by M. Veera Kumar. The complement of a  $\Delta$ -open set is  $\Delta$ -closed.

A set in a nano topological space is called  $n\Delta$ -open if it is the symmetric difference of two nano open sets were initiated, [10].

Preliminary concepts required in our work are briefly recalled in section 2. In section 3, we introduced the classes of  $m\Delta$ -lc-set,  $gm\Delta$ -lc-set,  $gm\Delta$ -lc\*-sets,  $gm\Delta$ -lc\*\*-sets and study some of its basic properties. In section 4, Finally we introduced and studied  $m\Delta$ LC-continuous,  $Gm\Delta$ LC-continuous map and  $Gm\Delta$ LCirresolute map.

## 2. Preliminaries

## Definition 2.1. [10]

A subset S of a space (U,  $\tau_R(X)$ ) is said to be nano  $\Delta$ -open set (in short,  $n\Delta$ -open) if S = (A - B)  $\cup$  (B - A), where A and B are nano-open subsets in U. The complement of micro- $\Delta$ -open sets is called nano- $\Delta$ -closed sets.

<sup>©</sup>Asia Mathematika, DOI: 10.5281/zenodo.10609824

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## Definition 2.2. [10]

The nano interior of a set A is denoted by nano  $\Delta$ -int(A) (briefly,  $n\Delta$  -int(A)) and is defined as the union of all  $n\Delta$  open sets contained in A. i.e.,  $n\Delta$  -int(A) =  $\bigcup$  {G : G is  $n\Delta$ -open and G  $\subseteq$  A }.

## Definition 2.3. [10]

The nano closure of a set A is denoted by nano  $\Delta$ -cl(A) (briefly,  $n\Delta$  -cl(A)) and is defined as the intersection of all  $n\Delta$  -closed sets containing A. i.e.,  $n\Delta$  -cl(A) =  $\cap$  {F : F is  $n\Delta$ -closed and A  $\subseteq$  F}.

### Definition 2.4. [2]

A subset U of a space (U,  $\tau_R(X)$ ) is called a generalized nano  $\Delta$ -closed (briefly,  $n\Delta g$ -closed) set if  $n\Delta$ cl(A)  $\subseteq$  T whenever A  $\subseteq$  T and T is  $n\Delta$  -open in (U,  $\tau_R(X)$ ). The complement of  $n\Delta q$ -closed set is called  $n\Delta q$ -open set.

## Proposition 2.1. [2]

Every  $n\Delta$  -closed set is  $n\Delta g$ -closed set but not convesely.

### Proposition 2.2. [2]

Every  $n\Delta$  -open set is  $n\Delta g$ -open set but not conversely.

### 3. Nano $\Delta$ Generalized-locally closed sets

**Definition 3.1.** A subset A of an nano topological space (U,  $\tau_R(X)$ ) is called an  $n\Delta$ -locally closed (briefly,  $n\Delta$ -lc) sets if  $A = S \cap G$  where S is  $n\Delta$ -open and G is  $n\Delta$ -closed.

The class of all  $n\Delta$ -locally closed sets in a nano topological space (U,  $\tau_R(X)$ ) is denoted by  $n\Delta LC(X)$ .

**Definition 3.2.** A subset A of an nano topological space  $(U, \tau_R(X))$  is called an generalized  $n\Delta$ -locally closed (briefly,  $n\Delta g$  -lc) sets if  $A = E \cap F$  where E is  $n\Delta g$  -open and F is  $n\Delta g$  -closed. The class of all  $n\Delta g$  -locally closed sets in nano topological spaces  $(U, \tau_R(X))$  is denoted by  $Gn\Delta LC(X)$ .

**Proposition 3.1.** Every  $n\Delta$ -closed (resp.  $n\Delta$ -open) set is  $n\Delta$ -lc-set but not conversely.

*Proof.* It follows from Definition 3.1.

**Proposition 3.2.** Every  $n\Delta g$  -closed (resp.  $n\Delta g$  -open) set is  $n\Delta g$ -lc-set but not conversely.

*Proof.* It follows from Definition 3.2.

**Proposition 3.3.** Every  $n\Delta$ -lc-set is  $n\Delta g$  -lc-set but not conversely.

*Proof.* It follows from Proposition 2.1 and 2.2.

**Example 3.1.** Let  $U = \{a, b, c\}$  with  $U/R = \{\{a\}, \{b, c\}\}$  and  $X = \{a\}$ . The nano topology  $\tau_R(X) = \{\phi, \{a\}, U\}$ . Then  $n\Delta$ -lc-sets are  $\phi$ ,  $\{b\}, \{a, c\}, U$ ,  $n\Delta g$  -lc-sets are  $\phi$ ,  $\{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, U$ . Here, the set  $\{b, c\}$  is  $n\Delta g$ -lc set but it is not  $n\Delta$ -lc set in  $(U, \tau_R(X))$ .

**Definition 3.3.** A space  $(U, \tau_R(X))$  is called a  $T_{\Delta 1}/2$ -space if every  $mg\Delta$ -closed set is  $n\Delta$ -closed.

**Theorem 3.1.** Let  $(U, \tau_R(X))$  be a  $T_{\Delta}1/2$ -space. Then  $Gn\Delta LC(X) = n\Delta LC(X)$ .

*Proof.* Since every  $n\Delta g$  -open set is  $n\Delta$ -open and every  $n\Delta g$  -closed set is  $n\Delta$ -closed in  $(U, \tau_R(X))$ ,  $Gn\Delta LC(X) \subseteq n\Delta LC(X)$  and hence  $Gn\Delta LC(X) = n\Delta LC(X)$ .

**Definition 3.4.** A subset A of a space  $(U, \tau_R(X))$  is called

- 1.  $n\Delta g$  -lc\*-set if  $A = O \cap P$ , where O is  $n\Delta g$  -open in  $(U, \tau_R(X))$  and P is  $n\Delta$ -closed in  $(U, \tau_R(X))$ .
- 2.  $n\Delta g$  -lc\*\*-set if  $A = R \cap S$ , where R is  $n\Delta$ -open in (U,  $\tau_R(X)$ ) and S is  $n\Delta g$  -closed in (U,  $\tau_R(X)$ ).

The class of all  $n\Delta g$  -lc\* (resp.  $n\Delta g$  -lc\*\*) sets in a nano topological space (U,  $\tau_R(X)$ ) is denoted by  $n\Delta g$  LC\* (X) (resp.  $n\Delta g$  LC\*\*(X)).

**Proposition 3.4.** Every  $n\Delta$ -lc-set is  $n\Delta g$ -lc<sup>\*</sup>-set but not conversely.

1000. It follows from Definitions $0.1$ and $0.4$ (1)
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**Proposition 3.5.** Every  $n\Delta$ -lc-set is  $n\Delta g$  -lc<sup>\*\*</sup>-set but not conversely.

*Proof.* It follows from Definitions 3.1 and 3.4(2).

**Proposition 3.6.** Every  $n\Delta g$  -lc<sup>\*</sup>-set is  $n\Delta g$  -lc-set but not conversely.

*Proof.* It follows from Definitions 3.2 and 3.4(1).

**Proposition 3.7.** Every  $n\Delta g$  -lc<sup>\*\*</sup>-set is  $n\Delta g$  -lc-set but not conversely.

*Proof.* It follows from Definitions 3.2 and 3.4 (2).

**Theorem 3.2.** For a subset A of  $(U, \tau_R(X))$  the following statements are equivalent:

- 1.  $A \in Gn\Delta LC(X)$ ,
- 2.  $A = S \cap n\Delta g cl(A)$  for some  $n\Delta g$  -open set S,
- 3.  $n\Delta g$  -cl(A) A is  $n\Delta g$  -closed,
- 4.  $A \cup (n\Delta g cl(A))^c$  is  $n\Delta g$  -open,
- 5.  $A \subseteq n\Delta g int(A \cup (n\Delta g cl(A))^c)$ .

*Proof.* (1)  $\Rightarrow$  (2). Let  $A \in Gn\Delta LC(X)$ . Then  $A = S \cap G$  where S is  $n\Delta g$  -open and G is  $n\Delta g$  -closed. Since  $A \subseteq G$ ,  $n\Delta g$  -cl(A)  $\subseteq$  G and so  $S \cap n\Delta g$  -cl(A)  $\subseteq$  A. Also  $A \subseteq S$  and  $A \subseteq n\Delta g$  -cl(A) implies  $A \subseteq S \cap n\Delta g$  -cl(A) and therefore  $A = S \cap n\Delta g$  -cl(A).

(2)  $\Rightarrow$  (3). A = S  $\cap$   $n\Delta g$  -cl(A) implies  $n\Delta g$  -cl(A)  $- A = n\Delta g$  -cl(A)  $\cap$  S<sup>c</sup> which is  $n\Delta g$  -closed since S<sup>c</sup> is  $n\Delta g$  -closed and  $n\Delta g$  -cl(A) is  $n\Delta g$  -closed.

(3)  $\Rightarrow$  (4). A  $\cup$   $(n\Delta g - cl(A))^c = (n\Delta g - cl(A) - A)^c$  and by assumption,  $(n\Delta g - cl(A) - A)^c$  is  $n\Delta g$ -open and so is A  $\cup$   $(n\Delta g - cl(A))^c$ .

(4)  $\Rightarrow$  (5). By assumption,  $A \cup (n\Delta g - cl(A))^c = n\Delta g - int(A \cup (n\Delta g - cl(A))^c)$  and hence  $A \subseteq n\Delta g - int(A \cup (n\Delta g - cl(A))^c)$ .

(5)  $\Rightarrow$  (1). By assumption and since  $A \subseteq n\Delta g - cl(A)$ ,  $A = n\Delta g - int(A \cup (n\Delta g - cl(A))^c) \cap n\Delta g - cl(A)$ . Therefore,  $A \in n\Delta g LC(X)$ .

**Theorem 3.3.** For a subset A of  $(U, \tau_R(X))$ , the following statements are equivalent:

- 1.  $A \in n\Delta g \ LC^*(U)$ ,
- 2.  $A = S \cap n\Delta cl(A)$  for some  $n\Delta g$  -open set S,
- 3.  $n\Delta cl(A) A$  is  $n\Delta g$  -closed,
- 4.  $A \cup (n\Delta cl(A))^c$  is  $n\Delta g$  -open.

*Proof.* (1)  $\Rightarrow$  (2). Let  $A \in n\Delta g$  LC \* (U). There exist an  $n\Delta g$  -open set S and a  $n\Delta$ -closed set G such that  $A = S \cap G$ . Since  $A \subseteq S$  and  $A \subseteq n\Delta cl(A)$ ,  $A \subseteq S \cap n\Delta cl(A)$ . Also since  $n\Delta cl(A) \subseteq G$ ,  $S \cap n\Delta cl(A) \subseteq S \cap G = A$ . Therefore  $A = S \cap n\Delta cl(A)$ .

(2)  $\Rightarrow$  (1). Since S is  $n\Delta g$  -open and  $n\Delta cl(A)$  is a  $n\Delta$ -closed set,  $A = S \cap n\Delta cl(A) \in n\Delta g LC^{*}(U)$ .

(2)  $\Rightarrow$  (3). Since  $n\Delta cl(A) - A = n\Delta cl(A) \cap S^c$ ,  $n\Delta cl(A) - A$  is  $n\Delta g$  -closed since  $S^c$  is  $n\Delta g$  -closed.

(3)  $\Rightarrow$  (2). Let S =  $(n\Delta cl(A) - A)^c$ . Then by assumption S is  $n\Delta g$  -open in (U,  $\tau_R(X)$ ) and A = S  $\cap n\Delta cl(A)$ .

(3)  $\Rightarrow$  (4). Let G =  $n\Delta cl(A)$  – A. Then G<sup>c</sup> = A  $\cup$   $(n\Delta cl(A))^c$  and A  $\cup$   $(n\Delta cl(A))^c$  is  $n\Delta g$  -open.

(4)  $\Rightarrow$  (3). Let S = A  $\cup$   $(n\Delta cl(A))^c$ . Then S<sup>c</sup> is  $n\Delta g$  -closed and S<sup>c</sup> =  $n\Delta cl(A)$  – A and so  $n\Delta cl(A)$  – A is  $n\Delta g$  -closed.

**Theorem 3.4.** Let A be a subset of  $(U, \tau_R(X))$ . Then  $A \in n\Delta g \ LC^{**}(X)$  if and only if  $A = S \cap n\Delta g$ -cl(A) for some  $n\Delta$ -open set S.

*Proof.* (1)  $\Rightarrow$  (2). Let  $A \in n\Delta g \ LC^{**}(X)$ . Then  $A = S \cap G$  where S is  $n\Delta$ -open and G is  $n\Delta g$  -closed. Since  $A \subseteq G$ ,  $n\Delta g \ -cl(A) \subseteq G$ . We obtain  $A = A \cap n\Delta g \ -cl(A) = S \cap G \cap n\Delta g \ -cl(A) = S \cap n\Delta g \ -cl(A)$ .

(2)  $\Rightarrow$  (1). Since S is  $n\Delta$ -open and  $n\Delta g$  -cl(A) is a  $n\Delta g$  -closed set, A = S  $\cap$   $n\Delta g$  -cl(A)  $\in$   $n\Delta g$  LC<sup>\*\*</sup>(X).

**Corollary 3.1.** Let A be a subset of  $(U, \tau_R(X))$ . If  $A \in n\Delta g \ LC^{**}(X)$ , then  $n\Delta g \ cl(A) - A$  is  $n\Delta g \ -closed$  and  $A \cup (n\Delta g \ -cl(A))^c$  is  $n\Delta g \ -open$ .

Proof. Let  $A \in n\Delta g \ LC^{**}(X)$ . Then by Theorem 3.4,  $A = S \cap n\Delta g \ -cl(A)$  for some  $n\Delta$ -open set S and  $n\Delta g \ -cl(A) - A = n\Delta g \ -cl(A) \cap S^c$  is  $n\Delta g \ -closed$  in  $(U, \tau_R(X))$ . If  $G = n\Delta g \ -cl(A) - A$ , then  $G^c = A \cup (n\Delta g \ -cl(A))^c$  and  $G^c$  is  $n\Delta g \ -open$  and so is  $A \cup (n\Delta g \ -cl(A))^c$ .

### 4. $G\Delta LC$ -Continuous and $G\Delta LC$ -Irresolute maps

**Definition 4.1.** A map  $f: (U, \tau_R(X)) \to (V, \tau_R(X)')$  is said to be  $n\Delta$  locally closed-continuous (briefly,  $n\Delta$ LC-continuous) if  $f^{-1}(V)$  is  $n\Delta$ LC-set in  $(U, \tau_R(X))$  for every  $n\Delta$ -open set V of  $(V, \tau_R(X)')$ .

**Definition 4.2.** A map f:  $(U, \tau_R(X)) \to (V, \tau_R(X)')$  is said to be  $G\Delta LC$ -continuous (resp.  $G\Delta LC^*$ continuous,  $G\Delta LC^{**}$ -continuous) if  $f^{-1}(V)$  is  $G\Delta LC$ -set (resp.  $G\Delta LC^*$ -set,  $G\Delta LC^{**}$ -set) in  $(U, \tau_R(X))$  for every  $n\Delta$ -open set V of  $(V, \tau_R(X)')$ .

**Theorem 4.1.** Let  $f: (U, \tau_R(X)) \to (V, \tau_R(X)')$  be a map. Then

- 1. If f is  $n\Delta$ -continuous, then it is  $n\Delta LC$ -continuous.
- 2. If f is  $n\Delta$ -continuous, then it is  $G\Delta LC$ -continuous.
- 3. If f is  $g\Delta$ -continuous, then it is  $G\Delta LC$ -continuous.

*Proof.* (1) It is an immediate consequence of Proposition 3.1.

(2) It is an immediate consequence of Proposition 3.1 and 3.3.

(3) It is an immediate consequence of Propositions 3.2.

**Theorem 4.2.** Let  $f: (U, \tau_R(X)) \to (V, \tau_R(X)')$  be a map. Then

- 1. If f is  $n\Delta LC$ -continuous, then it is  $G\Delta LC$ -continuous.
- 2. If f is  $n\Delta LC$ -continuous, then it is  $G\Delta LC^*$ -continuous.
- 3. If f is  $n\Delta LC$ -continuous, then it is  $G\Delta LC^{**}$ -continuous.
- 4. If f is  $G\Delta LC^*$ -continuous, then it is  $G\Delta LC$ -continuous.
- 5. If f is  $G \Delta L C^{**}$ -continuous, then it is  $G \Delta L C$ -continuous.

*Proof.* (1) It is an immediate consequence of Proposition 3.3.

- (2) It is an immediate consequence of Proposition 3.4.
- (3) It is an immediate consequence of Propositions 3.5.
- (4) It is an immediate consequence of Propositions 3.6.
- (5) It is an immediate consequence of Propositions 3.2.

**Definition 4.3.** A map  $f : (U, \tau_R(X)) \to (V, \tau_R(X)')$  is said to be  $G\Delta LC$  -irresolute (resp.  $G\Delta LC$  \*irresolute,  $G\Delta LC$  \*\*-irresolute) if  $f^{-1}(V)$  is  $G\Delta LC$  -set (resp.  $G\Delta LC$  \*-set,  $G\Delta LC$  \*\*-set) in  $(U, \tau_R(X))$ for every  $G\Delta LC$  -set (resp.  $G\Delta LC$  \*-set,  $G\Delta LC$  \*\*-set) V of  $(V, \tau_R(X)')$ .

**Theorem 4.3.** Let  $f: (U, \tau_R(X)) \to (V, \tau_R(X)')$  be a map. Then

- 1. If f is  $G\Delta LC$  -irresolute then it is  $G\Delta LC$  -continuous.
- 2. If f is  $G\Delta LC^*$ -irresolute then it is  $G\Delta LC^*$ -continuous.
- 3. If f is  $G\Delta LC$  \*\* -irresolute then it is  $G\Delta LC$  \*\* -continuous.

Proof. (1) Let  $f: (U, \tau_R(X)) \to (V, \tau_R(X)')$  be a  $G\Delta LC$  -irresolute map. Let V be a  $n\Delta$ -open set of  $(V, \tau_R(X)')$ . Since every  $n\Delta$ -open set is  $g\Delta$ -open and  $g\Delta$ -open set is  $g\Delta$ -lc-set [by the Proposition 2.2 and Proposition 3.2], V is  $G\Delta LC$  -set of  $(V, \tau_R(X)')$ . Since f is  $G\Delta LC$  -irresolute, then  $f^{-1}(V)$  is a  $G\Delta LC$  -set of  $(U, \tau_R(X))$ . Therefore f is  $G\Delta LC$  -continuous.

(2) Let  $f: (U, \tau_R(X)) \to (V, \tau_R(X)')$  be a  $G\Delta LC$  \*-irresolute map. Let V be a  $n\Delta$ -open set of  $(V, \tau_R(X)')$ . Since every  $n\Delta$ -open set is  $n\Delta$ -lc set and  $n\Delta$ -lc-set is  $g\Delta$ -lc\*-set [by Proposition 3.1 and Proposition 3.4], V is  $G\Delta LC$  \*-set of  $(V, \tau_R(X)')$ . Since f is  $G\Delta LC$  \*-irresolute, then  $f^{-1}(V)$  is a  $G\Delta LC^*$ -set of  $(U, \tau_R(X))$ . Therefore f is  $G\Delta LC^*$ -continuous.

(3) Let f: (U,  $\tau_R(X)$ )  $\rightarrow$  (V,  $\tau_R(X)'$ ) be a  $G\Delta LC$  \*\*-irresolute map. Let V be a  $n\Delta$ -open set of (V,  $\tau_R(X)'$ ).

Since every  $n\Delta$ -open set is  $n\Delta$ -lc-set and  $n\Delta$ -lc-set is  $g\Delta$ -lc<sup>\*\*</sup>-set [by Proposition 3.1 and Proposition 3.5], A is  $G\Delta LC$  <sup>\*\*</sup>-set of (V,  $\tau_R(X)'$ ). Since f is  $G\Delta LC$  <sup>\*\*</sup>-irresolute, then f<sup>-1</sup>(V) is a  $G\Delta LC$  <sup>\*\*</sup>-set of (U,  $\tau_R(X)$ ) . Therefore f is  $G\Delta LC$  <sup>\*\*</sup>-continuous.

**Theorem 4.4.** Let  $f: (U, \tau_R(X)) \to (V, \tau_R(X)')$  and  $g: (V, \tau_R(X)') \to (Z, \gamma)$  be any two maps. Then

- 1.  $g \circ f$  is  $G \Delta LC$ -continuous if g is  $n \Delta$ -continuous and f is  $G \Delta LC$ -continuous.
- 2.  $g \circ f$  is  $G \Delta LC$ -irresolute if both f and g are  $G \Delta LC$ -irresolute.
- 3.  $g \circ f$  is  $G \Delta LC$ -continuous if g is  $G \Delta LC$ -continuous and f is  $G \Delta LC$ -irresolute.

Proof. (1) Since g is a  $n\Delta$ -continuous from  $(V, \tau_R(X)') \rightarrow (Z, \gamma)$ , for any  $n\Delta$ -open set z as a subset of Z, we get  $g^{-1}(z) = G$  is a  $n\Delta$ -open set in  $(V, \tau_R(X)')$ . As f is a  $G\Delta$ LC-continuous map. We get  $(g \circ f)^{-1}(z) = f^{-1}(g^{-1}(z)) = f^{-1}(G) = S$  and S is a  $G\Delta$ LC-set in  $(U, \tau_R(X))$ , since every  $n\Delta$ -open set is  $g\Delta$ -open set and  $g\Delta$ -open set is  $g\Delta$ -lc-set [by the Proposition 2.2 and Proposition 3.2]. Hence  $(g \circ f)$  is a  $G\Delta$ LC-continuous map.

(2) Consider two  $G\Delta LC$ -irresolute maps,  $f: (U, \tau_R(X)) \to (V, \tau_R(X)')$  and  $g: (V, \tau_R(X)') \to (Z, \gamma)$  is a  $G\Delta LC$ -irresolute maps. As g is consider to be a  $G\Delta LC$ -irresolute map, by Definition 4.3, for every  $g\Delta$ -lc-set  $z \subseteq (Z, \gamma), g^{-1}(z) = G$  is a  $g\Delta$ -lc-set in  $(V, \tau_R(X)')$ . Again since f is  $G\Delta LC$ -irresolute,  $(g \circ f)^{-1}(z) = f^{-1}(g^{-1}(z)) = f^{-1}(G) = S$  and S is a  $g\Delta$ -lc-set in  $(U, \tau_R(X))$ . Hence  $(g \circ f)$  is a  $G\Delta LC$ -irresolute map.

(3) Let g be a  $G\Delta LC$ -continuous map from  $(V, \tau_R(X)') \to (Z, \gamma)$  and z subset of Z be a  $n\Delta$ -open set. Therefore  $g^{-1}(z) = G$  is a  $g\Delta$ -lc-set in  $(V, \tau_R(X)')$ , since every  $n\Delta$ -open set is  $g\Delta$ -open set and  $g\Delta$ -open set is  $g\Delta$ -lc-set [by the Proposition 2.2 and Proposition 3.2]. Also since f is  $G\Delta LC$ -irresolute, we get  $(g \circ f)^{-1}(z) = f^{-1}(g^{-1}(z)) = f^{-1}(G) = S$  and S is a  $g\Delta$ -lc-set in  $(U, \tau_R(X))$ . Hence  $(g \circ f)$  is a  $G\Delta LC$ -continuous map.  $\Box$ 

## Conclusion

In this paper, we introduce the classes of  $n\Delta$ -lc-set,  $g\Delta$ -lc-set,  $g\Delta$ -lc\*-sets,  $g\Delta$ -lc\*-sets and study some of its basic properties. Finally we introduced and studied  $n\Delta$ LC-continuous,  $G\Delta$ LC-continuous map and  $G\Delta$ LC-irresolute map. In future, we have extended this work in various nano topological fields with some applications.

### Acknowledgment

I thank to referees for giving their useful suggestions and help to improve this manuscript.

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