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Comment on the primals defined in some examples of primal topological spaces

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Abstract: In this comment, we mention and correct some examples of primals in the following articles cited in [1], [2] and [3], by giving remarks and correct examples. Finally, the main reason for these incorrect examples has been mentioned.

Key words: Primal, primal topological space, closure operator.

1. Introduction

Throughout this paper, the universe, primal, and power set of of the universe are denoted by Ξ, $\mathcal{P}$ and 2^{Ξ}, respectively.

Definition 1.1. [4] A collection $\mathcal{P} \subseteq 2^{\Xi}$ is called a primal on Ξ if it satisfies the following conditions:

(1) $\Xi \notin \mathcal{P}$,

(2) if $\Lambda \in \mathcal{P}$ and $\Gamma \subseteq \Lambda$, then $\Gamma \in \mathcal{P}$,

(3) if $\Lambda \cap \Gamma \in \mathcal{P}$, then $\Lambda \in \mathcal{P}$ or $\Gamma \in \mathcal{P}$.

Corollary 1.1. *[4] The family $\mathcal{P} \subseteq 2^{\Xi}$ is a primal on Ξ if and only if it satisfies the following conditions:*

1) $\Xi \notin \mathcal{P}$,

2) *if* $\Gamma \notin \mathcal{P}$ *and* $\Gamma \subseteq \Lambda$, *then* $\Lambda \notin \mathcal{P}$,

3) *if* $\Lambda \notin \mathcal{P}$ *and* $\Gamma \notin \mathcal{P}$, *then* $\Lambda \cap \Gamma \notin \mathcal{P}$.

Definition 1.2. [4] A topological space (Ξ, τ) with a primal $\mathcal{P}$ on Ξ is called a primal topological space and denoted by $(\Xi, \tau, \mathcal{P})$.

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Definition 1.3. [4] Let $(\Xi, \tau, \mathcal{P})$ be a primal topological space. The family of all open neighborhoods of a point ξ of Ξ will be denoted by $O(\Xi, \xi)$. We consider a map $(\cdot)^\diamond : 2^\Xi \to 2^\Xi$ as $\Lambda^\diamond(\Xi, \tau, \mathcal{P}) = \{\xi \in \Xi | (\forall U \in O(\Xi, \xi))(\Lambda^c \cup U^c \in \mathcal{P})\}$ for any subset Λ of Ξ. We can also write $\Lambda^\diamond$ as $\Lambda^\diamond(\Xi, \tau, \mathcal{P})$ to specify the primal as per our requirements.

The remarks and counter-examples bellow give the correct statements for the corresponding statements on the cited articles.

2. The corrections

Remark 2.1. *Let Ξ be ANY set. The set Ξ could be empty set as well. Consider the empty family $\mathcal{P} = \emptyset$. By the way, we would like to recall that the empty family $(\emptyset)$ and the family containing the empty set $(\{\emptyset\})$ are not the same.*

Now, let $\mathcal{P} = \emptyset$. We will prove that the empty family, namely $\mathcal{P} = \emptyset$, is a primal on ANY set Ξ.

$$\mathcal{P} \text{ is a primal on } \Xi :\Leftrightarrow \begin{cases} \mathbf{P_1}) & \Xi \notin \mathcal{P} \\ \mathbf{P_2}) & (\Lambda \in \mathcal{P})(\Gamma \subseteq \Lambda) \Rightarrow \Gamma \in \mathcal{P} \\ \mathbf{P_3}) & (\Lambda \notin \mathcal{P})(\Gamma \notin \mathcal{P}) \Rightarrow \Lambda \cap \Gamma \notin \mathcal{P} \end{cases}$$

If we show that these 3 propositions are true, then we show that the empty family is a primal on the set Ξ.

$\mathbf{P}_1$) $\Xi \notin \emptyset$. *This is obvious.*

$\mathbf{P_2}$) *Now let us show that the proposition*

$$(\Lambda \in \emptyset)(\Gamma \subseteq \Lambda) \Rightarrow \Gamma \in \emptyset \ldots (*)$$

is true.

$$\left[\underbrace{(\Lambda \in \emptyset)}_{False}(\underbrace{\Gamma \subseteq \Lambda}_{p}) \Rightarrow \underbrace{\Gamma \in \emptyset}_{False}\right] \equiv [\underbrace{(\ False \ \wedge p)}_{False} \Rightarrow \ False \ ]$$
$$\equiv [\ False \ \Rightarrow \ False \ ]$$
$$\equiv \ True$$

Then, the proposition $(*)$ is true.

$\mathbf{P_3}$) *Now let us show that the proposition*

$$(\Lambda \notin \emptyset)(\Gamma \notin \emptyset) \Rightarrow \Lambda \cap \Gamma \notin \emptyset \ldots (**)$$

is true.

$$\left[\underbrace{(\Lambda \notin \emptyset)}_{True}(\underbrace{\Gamma \notin \emptyset}_{True}) \Rightarrow \underbrace{\Lambda \cap \Gamma \notin \emptyset}_{True}\right] \equiv [\underbrace{(\ True \ \wedge True)}_{True} \Rightarrow \ True \ ]$$
$$\equiv [\ True \ \Rightarrow \ True \ ]$$
$$\equiv \ True$$

Then, the proposition $(**)$ is true.

Therefore, the family $\mathcal{P} = \emptyset$ is a primal on ANY set Ξ. As a result

All primals on $\Xi = \emptyset$.

$$\mathcal{P}_1 = \emptyset = 2^{\Xi} \backslash \{\Xi\}.$$

All primals on $\Xi = \{\xi_1\}$.

$$\begin{aligned} \mathcal{P}_1 &= \emptyset \\ \mathcal{P}_2 &= \{\emptyset\} = 2^{\Xi} \backslash \{\Xi\}. \end{aligned}$$

All primals on $\Xi = \{\xi_1, \xi_2\}$.

$$\begin{aligned} \mathcal{P}_1 &= \emptyset \\ \mathcal{P}_2 &= \{\emptyset, \{\xi_1\}\} \\ \mathcal{P}_3 &= \{\emptyset, \{\xi_2\}\} \\ \mathcal{P}_4 &= \{\emptyset, \{\xi_1\}, \{\xi_2\}\} = 2^{\Xi} \backslash \{\Xi\}. \end{aligned}$$

All primals on $\Xi = \{\xi_1, \xi_2, \xi_3\}$.

$$\begin{aligned} \mathcal{P}_1 &= \emptyset \\ \mathcal{P}_2 &= \{\emptyset, \{\xi_1\}, \{\xi_2\}, \{\xi_1, \xi_2\}\} \\ \mathcal{P}_3 &= \{\emptyset, \{\xi_1\}, \{\xi_3\}, \{\xi_1, \xi_3\}\} \\ \mathcal{P}_4 &= \{\emptyset, \{\xi_2\}, \{\xi_3\}, \{\xi_2, \xi_3\}\} \\ \mathcal{P}_5 &= \{\emptyset, \{\xi_1\}, \{\xi_2\}, \{\xi_3\}, \{\xi_1, \xi_2\}, \{\xi_1, \xi_3\}\} \\ \mathcal{P}_6 &= \{\emptyset, \{\xi_1\}, \{\xi_2\}, \{\xi_3\}, \{\xi_1, \xi_2\}, \{\xi_2, \xi_3\}\} \\ \mathcal{P}_7 &= \{\emptyset, \{\xi_1\}, \{\xi_2\}, \{\xi_3\}, \{\xi_1, \xi_3\}, \{\xi_2, \xi_3\}\} \\ \mathcal{P}_8 &= \{\emptyset, \{\xi_1\}, \{\xi_2\}, \{\xi_3\}, \{\xi_1, \xi_2\}, \{\xi_2, \xi_3\}, \{\xi_1, \xi_3\}\} = 2^{\Xi} \backslash \{\Xi\}. \end{aligned}$$

Corollary 2.1. *According to Remark 2.1, Theorem 15 part (i) in [1] is satisfied for any set* Ξ.

Remark 2.2. *In Example 2 in [2], the primal is defined as* $\mathcal{P}_f$ = *all finite subsets of the real line* $\mathbb{R}$ *whose complement is not finite. Clearly,* $\mathcal{P}_f$ *is not a primal, since for any* $\xi \in \mathbb{R}$, *we have* $\{\xi\} \in \mathcal{P}_f$ *and* $\{\xi\} = (-\infty, 1] \cap [1, \infty)$ *but neither* $(-\infty, 1]$ *nor* $[\xi, \infty) \in \mathcal{P}_f$.

Remark 2.3. *In Example 1 in [3], the primal is defined on a universe* $\Xi = \{\xi_1, \xi_2, \xi_3, \xi_4\}$ *as* $\mathcal{P} = \{\phi, \{\xi_3\}, \{\xi_4\}\}$. *This primal is incorrect, since for example* $\{\xi_3\} = \{\xi_1, \xi_3\} \cap \{\xi_2, \xi_3\} \in \mathcal{P}$ *but neither* $\{\xi_1, \xi_3\}$ *nor* $\{\xi_2, \xi_3\} \in \mathcal{P}$. *Consequentially, Example 4. is incorrect. Now, we give the following correction:*

Example 2.1. *Suppose that* $(\Xi, \mathfrak{g}, \mathcal{P})$ *is a GPT space, where* $\Xi = \{\xi_1, \xi_2, \xi_3, \xi_4\}$, $\mathfrak{g} = \{\phi, \{\xi_1\}, \{\xi_2\}, \{\xi_3\}, \{\xi_1, \xi_2\}, \{\xi_2, \xi_3\}\{\xi_1, \xi_3\}, \{\xi_1, \xi_2, \xi_3\}, \Xi\}$ *and* $\mathcal{P} = 2^{\Xi} \backslash \{\Xi\}$. *Consider* $E = \{\xi_1, \xi_2, \xi_4\}$. *Thus,* E *is* $(\mathfrak{g}, \mathcal{P}) - \beta$*-open. By using the same primal, Example 4 can be corrected.*

Remark 2.4. *In Example 2 in [3], the primal is defined on a universe* $\Xi = \{\xi_1, \xi_2, \xi_3, \xi_4\}$ *as* $\mathcal{P} = \{\phi, \{\xi_2\}, \{\xi_4\}\}$. *This primal is incorrect, since for example* $\{\xi_2\} = \{\xi_1, \xi_2\} \cap \{\xi_2, \xi_3\} \in \mathcal{P}$ *but neither* $\{\xi_1, \xi_2\}$ *nor* $\{\xi_2, \xi_3\} \in \mathcal{P}$. *Consequentially, Example 5. is incorrect. Now, we give the following correction:*

Example 2.2. *Suppose that* $(\Xi, \mathfrak{g}, \mathcal{P})$ *is a GPT space, where* $\Xi = \{\xi_1, \xi_2, \xi_3, \xi_4\}$, $\mathfrak{g} = \{\phi, \{\xi_1, \xi_2\}, \{\xi_2, \xi_3\}, \{\xi_1, \xi_2, \xi_3\}, \Xi\}$ *and* $\mathcal{P} = 2^{\Xi} \backslash \{\Xi\}$. *Consider* $E = \{\xi_1, \xi_3, \xi_4\}$. *Thus,* E *is* $(\mathfrak{g}, \mathcal{P})$*-semi-open. By using the same primal, Example 5 can be corrected.*

Remark 2.5. *In Example 3 in [3], the primal is defined on a universe* $\Xi = \{\xi_1, \xi_2, \xi_3, \xi_4\}$ *as* $\mathcal{P} = \{\phi, \{\xi_3\}, \{\xi_4\}\}$. *This primal is incorrect, since for example* $\{\xi_4\} = \{\xi_1, \xi_4\} \cap \{\xi_2, \xi_4\} \in \mathcal{P}$ *but neither* $\{\xi_1, \xi_4\}$ *nor* $\{\xi_2, \xi_4\} \in \mathcal{P}$. *Consequentially, Example 6. is incorrect. Now, we give the following correction:*

Example 2.3. *Suppose that* $(\Xi, \mathfrak{g}, \mathcal{P})$ *is a GPT space, where* $\Xi = \{\xi_1, \xi_2, \xi_3, \xi_4\}$, $\mathfrak{g} = \{\phi, \{\xi_1, \xi_2, \xi_3\}\}$, *and* $\mathcal{P} = 2^{\Xi} \backslash \{\Xi\}$. *Consider* $E = \{\xi_1, \xi_2\}$. *Thus,* E *is* $(\mathfrak{g}, \mathcal{P})$*-pre-open. By using the same primal, Example 6 can be corrected.*

Remark 2.6. *In Example 7 in [3], the primal is defined on a universe* $\Xi = \{\xi_1, \xi_2, \xi_3, \xi_4\}$ *as* $\mathcal{P} = \{\phi, \{\xi_1\}, \{\xi_3\}, \{\xi_4\}, \{\xi_1, \xi_3\}, \{\xi_1, \xi_4\}, \{\xi_3, \xi_4\}\}$. *This primal is incorrect, since* $\phi = \{\xi_1, \xi_3, \xi_4\} \cap \{\xi_2\} \in \mathcal{P}$ *but neither* $\{\xi_1, \xi_3, \xi_4\}$ *nor* $\{\xi_2\} \in \mathcal{P}$. *Now, it is corrected as below:*

Example 2.4. *Suppose that* $(\Xi, \mathfrak{g}, \mathcal{P})$ *is a GPT space, where* $\Xi = \{\xi_1, \xi_2, \xi_3, \xi_4\}$, $\mathfrak{g} = \{\phi, \{\xi_1, \xi_2\}, \{\xi_2, \xi_3\}, \{\xi_1, \xi_2, \xi_3\}\}$ *and* $\mathcal{P} = \{\phi, \{\xi_1\}, \{\xi_3\}, \{\xi_4\}, \{\xi_1, \xi_3\}, \{\xi_1, \xi_4\}, \{\xi_3, \xi_4\}, \{\xi_1, \xi_3, \xi_4\}\}$. *Consider* $E = \{\xi_2, \xi_3, \xi_4\}$. *Hence, we have* $cl^{\diamond}(E) = \Xi$ *and* $i_{\mathfrak{g}}(\Xi) = \{\xi_1, \xi_2, \xi_3\}$. *Therefore,* E *is* $(\mathfrak{g}, \mathcal{P})$*-dense, Thus, which means it is not a* $(\mathfrak{g}, \mathcal{P})$*-pre-open set.*

Remark 2.7.

(1) Similarly, the defined primals in Example 7 in [1] and Examples 8,10,12,13 and 14 in [3] are not correct. Anyone can add correct primals to modify these examples and obtain the required aim of them.

(2) It should be noted that the essential reason for all the previous incorrect examples is that the defined primal in the given universe doesn't meet axiom (3) mentioned in definition 1.1 or corollary 1.1.

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