Asia Mathematika
Volume: 8 Issue: 2, (2024) Pages: 23 – 30
Available online at **www.asiamath.org**

On micro forms of Δ-open sets, Δ-continuous maps and generalized micro Δ-continuous in micro topological spaces

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Received: 22 Jun 2024 • **Accepted:** 16 Sep 2024 • **Published Online:** 20 Sep 2024

Abstract: This article introduces and studied $m\Delta$-open sets in micro topological spaces. We offer a new class of sets called $gm\Delta$-closed sets in micro topological spaces and we study some of its basic properties. The idea of introducing new classes of open sets are called micro semi Δ-open, micro α-Δ-open, micro pre Δ-open, micro b-Δ-open micro β-Δ-open, micro regular Δ-open sets, micro semi Δ-continuous, micro α-Δ-continuous, micro pre Δ-continuous, micro b-Δ-continuous micro β-Δ-continuous and micro regular Δ-continuous in micro topological spaces. We introduce $m\Delta$-continuous maps, $gm\Delta$-continuous maps, $m\Delta$-irresolute maps, $gm\Delta$-irresolute maps, contra $m\Delta$-continuous maps and contra $gm\Delta$-continuous maps in micro topological spaces and discuss some of their properties.

Key words: Δ-closed, $m\Delta$-closed, $gm\Delta$-closed, $gm\Delta$-continuous maps, $gm\Delta$-irresolute maps and contra $gm\Delta$-continuous maps

1. Introduction

Several notions of open-like and closed-like sets in micro topological spaces were introduced and studied. The beginning was with S. Chandrasekar who initiated the notion of micro forms of open sets, [1, 2]. After that micro β-open sets and micro b-open sets in micro topological spaces were introduced H. Z. Ibrahim, [7, 8]. Furthermore, the notions of micro regular open sets and micro π-open sets were initiated, [3]. We introduced and studied the notion of micro Δ-open sets in micro topological spaces,[5]. The concept of micro continuity in micro topological spaces was extended to generalized micro Δ-continuity, [6].

A set in a topological space is called Δ-open if it is the symmetric difference of two open sets. The notion of Δ-open sets appeared in [10] and in [4]. However, it was pointed out in [10] and in [4] that the notion of Δ-open sets is due to a preprint by M. Veera Kumar. The complement of a Δ-open set is Δ-closed.

A set in a micro topological space is called $m\Delta$-open if it is the symmetric difference of two micro open sets were initiated, [5].

Preliminary concepts required in our work are briefly recalled in section 2. In section 3, the idea of introducing new classes of open sets are called micro semi Δ-open, micro α-Δ-open, micro pre Δ-open, micro b-Δ-open, micro β-Δ-open, micro regular Δ-open sets, micro semi Δ-continuous, micro α-Δ-continuous, micro pre Δ-continuous, micro b-Δ-continuous micro β-Δ-continuous and micro regular Δ-continuous in micro topological spaces and also the concept of $gm\Delta$-continuous maps and $gm\Delta$-irresolute maps. We introduce contra $gm\Delta$-continuous map in micro topological spaces and discuss some of their properties.

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2. Preliminaries

Definition 2.1. [9] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements in the same equivalence class are indiscernible. The pair (U, R) is said to be the approximation space. Let X $\subseteq$ U.

1. The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$.
   That is, $L_R(X) = \bigcup x_\epsilon U\{R(X) : R(X) \subseteq X\} where R(x)$ denotes the equivalence class determined by X.
2. The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$.
   That is, $U_R(X) = \bigcup x_\epsilon U\{R(X) : R(X) \cap X \neq \phi\}$
3. The boundary region of X with respect to R is the set of all objects, which can be neither in nor as not-X with respect to R and it is denoted by $B_R(X)$. That is, $B_R(X) = U_R(X) - L_R(X)$

Definition 2.2. [9] If (U, $\tau_R(X)$) is the nano topological space with respect to X where X $\subseteq$ U and if A $\subseteq$ U, then

1. The nano interior of the set A is defined as the union of all nano open subsets contained in A and it is denoted by nint(A). That is, nint(A) is the largest nano open subset of A.
2. The nano closure of the set A is defined as the intersection of all nano closed sets containing A and it is denoted by ncl(A). That is, ncl(A) is the smallest nano closed set containing A.

Definition 2.3. [1] Let (U, $\tau_R(X)$) be a nano topological space. Then, $\mu_R(X)$ = {N $\cup$ ($\acute{N} \cap \mu$) : $N, \acute{N} \in \tau_R(X)$ and $\mu \notin \tau_R(X)$} is called the Micro topology on U with respect to X. The triplet (U, $\tau_R(X)$, $\mu_R(X)$) is called Micro topological space and the elements of $\mu_R(X)$ are called Micro open sets and the complement of a Micro open set is called a Micro closed set.

Definition 2.4. [1] The Micro interior of a set A is denoted by Micro-int(A) (briefly, m-int(A)) and is defined as the union of all Micro open sets contained in A. i.e., Mic-int(A) = $\cup$ {G : G is Micro open and G $\subseteq$ A }.

Definition 2.5. [1] The Micro closure of a set A is denoted by Micro-cl(A) (briefly, m-cl(A)) and is defined as the intersection of all Micro closed sets containing A. i.e., Mic-cl(A) = $\cap$ {F : F is Micro closed and A $\subseteq$ F}.

Definition 2.6. [4, 10, 11] A subset A of a space (X, τ) is called Δ-open if A = (B $-$ C) $\cup$ (C $-$ B), where B and C are open subsets of X. Δ-closed sets are the complement of Δ-open sets.

Definition 2.7. [5] A subset S of a space (U, $\tau_R(X)$, $\mu_R(X)$) is said to be micro Δ-open set (in short, $m\Delta$-open) if S = (A $-$ B) $\cup$ (B $-$ A), where A and B are micro-open subsets in U. The complement of micro-Δ-open sets is called micro -Δ-closed sets.

Definition 2.8. [5] The micro interior of a set A is denoted by micro Δ-int(A) (briefly, $m\Delta$-int(A)) and is defined as the union of all $m\Delta$ open sets contained in A. i.e., $m\Delta$-int(A) = $\cup$ {G : G is $m\Delta$-open and G $\subseteq$ A }.

Definition 2.9. [5] The micro closure of a set A is denoted by micro Δ-cl(A) (briefly, $m\Delta$-cl(A)) and is defined as the intersection of all $m\Delta$-closed sets containing A. i.e., $m\Delta$-cl(A) = $\cap$ {F : F is $m\Delta$-closed and A $\subseteq$ F}.

Definition 2.10. [6] A subset A of a space (U, $\tau_R(X)$, $\mu_R(X)$) is called a generalized micro Δ-closed (briefly, gm-Δ-closed) set if $m\Delta$cl(A) $\subseteq$ T whenever A $\subseteq$ T and T is $m\Delta$-open in (U, $\tau_R(X)$).
gm-Δ-open sets are the complement of gm-Δ -closed sets.

Proposition 2.1. *[6] Every $m\Delta$-closed set is gm-Δ-closed but not conversely.*

Proof. Let A be a $m\Delta$-closed set and T be any $m\Delta$-open set containing A. Since A is $m\Delta$-closed, we have $m\Delta$cl(A) = A $\subseteq$ T. Hence A is gm-Δ-closed. ☐

The converse of Proposition 2.1 need not be true as seen from the following example.

Example 2.1. *Let U = {1, 2, 3} with U/ R= {{1, 2, 3}} and X= {2, 3}. The nano topology $\tau_R(X)$ consists of {ϕ, U}. If μ = {1, 3} then the micro topology $\mu_R(X)$ consists of {ϕ, {1, 3}, U}. Then mΔ-closed sets are {ϕ, {2}, {1, 3}, U} and gm-Δ-closed sets are {ϕ, {1}, {2}, {3}, {1, 2},{1, 3}, {2, 3}, U}. Here, H = {1, 2} is gm-Δ-closed set but it is not mΔ-closed.*

3. On Micro Forms of Δ-Open Sets, Continuous maps, gm-Δ -Continuous maps and gm-Δ -Irresolute maps

Definition 3.1. A subset S of a space (U, $\tau_R(X)$, $\mu_R(X)$) is called

1. micro semi-Δ-open set (in short, $ms\Delta$-open) if S = (A − B) $\cup$ (B − A), where A and B are micro semi-open subsets in U.
2. micro α-Δ-open set (in short, m$\alpha\Delta$-open) if S = (A − B) $\cup$ (B − A), where A and B are micro α-open subsets in U.
3. micro pre-Δ-open set (in short, $mp\Delta$-open) if S = (A − B) $\cup$ (B − A), where A and B are micro pre-open subsets in U.
4. micro b-Δ-open set (in short, $mb\Delta$-open) if S = (A − B) $\cup$ (B − A), where A and B are micro b-open subsets in U.
5. micro β-Δ-open set (in short, m$\beta\Delta$-open) if S = (A − B) $\cup$ (B − A), where A and B are micro β-open subsets in U.
6. micro regular-Δ-open set (in short, $mr\Delta$-open) if S = (A − B) $\cup$ (B − A), where A and B are micro regualr-open subsets in U.

The complements of the above mentioned micro Δ-open sets are called their respective micro $sm\Delta$-closed sets.

Proposition 3.1. *Le (U, $\tau_R(X)$, $\mu_R(X)$) be micro topological spaces. Then*

1. *Every $m\Delta$-open set is micro semi-Δ-open.*
2. *Every micro semi-open set is micro semi-Δ-open.*

But the converse implications are not true in general. Following is an example:

Example 3.1. *Let U = {1, 2, 3} with U/ R= {{1}, {2, 3}} and X= {1}. The nano topology* $\tau_R(X)$ *consists of* {ϕ, {1}, U}. *If* μ = {1, 3} *then the micro topology* $\mu_R(X)$ *consists of* {ϕ, {1}, {1, 3}, U}. *Then m*Δ*-open sets are* ϕ, {1}, {2}, {3}, {1, 3}, {2, 3}, *U, micro semi-open sets are* ϕ, {1}, {1, 2}, {1, 3}, *U and micro semi* Δ*-open sets are* ϕ, {1}, {2}, {3}, {1, 2}, {1, 3}, {2, 3}, *U. Here, P* = {2, 3} *is micro semi* Δ*-open. But is neither m*Δ*-open nor micro semi-open.*

Proposition 3.2. *Le (U,* $\tau_R(X)$, $\mu_R(X)$*) be micro topological spaces. Then*

1. *Every m*Δ*-open set is micro* α*-*Δ*-open.*
2. *Every m*Δ*-open set is micro pre* Δ*-open.*
3. *Every m*Δ*-open set is micro b-*Δ*-open.*
4. *Every m*Δ*-open set is micro* β*-*Δ*-open.*
5. *Every micro* α*-m*Δ*-open set is micro pre-*Δ*-open.*
6. *Every micro semi-*Δ*-open set is micro b-*Δ*-open.*
7. *Every micro* α*-open set is micro* α*-*Δ*-open.*
8. *Every micrp pre-open set is micro pre* Δ*-open.*
9. *Every micro b-open set is micro b-*Δ*-open.*
10. *Every micro* β*-open set is micro* β*-*Δ*-open.*
11. *Every micro regular-open set is micro regular-*Δ*-open.*

But the converse of above implications are not true in general. Following is an example:

Example 3.2. *Let U = {1, 2, 3} with U/ R= {{1}, {2, 3}} and X= {1}. The nano topology* $\tau_R(X)$ *consists of* {ϕ, {1}, U}. *If* μ = {1, 3} *then the micro topology* $\mu_R(X)$ *consists of* {ϕ, {1}, {1, 3}, U}. *Then m*Δ*-open sets are* ϕ, {1}, {2}, {3}, {1, 3}, {2, 3}, *U , micro* α*-*Δ*-open sets are* ϕ, {1}, {2}, {3}, {1, 2}, {1, 3}, {2, 3}, *U., micro pre-*Δ*-open sets are* ϕ, {1}, {2}, {3}, {1, 2}, {1, 3}, {2, 3}, *U., micro b-*Δ*-open sets are* ϕ, {1}, {2}, {3}, {1, 2}, {1, 3}, {2, 3}, *U., micro* β*-*Δ*-open sets are* ϕ, {1}, {2}, {3}, {1, 2}, {1, 3}, {2, 3}, *U., micro* α*-open sets are* ϕ, {1}, {1, 2}, {1, 3}, *U., micro pre-open sets are* ϕ, {1}, {1, 2}, {1, 3}, *U., micro b-open sets are* ϕ, {1}, {1, 2}, {1, 3}, *U., micro* β*-open sets are* ϕ, {1}, {1, 2}, {1, 3}, *U. Here, (i) Q* = {1, 2} *is micro* α*-*Δ*-open, micro pre* Δ*-open, micro b-*Δ*-open, micro* β*-*Δ*-open sets but is not m*Δ*-open. (ii) R* = {2, 3} *is micro* α*-*Δ*-open, micro pre* Δ*-open, micro b-*Δ*-open, micro* β*-*Δ*-open sets . But is neigher micro* α*-open nor micro pre-open nor micro b-open nor micro-*β*-open.*

Example 3.3. *Let U = {1, 2, 3} with U/ R= {{1, 2, 3}} and X= {2, 3}. The nano topology* $\tau_R(X)$ *consists of* {ϕ, U}. *If* μ = {1, 3} *then the micro topology* $\mu_R(X)$ *consists of* {ϕ, {1, 3}, U}. *Then m*Δ*-closed sets are* ϕ, {2}, {1, 3}, *U, micro* α*-*Δ*-open sets are* ϕ, {2}, {1, 3}, *U., micro pre-*Δ*-open sets are* ϕ, {1}, {2}, {3}, {1, 2}, {1, 3}, {2, 3}, *U. Here, L* = {1, 2} *is micro pre-*Δ*-open. But is neigher m*Δ*-open nor micro* α*-*Δ*-open.*

Example 3.4. *Let U = {1, 2, 3} with U/ R= {{1, 2, 3}} and X= {1, 3}. The nano topology* $\tau_R(X)$ *consists of* {ϕ, U}. *If* μ = {2, 3} *then the micro topology* $\mu_R(X)$ *consists of* {ϕ, {2, 3}, U}. *Then m*Δ*-closed sets are* ϕ, *{1}, {2, 3}, U, micro semi-Δ-open sets are ϕ, {1}, {2, 3}, U., micro b-Δ-open sets are ϕ, {1}, {2}, {3}, {1, 2}, {1, 3}, {2, 3}, U. Here, M = {1, 3} is micro b-Δ-open. But is neigher mΔ-open nor micro semi-Δ-open.*

Example 3.5. *Let U = {1, 2, 3} with U/ R= {{1}, {2, 3}} and X= {1}. The nano topology $\tau_R(X)$ consists of {ϕ, {1}, U}. If μ = {2} then the micro topology $\mu_R(X)$ consists of {ϕ, {1}, {2}, {1, 2}, U}. Then mΔ-open sets are ϕ, {1}, {2}, {3}, {1, 2}, {1, 3}, {2, 3}, U , micro regular-Δ-open sets are ϕ, {1}, {2}, {3}, {1, 2}, {1, 3}, {2, 3}, U., micro regular-open sets are ϕ, {1}, {2}, {1, 2}, U. Here, N = {2, 3} is micro regular-Δ-open sets but is not micro-regular-open.*

Definition 3.2. A map f : (U, $\tau_R(X)$, $\mu_R(X)$) $\rightarrow$ (V, $\tau_R(X)'$, $\mu_R(X)'$) is called

1. $m\Delta$-continuous if f^{-1}(G) is a $m\Delta$-open set of (U, $\tau_R(X)$, $\mu_R(X)$) for every $m\Delta$-open set G of (V, $\tau_R(X)'$, $\mu_R(X)'$).
2. micro semi-Δ-continuous if f^{-1}(G) is a micro semi-Δ-open set of (U, $\tau_R(X)$, $\mu_R(X)$) for every $m\Delta$-open set G of (V, $\tau_R(X)'$, $\mu_R(X)'$).
3. micro α-Δ-continuous if f^{-1}(G) is a micro α-Δ-open set of (U, $\tau_R(X)$, $\mu_R(X)$) for every $m\Delta$-open set G of (V, $\tau_R(X)'$, $\mu_R(X)'$).
4. micro pre-Δ-continuous if f^{-1}(G) is a micro pre-Δ-open set of (U, $\tau_R(X)$, $\mu_R(X)$) for every $m\Delta$-open set G of (V, $\tau_R(X)'$, $\mu_R(X)'$).
5. micro b-Δ-continuous if f^{-1}(G) is a micro b-Δ-open set of (U, $\tau_R(X)$, $\mu_R(X)$) for every $m\Delta$-open set G of (V, $\tau_R(X)'$, $\mu_R(X)'$).
6. micro β-Δ-continuous if f^{-1}(G) is a micro β-Δ-open set of (U, $\tau_R(X)$, $\mu_R(X)$) for every $m\Delta$-open set G of (V, $\tau_R(X)'$, $\mu_R(X)'$).
7. micro regular-Δ-continuous if f^{-1}(G) is a micro regular-Δ-open set of (U, $\tau_R(X)$, $\mu_R(X)$) for every $m\Delta$-open set G of (V, $\tau_R(X)'$, $\mu_R(X)'$).

Theorem 3.1. *Le (U, $\tau_R(X)$, $\mu_R(X)$) be micro topological spaces. Then*

1. *Every mΔ-continuous is micro semi-Δ-continuous but not conversely.*
2. *Every mΔ-continuous is micro α-Δ-continuous but not conversely.*
3. *Every mΔ-continuous is micro pre-Δ-continuous but not conversely.*
4. *Every mΔ-continuous is micro b-Δ-continuous but not conversely.*
5. *Every mΔ-continuous is micro β-Δ-continuous but not conversely.*
6. *Every micro α-mΔ-continuous is micro pre-Δ-continuous but not conversely.*
7. *Every micro semi-Δ-continuous is micro b-Δ-continuous but not conversely.*
8. *Every micro regular-continuous is micro regular-Δ-continuous but not conversely.*

Proof. The proof follows from Proposition 3.1 and 3.2. □

Definition 3.3. A map f : (U, $\tau_R(X)$, $\mu_R(X)$) $\to$ (V, $\tau_R(X)'$, $\mu_R(X)'$) is called gm-Δ -continuous if f^{-1}(G) is a gm-Δ -closed set of (U, $\tau_R(X)$, $\mu_R(X)$) for every $m\Delta$-closed set G of (V, $\tau_R(X)'$, $\mu_R(X)'$) .

Theorem 3.2. *Every $m\Delta$-continuous is gm-Δ -continuous but not conversely.*

Proof. The proof follows from Proposition 2.1. □

Theorem 3.3. *If f : (U, $\tau_R(X)$, $\mu_R(X)$) $\to$ (V, $\tau_R(X)'$, $\mu_R(X)'$) is gm-Δ -continuous and g : (V, $\tau_R(X)'$, $\mu_R(X)'$) $\to$ (W, $\tau_R(X)''$) is $m\Delta$- continuous then g $\circ$ f : (U, $\tau_R(X)$, $\mu_R(X)$) $\to$ (W, $\tau_R(X)''$) is gm-Δ -continuous.*

Proof. Let K be $m\Delta$-closed set in W. Since g is $m\Delta$-continuous, g^{-1}(K) is $m\Delta$-closed in V. Since f is gm-Δ -continuous, $(g \circ f)^{-1}$(G) = $f^{-1}(g^{-1}$(K)) is gm-Δ -closed in U. Therefore g $\circ$ f is gm-Δ -continuous. □

Proposition 3.3. *A map f : (U, $\tau_R(X)$, $\mu_R(X)$) $\to$ (V, $\tau_R(X)'$, $\mu_R(X)'$) is gm-Δ -continuous if and only if f^{-1} (G) is gm-Δ -open in (U, $\tau_R(X)$, $\mu_R(X)$) for every $m\Delta$-open set G in (V, $\tau_R(X)'$, $\mu_R(X)'$) .*

Proof. Let f : (U, $\tau_R(X)$, $\mu_R(X)$) $\to$ (V, $\tau_R(X)'$, $\mu_R(X)'$) be gm-Δ -continuous and G be an $m\Delta$-open set in (V, $\tau_R(X)'$, $\mu_R(X)'$) . Then G^c is $m\Delta$-closed in (V, $\tau_R(X)'$, $\mu_R(X)'$) and since f is gm-Δ -continuous, $f^{-1}(G^c)$ is gm-Δ -closed in (U, $\tau_R(X)$, $\mu_R(X)$). But $f^{-1}(G^c)$ = $f^{-1}((G))^c$ and so f^{-1}(G) is gm-Δ -open in (U, $\tau_R(X)$, $\mu_R(X)$).
Conversely, assume that f^{-1}(G) is gm-Δ -open in (U, $\tau_R(X)$, $\mu_R(X)$) for each $m\Delta$-open set G in (V, $\tau_R(X)'$, $\mu_R(X)'$) . Let F be a $m\Delta$-closed set in (V, $\tau_R(X)'$, $\mu_R(X)'$) . Then F^c is $m\Delta$-open in (V, $\tau_R(X)'$, $\mu_R(X)'$) and by assumption, $f^{-1}(F^c)$ is gm-Δ -open in (U, $\tau_R(X)$, $\mu_R(X)$). Since $f^{-1}(F^c)$ = $f^{-1}((F))^c$, we have f^{-1}(F) is $m\Delta$-closed in (U, $\tau_R(X)$, $\mu_R(X)$) and so f is gm-Δ -continuous. □

Definition 3.4. A map f : (U, $\tau_R(X)$, $\mu_R(X)$) $\to$ (V, $\tau_R(X)'$, $\mu_R(X)'$) is called

1. $m\Delta$-irresolute if f^{-1}(G) is a micro semi Δ-closed set of (U, $\tau_R(X)$, $\mu_R(X)$) for every micro semi Δ-closed set G of (V, $\tau_R(X)'$, $\mu_R(X)'$) .
2. gm-Δ -irresolute if f^{-1}(G) is a gm-Δ -closed set of (U, $\tau_R(X)$, $\mu_R(X)$) for every gm-Δ -closed set G of (V, $\tau_R(X)'$, $\mu_R(X)'$) .

Example 3.6. *Let U = {1, 2, 3} with U/ R= {{1}, {2, 3}} and X= {1}. The nano topology $\tau_R(X)$ consists of {ϕ, {1}, U}. If μ = {1, 3} then the micro topology $\mu_R(X)$ consists of {ϕ, {1}, {1, 3}, U}. Then micro semi-closed sets are ϕ, {2}, {3}, {2, 3}, U and micro semi Δ-closed sets are ϕ, {1}, {2}, {3}, {1, 2}, {1, 3}, {2, 3}, U. Let V = {1,2, 3} with V/ R= {{1, 2, 3}} and X = {2, 3}. The micro topology $\tau_R(X)'$ = {ϕ, V}. If μ = {1, 3} then the micro topology $\mu_R(X)$ = {ϕ, {1, 3}, V}. Then micro semi-closed sets are ϕ, {2}, V and micro semi Δ-closed sets are ϕ, {2}, {1, 3}, V. Define f : (U, $\tau_R(X)$, $\mu_R(X)$) $\to$ (V, $\tau_R(X)'$, $\mu_R(X)'$) to be the identity map. Then f is $m\Delta$-irresolute.*

Example 3.7. *Let U = {1, 2, 3} with U/ R= {{1, 2, 3}} and X= {2, 3}. The nano topology $\tau_R(X)$ consists of {ϕ, U}. If μ = {1, 3} then the micro topology $\mu_R(X)$ consists of {ϕ, {1, 3}, U}. Then $m\Delta$-closed sets are ϕ, {2}, {1, 3}, U and gm-Δ -closed sets are ϕ, {1}, {2}, {3}, {1, 2}, {1, 3}, {2, 3}, U. Let V = {1,2, 3} with*

V/ R= {{1}, {2, 3}} and X = {1}. The micro topology $\tau_R(X)'$ consists of {ϕ, {1}, V}. If μ = {1, 3} then the micro topology $\mu_R(X)$ consists of {ϕ, {1}, {1, 3}, V}. Then $m\Delta$-closed sets are ϕ, {1}, {2}, {1, 2}, {1, 3}, {2, 3}, V and gm-Δ -closed sets are ϕ, {1}, {2}, {1, 2}, {1, 3}, {2, 3}, V. Define f : (U, $\tau_R(X)$, $\mu_R(X)$) $\to$ (V, $\tau_R(X)'$, $\mu_R(X)'$) to be the identity map. Then f is gm-Δ -irresolute.

Theorem 3.4. *Every gm-Δ -irresolute map is gm-Δ -continuous but not conversely.*

Proof. Let f : (U, $\tau_R(X)$, $\mu_R(X)$) $\to$ (V, $\tau_R(X)'$, $\mu_R(X)'$) be a gm-Δ -irresolute map. Let G be a $m\Delta$-closed set of (V, $\tau_R(X)'$, $\mu_R(X)'$) . Then by the Proposition 2.1, G is gm-Δ -closed. Since f is gm-Δ -irresolute, then f^{-1}(G) is a gm-Δ -closed set of (U, $\tau_R(X)$, $\mu_R(X)$). Therefore f is gm-Δ -continuous. □

Theorem 3.5. *Let f : (U, $\tau_R(X)$, $\mu_R(X)$) $\to$ (V, $\tau_R(X)'$, $\mu_R(X)'$) and g : (V, $\tau_R(X)'$, $\mu_R(X)'$) $\to$ (W, $\tau_R(X)''$) be two arbitrary maps. Then*

1. *g $\circ$ f is gm-Δ -continuous if g is $m\Delta$-continuous and f is gm-Δ -continuous.*
2. *g $\circ$ f is gm-Δ -irresolute if both f and g are gm-Δ -irresolute.*
3. *g $\circ$ f is gm-Δ -continuous if g is gm-Δ -continuous and f is gm-Δ -irresolute.*

Proof. Omitted. □

Definition 3.5. A map f : (U, $\tau_R(X)$, $\mu_R(X)$) $\to$ (V, $\tau_R(X)'$, $\mu_R(X)'$) is called

1. contra-$m\Delta$-continuous if f^{-1}(G) is a $m\Delta$-closed set of (U, $\tau_R(X)$, $\mu_R(X)$) for every $m\Delta$-open set G of (V, $\tau_R(X)'$, $\mu_R(X)'$) .
2. contra-gm-Δ -continuous if f^{-1}(G) is a gm-Δ -closed set of (U, $\tau_R(X)$, $\mu_R(X)$) for every $m\Delta$-open set G of (V, $\tau_R(X)'$, $\mu_R(X)'$) .

Proposition 3.4. *Every contra-$m\Delta$-continuous is contra-gm-Δ -continuous but not conversely.*

Proof. Let f : (U, $\tau_R(X)$, $\mu_R(X)$) $\to$ (V, $\tau_R(X)'$, $\mu_R(X)'$) be a contra $m\Delta$-continuous map and let G be any $m\Delta$-open set in (V, $\tau_R(X)'$, $\mu_R(X)'$) . Then, f^{-1}(G) is $m\Delta$-closed in U. Since every $m\Delta$-closed set is gm-Δ -closed, f^{-1}(G) is gm-Δ -closed in U. Therefore f is contra-gm-Δ -continuous. □

Example 3.8. *Let U = {1, 2, 3} with U/ R= {{1, 2, 3}} and X= {2, 3}. The nano topology $\tau_R(X)$ consists of {ϕ, U}. If μ = {1, 3} then the micro topology $\mu_R(X)$ consists of {ϕ, {1, 3}, U}. Then $m\Delta$-closed sets are ϕ, {2}, {1, 3}, U and gm-Δ -closed sets are ϕ, {1}, {2}, {3}, {1, 2}, {1, 3}, {2, 3}, U. Let V = {1,2, 3} with V/ R= {{1}, {2, 3}} and X = {1}. The micro topology $\tau_R(X)'$ consists of {ϕ, {1}, V}. If μ = {1, 3} then the micro topology $\mu_R(X)$ consists of {ϕ, {1}, {1, 3}, V}. Then $m\Delta$-closed sets are ϕ, {1}, {2}, {1, 2}, {1, 3}, {2, 3}, V and gm-Δ -closed sets are ϕ, {1}, {2}, {1, 2}, {1, 3}, {2, 3}, V. Define f : (U, $\tau_R(X)$, $\mu_R(X)$) $\to$ (V, $\tau_R(X)'$, $\mu_R(X)'$) to be the identity map. Then f is contra-gm-Δ -continuous but not contra $m\Delta$-continuous, since f^{-1}({2, 3}) = {2, 3} is not $m\Delta$-closed in (U, $\tau_R(X)$, $\mu_R(X)$).*

Conclusion

In this article, the idea of introducing new classes of open-like sets micro semi Δ-open, micro α-Δ-open, micro pre-Δ-open, micro β-Δ-open, micro regular-Δ-open sets, micro semi Δ-continuous, micro α-Δ-continuous, micro pre-Δ-continuous, micro b-Δ-continuous micro β-Δ-continuous and micro regular-Δ-continuous in micro topological spaces We introduced $m\Delta$-continuous maps, gm-Δ -continuous maps, $m\Delta$-irresolute maps, gm-Δ -irresolute maps, contra $m\Delta$-continuous maps and contra-gm-Δ -continuous maps in micro topological spaces and discuss some of their properties. In future, we have extended this work in various micro topological fields with some applications.

Acknowledgment

I thank to referees for giving their useful suggestions and help to improve this manuscript.

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