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On $\delta g\alpha$ Continuous Maps in Topological Spaces

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Abstract: This research paper's goal is to present and examine an innovative type of continuous map is named as $\delta g\alpha$ continuous map in TSs. Also discuss some basic properties of this continuous map. Further investigate the relationship between the newly defined map and the existing continuous map with suitable examples.

Key words: CS, OS, $\delta g\alpha$ CS, $\delta g\alpha$ OS, $\delta g\alpha$ continuous map, $\delta g\alpha$ irresolute map

1. Introduction

Continuous function plays a vital role in Topology. Many mathematicians have introduced and studied various stronger and weaker forms of the continuous functions. Levine [1] introduced and studied the weaker forms of continuity, namely semi-continuity, in the year 1963. R. Devi et al. [2–4, 6, 15–17] introduced generalized continuous maps in TSs and generalized α-continuous maps in TS(TSs) during the years 1991-1997. In the year 1961, Levine N [5] introduced the concepts of decomposition of continuity in TSs. D. Sivaraj and V.E. Sasikala [8, 11] introduced the study on Soft $\alpha-$OSs and Soft Pre-OSs in 2017 and also introduced Beta Generalized CSs in TSs in the year 2022. In 2002, A. Csaszar [13] introduced Generalized topology and generalized continuity. M. Lellis Thivagar [18] developed the Generalization of pairwise α-continuous functions. N. Biswas [19] studied characterizations of semi-continuous functions. V.E. Sasikala, D. Sivaraj, and R. Thirumalaisamy [20–22] studied notes on soft g-CSs.

Dontchev J and Jafari S, Noiri T [4, 9] defined the concept of contra-pre continuous functions. S.N. Maheswari and S.S. Thakur introduced the notion of α-irresolute mappings. V.E. Sasikala, D. Sivaraj, and A.P. Ponraj [14, 23] introduced soft semi weakly generalized CS in soft TSs and also defined the Soft swg-Separation Axioms in Soft TSs. Y. Gnanambal et al. [22] introduced the concept of gpr-continuous functions in TSs.

2. Preliminaries

Definition 2.1: A function $f:(I,\tau_1)\rightarrow(J,\tau_2)$ is claimed to be:

1. Semi continuous [2] if $f^{-1}(L)$ in semi-OS is (I,τ_1) for all OS L of (J,τ_2).

2. Pre continuous [14] if $f^{-1}(L)$ in pre-CS is (I,τ_1) for all CS L of (J,τ_2).

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3. α-continuous [6] if $f^{-1}(L)$ in α-CS is (I, τ_1) for all CS L of (J, τ_2).
4. β-continuous [5] if $f^{-1}(L)$ in semi pre-OS is (I, τ_1) for all OS L of (J, τ_2).
5. g-continuous [4] if $f^{-1}(L)$ in g-CS is (I, τ_1) for all CS L of (J, τ_2).
6. gα-continuous [3] if $f^{-1}(L)$ in gα-CS is (I, τ_1) for all CS L of (J, τ_2).
7. αg-continuous [3] if $f^{-1}(L)$ in αg-CS is (I, τ_1) for all CS L of (J, τ_2).
8. gp-continuous [9] if $f^{-1}(L)$ in gp-CS is (I, τ_1) for all CS L of (J, τ_2).
9. gpr-continuous [15] if $f^{-1}(L)$ in gpr-CS is (I, τ_1) for all CS L of (J, τ_2).

Definition 2.2: A function $f : (I, \tau_1) \to (J, \tau_2)$ is said to be:

1. Irresolute [1] if $f^{-1}(L)$ in semi OS is (I, τ_1) for all semi OS L of (J, τ_2).
2. gp-irresolute [9] if $f^{-1}(L)$ in gp-CS is (I, τ_1) for all gp-CS L of (J, τ_2).
3. α-irresolute [18] if $f^{-1}(L)$ in α-OS is (I, τ_1) for all α-OS L of (J, τ_2).

3. $\delta g\alpha$-Continuous Functions

This part presents the new class of $\delta g\alpha$ irresolute and continuous maps and examines some of their characteristics. Also, we provided some characterizations of $\delta g\alpha$-continuous mappings and $\delta g\alpha$ irresolute mappings in TSs.

Definition 3.1: Let (I, τ_1) and (J, τ_2) be any two topological spaces (2-TSs). A function $f : (I, \tau_1) \to (J, \tau_2)$ is called $\delta g\alpha$ Continuous if each CS's inverted image in J is a $\delta g\alpha$ CS in I, i.e., if $f^{-1}(L)$ is a $\delta g\alpha$ CS in (I, τ_1) for all CS L in (J, τ_2).

Example 3.2: Let $I = \{o_1, p_2, q_3\}$ with $\tau_1 = \{I, \emptyset, \{o_1\}, \{p_2\}, \{o_1, p_2\}\}$ and the CSs are $\{I, \emptyset, \{p_2, q_3\}, \{o_1, q_3\}, \{q_3\}\}$. Then, the $\delta g\alpha$-CSs are $\{\emptyset, I, \{o_1\}, \{p_2\}, \{q_3\}, \{o_1, p_2\}, \{p_2, q_3\}, \{o_1, q_3\}\}$. Let $J = \{o_1, p_2, q_3\}$ with $\tau_2 = \{J, \emptyset, \{o_1\}, \{p_2\}, \{o_1, p_2\}, \{o_1, q_3\}\}$ and the CSs are $\{\emptyset, J, \{p_2, q_3\}, \{o_1, q_3\}, \{q_3\}, \{p_2\}\}$. Then, the $\delta g\alpha$-CSs are $\{J, \emptyset, \{o_1\}, \{q_3\}, \{o_1, q_3\}\}$. Let $f : (I, \tau_1) \to (J, \tau_2)$ be defined by $f(o_1) = p_2, \quad f(p_2) = q_3, \quad f(q_3) = o_1$ Then, $f^{-1}(p_2) = \{o_1\}, \quad f^{-1}(q_3) = \{p_2\}, \quad f^{-1}(o_1) = \{q_3\}$ Here, the inverse images of CSs in V are as follows: $f(\{q_3\}) = \{q_3\}, \quad f(\{p_2\}) = \{p_2\}, \quad f(\{o_1, q_3\}) = \{o_1, q_3\}$ Then, $f^{-1}(\{q_3\}) = \{q_3\}, \quad f^{-1}(\{p_2\}) = \{p_2\}, \quad f^{-1}(\{o_1, q_3\}) = \{p_1, q_3\}$ are $\delta g\alpha$-CSs in (I, τ_1), thus f is $\delta g\alpha$-continuous.

Theorem 3.3: Let (I, τ_1) and (J, τ_2) be any two topological spaces (2-TSs). Let $f : (I, \tau_1) \to (J, \tau_2)$ be an f-continuous function, then $\delta g\alpha$ is continuous.

Proof: Let L be a CS in (J, τ_2). Then $f^{-1}(L)$ is a CS in (I, τ_1) since f is continuous. However, every CS is a $\delta g\alpha$ CS. Therefore, $f^{-1}(L)$ is a $\delta g\alpha$ CS. Hence, f is $\delta g\alpha$ continuous. The following example illustrates that the converse of the above theorem is not true.

Illustration 3.4:
Let $I = \{l_1, l_2, l_3\}$ with $\tau_1 = \{I, \emptyset, \{l_1\}, \{l_2\}, \{l_1, l_2\}\}$ and the CSs are $\{I, \emptyset, \{l_2, l_3\}, \{l_1, l_3\}, \{l_3\}\}$. Then $\delta g\alpha$ CSs are $\{\emptyset, I, \{l_1\}, \{l_2\}, \{l_3\}, \{l_1, l_2\}, \{l_2, l_3\}, \{l_1, l_3\}\}$. Let $J = \{o_1, o_2, o_3\}$ with $\tau_2 = \{J, \emptyset, \{o_1\}, \{o_2\}, \{o_1, o_2\}, \{o_1, o_3\}\}$ and the CSs are $\{\emptyset, J, \{o_2, o_3\}, \{o_1, o_3\}, \{o_3\}, \{o_2\}\}$. Then $\delta g\alpha$ CSs are $\{J, \emptyset, \{o_1\}, \{o_3\}, \{o_1, o_3\}\}$. Let

$f:(I,\tau_1)\to(J,\tau_2)$ be defined by $f(l_1)=o_2$, $f(l_2)=o_3$, $f(l_3)=o_1$. Then the inverse mappings are $f^{-1}(o_2)=\{l_1\}$, $f^{-1}(o_3)=\{l_2\}$, $f^{-1}(o_1)=\{l_3\}$. Here, the inverse images of CSs in J are $f^{-1}(\{l_3\})=\{o_3\}$, $f^{-1}(\{l_2\})=\{o_2\}$, $f^{-1}(\{l_1,l_3\})=\{o_1,o_3\}$. By Example 3.2, f is $\delta g\alpha$-continuous but not continuous since the inverse image $f^{-1}(\{o_2\})=\{l_2\}$ is not a CS in (I,τ_1).

Theorem 3.5: Let (I,τ_1) and (J,τ_2) be any two topological spaces (2-TSs). Let $f:(I,\tau_1)\to(J,\tau_2)$ be a $\delta g\alpha$-continuous function. Then f is continuous if (I,τ_1) exists as a $\delta g\alpha$-space.

Proof: Let $f:(I,\tau_1)\to(J,\tau_2)$ be a $\delta g\alpha$-continuous function. Let L be any closed set (CS) in (J,τ_2). Then $f^{-1}(L)$ is a $\delta g\alpha$-CS in (I,τ_1). Since (I,τ_1) is a $\delta g\alpha$-space, it follows that $f^{-1}(L)$ is a CS in (I,τ_1). Hence, f is continuous.

Theorem 3.6: Let (I,τ_1) and (J,τ_2) be any two topological spaces (2-TSs). Let $f:(I,\tau_1)\to(J,\tau_2)$ be a δ-continuous function. Then f is $\delta g\alpha$-continuous.

Proof: Let L be any closed set (CS) in (J,τ_2). Since f is δ-continuous, $f^{-1}(L)$ is a δ-CS in (I,τ_1). However, every δ-CS is also a $\delta g\alpha$-CS, which implies that $f^{-1}(L)$ is a $\delta g\alpha$-CS in (I,τ_1).

$\therefore$ f is $\delta g\alpha$-continuous.

The following example illustrates that the converse of the above theorem is not true.

Example 3.7: Let $I=\{l_1,m_2,n_3,o_4\}$ with $\tau_1=\{I,\emptyset,\{l_1\},\{m_2\},\{l_1,m_2\},\{l_1,m_2,n_3\}\}$ and the δ-CSs are $\{\emptyset,I,\{m_2,n_3,o_4\},\{l_1,m_3,o_4\},\{n_3,o_4\},\{o_4\}\}$. Then $\delta g\alpha$-CSs are $\{\emptyset,I,\{l_1\},\{n_3\}\}$. Let $J=\{p_1,p_2,p_3,p_4\}$ with $\tau_2=\{J,\emptyset,\{p_1\},\{p_2\},\{p_1,p_2\},\{p_1,p_3\},\{p_1,p_4\},\{p_1,p_2,p_3\},\{p_1,p_2,p_4\},\{p_1,p_3,p_4\}\}$ and the CSs are $\{\emptyset,J,\{p_2,p_3,p_4\},\{p_1,p_3,p_4\},\{p_3,p_4\},\{p_2,p_4\},\{p_2,p_3\},\{p_4\},\{p_3\},\{p_2\}\}$. Let $f:(I,\tau_1)\to(J,\tau_2)$ be defined by $f(l_1)=p_2$, $f(m_2)=p_4$, $f(n_3)=p_1$, $f(o_4)=p_3$. Then the inverse mappings are $f^{-1}(p_2)=\{l_1\}$, $f^{-1}(p_4)=\{m_2\}$, $f^{-1}(p_1)=\{n_3\}$, $f^{-1}(p_3)=\{o_4\}$. Here, f is $\delta g\alpha$-continuous but not δ-continuous since the inverse image $f^{-1}(\{p_2\})=\{l_1\}$ is $\delta g\alpha$-continuous but not a δ-CS in (I,τ_1).

Theorem 3.8: Let (I,τ_1) and (J,τ_2) be any two topological spaces (2-TSs). Let $f:(I,\tau_1)\to(J,\tau_2)$ be a $\delta g\alpha$-continuous function. Then f is δ-continuous.

Proof: Let L be any closed set (CS) in (J,τ_2). Since f is $\delta g\alpha$-continuous, $f^{-1}(L)$ is a $\delta g\alpha$-CS in (I,τ_1). But every $\delta g\alpha$-CS is also a δ-CS.

$\therefore$ $f^{-1}(L)$ is a δ-CS in (I,τ_1). Hence, f is δ-continuous.

The following example illustrates that the above theorem's converse is untrue.

Illustration 3.9: Let $I=\{o_1,p_2,q_3,r_4\}$ with $\tau_1=\{I,\emptyset,\{o_1\},\{p_2\},\{o_1,p_2\},\{o_1,p_2,q_3\}\}$ and the δ-CSs are $\{\emptyset,I,\{p_2,q_3,r_4\},\{o_1,q_3,r_4\},\{q_3,r_4\},\{r_4\}\}$. Then $\delta g\alpha$-CSs are $\{\emptyset,I,\{o_1\},\{q_3\}\}$. Let $J=\{r_1,r_2,r_3,r_4\}$ with $\tau_2=\{J,\emptyset,\{r_1\},\{r_2\},\{r_1,r_2\},\{r_1,r_3\},\{r_1,r_4\},\{r_1,r_2,r_3\},\{r_1,r_2,r_4\},\{r_1,r_3,r_4\}\}$ and the CSs are $\{\emptyset,J,\{r_2,r_3,r_4\},\{r_1,r_3,r_4\},\{r_3,r_4\},\{r_2,r_4\},\{r_2,r_3\},\{r_4\},\{r_3\},\{r_2\}\}$. Let $f:(I,\tau_1)\to(J,\tau_2)$ be defined by $f(o_1)=r_2$, $f(p_2)=r_4$, $f(q_3)=r_1$, $f(r_4)=r_3$. Then the inverse mappings are $f^{-1}(r_2)=\{o_1\}$, $f^{-1}(r_4)=\{p_2\}$, $f^{-1}(r_1)=\{q_3\}$, $f^{-1}(r_3)=\{r_4\}$. Here, f is δ-continuous but not $\delta g\alpha$-continuous since the inverse image $f^{-1}(\{r_1,r_3,r_4\})=\{p_2,q_3,r_4\}$ is δ-continuous but not a $\delta g\alpha$-CS in (I,τ_1).

Theorem 3.10: Let (I,τ_1) and (J,τ_2) be any two topological spaces (2-TSs). Let $f:(I,\tau_1)\to(J,\tau_2)$ be an α-continuous function. Then f is $\delta g\alpha$-continuous.

Proof: Let L be any closed set (CS) in (J,τ_2). Since f is α-continuous, $f^{-1}(L)$ is an α-CS in (I,τ_1). But every α-CS is also a $\delta g\alpha$-CS.

$\therefore$ $f^{-1}(L)$ is a $\delta g\alpha$-CS in (I,τ_1). Hence, f is $\delta g\alpha$-continuous.

The following example illustrates that the converse of the above theorem is not true.

Illustration 3.11: Let $I=\{k_1,k_2,k_3\}$ with $\tau_1=\{I,\emptyset,\{k_1\},\{k_2\},\{k_1,k_2\}\}$ and the α-CSs are $\{\emptyset,I,\{k_3\},\{k_2,k_3\},\{k_1,k_3\}\}$. Then $\delta g\alpha$-CSs are $\{\emptyset,I,\{k_1\},\{k_2\},\{k_3\},\{k_1,k_2\},\{k_2,k_3\},\{k_1,k_3\}\}$. Let $J=\{k_1,k_2,k_3\}$ with $\tau_2=\{J,\emptyset,\{k_1\},\{k_2\},\{k_1,k_2\},\{k_1,k_3\}\}$ and the CSs are $\{\emptyset,J,\{k_2,k_3\},\{k_1,k_3\},\{k_3\},\{k_2\}\}$. Then $\delta g\alpha$-CSs are $\{J,\emptyset,\{k_1\},\{k_3\},\{k_1,k_3\}\}$. Let $f:(I,\tau_1)\to(J,\tau_2)$ be defined by $f(k_1)=k_3,\quad f(k_2)=k_2,\quad f(k_3)=k_1$. Then the inverse mappings are $f^{-1}(k_3)=\{k_1\},\quad f^{-1}(k_2)=\{k_2\},\quad f^{-1}(k_1)=\{k_3\}$. Here, f is $\delta g\alpha$-continuous but not α-continuous since the inverse image $f^{-1}(\{k_2\})=\{k_2\}$ is not an α-CS in (I,τ_1).

Theorem 3.12: Let (I,τ_1) and (J,τ_2) be any two topological spaces. Let $f:(I,\tau_1)\to(J,\tau_2)$. If f is a g-continuous function, then f is $\delta g\alpha$-continuous.

Proof: Let L be any CS in (J,τ_2). Then $f^{-1}(L)$ is a g-CS in (I,τ_1) as f is g-continuous. But every g-CS is a $\delta g\alpha$-CS, hence $f^{-1}(L)$ is a $\delta g\alpha$-CS in (I,τ_1).

$\therefore$ f is $\delta g\alpha$-continuous.

The following example demonstrates that the converse of the above theorem does not hold.

Illustration 3.13: Let $I=\{k_1,k_2,k_3\}$ with $\tau_1=\{I,\emptyset,\{k_1\},\{k_2\},\{k_1,k_2\}\}$ and the g-CSs are $\{\emptyset,I,\{k_3\},\{k_2,k_3\},\{k_1,k_3\}\}$. Then $\delta g\alpha$-CSs are $\{\emptyset,I,\{k_1\},\{k_2\},\{k_3\},\{k_1,k_2\},\{k_2,k_3\},\{k_1,k_3\}\}$. Let $J=\{k_1,k_2,k_3\}$ with $\tau_2=\{J,\emptyset,\{k_1\},\{k_2\},\{k_1,k_2\},\{k_1,k_3\}\}$ and the CSs are $\{\emptyset,J,\{k_2,k_3\},\{k_1,k_3\},\{k_3\},\{k_2\}\}$. Let $f:(I,\tau_1)\to(J,\tau_2)$ be defined by $f(k_1)=k_3,\quad f(k_2)=k_2,\quad f(k_3)=k_1$. Then the inverse mappings are $f^{-1}(k_3)=\{k_1\},\quad f^{-1}(k_2)=\{k_2\},\quad f^{-1}(k_1)=\{k_3\}$. Here, f is $\delta g\alpha$-continuous but not g-continuous since the inverse image $f^{-1}(\{k_2\})=\{k_2\}$ is not a g-CS in (I,τ_1).

Theorem 3.14: Let (I,τ_1) and (J,τ_2) be any two topological spaces (2-TSs). If a function $f:(I,\tau_1)\to(J,\tau_2)$ is δg-continuous, then f is $\delta g\alpha$-continuous.

Proof: Let B be a closed set (CS) in (J,τ_2). Since f is δg-continuous, $f^{-1}(B)$ is a δg-CS in (I,τ_1). But every δg-CS is also a $\delta g\alpha$-CS.

$\therefore$ $f^{-1}(B)$ is a $\delta g\alpha$-CS in (I,τ_1). Hence, f is $\delta g\alpha$-continuous.

The following example illustrates that the converse of the above theorem is not true.

Illustration 3.15: Let $I=\{o_1,o_2,o_3,o_4\}$ with $\tau_1=\{I,\emptyset,\{o_1\},\{o_2\},\{o_1,o_2\},\{o_1,o_2,o_3\}\}$ and the CSs are $\{\emptyset,I,\{o_2,o_3,o_4\},\{o_1,o_3,o_4\},\{o_3,o_4\},\{o_4\}\}$. The δg-CS is $\{\emptyset,I\}$. Then $\delta g\alpha$-CSs are $\{\emptyset,I,\{o_1\},\{o_3\}\}$. Let $J=\{o_1,o_2,o_3,o_4\}$ with $\tau_2=\{J,\emptyset,\{o_1\},\{o_2\},\{o_1,o_2\},\{o_1,o_3\},\{o_1,o_4\},\{o_1,o_2,o_3\},\{o_1,o_2,o_4\},\{o_1,o_3,o_4\}\}$ and the CSs are $\{\emptyset,J,\{o_2,o_3,o_4\},\{o_1,o_3,o_4\},\{o_3,o_4\},\{o_2,o_4\},\{o_2,o_3\},\{o_4\},\{o_3\},\{o_2\}\}$. Then $\delta g\alpha$-CSs are $\{J,\emptyset,\{o_1\},\{o_2\},\{o_3\},\{o_1,o_2,o_3\}\}$. Let $f:(I,\tau_1)\to(J,\tau_2)$ be defined by $f(o_1)=o_2,\quad f(o_2)=o_4,\quad f(o_3)=o_3,\quad f(o_4)=o_1$.

Then the inverse mappings are $f^{-1}(o_2)=\{o_1\},\quad f^{-1}(o_4)=\{o_2\},\quad f^{-1}(o_3)=\{o_3\},\quad f^{-1}(o_1)=\{o_4\}$. Here, f is $\delta g\alpha$-continuous but not δg-continuous since the inverse image $f^{-1}(\{o_3\})=\{o_3\}$ is $\delta g\alpha$-continuous but not a δg-CS in (I,τ_1).

Theorem 3.16: Let (I,τ_1) and (J,τ_2) be any two topological spaces (2-TSs). If a function $f:(I,\tau_1)\to(J,\tau_2)$ is $g\delta$-continuous, then f is $\delta g\alpha$-continuous.

Proof: Let B be a closed set (CS) in (J,τ_2). Since f is $g\delta$-continuous, $f^{-1}(B)$ is a $g\delta$-CS in (I,τ_1). But every $g\delta$-CS is also a $\delta g\alpha$-CS$\Rightarrow f^{-1}(B)$ is a $\delta g\alpha$-CS in (I,τ_1).

$\therefore$ f is $\delta g\alpha$-continuous.

The following example illustrates that the above theorem's converse is untrue.

Illustration 3.17: Let $I = \{n_1, m_2, o_3, p_4\}$ with $\tau_1 = \{I, \emptyset, \{n_1\}, \{m_2\}, \{n_1, m_2\}, \{n_1, m_2, o_3\}\}$ and the closed sets (CSs) are $\{\emptyset, I, \{m_2, o_3, p_4\}, \{n_1, o_3, p_4\}, \{o_3, p_4\}, \{p_4\}\}$. The $g\delta$-CSs are $\{\emptyset, I\}$ and the $\delta g\alpha$-CSs are $\{\emptyset, I, \{n_1\}, \{o_3\}\}$. Let $J = \{n_1, m_2, o_3, p_4\}$ with $\tau_2 = \{J, \emptyset, \{n_1\}, \{m_2\}, \{n_1, m_2\}, \{n_1, o_3\}, \{n_1, p_4\}, \{n_1, m_2, o_3\}, \{n_1, m_2, p_4\}, \{n_1, o_3, p_4\}\}$ and the CSs are $\{\emptyset, J, \{m_2, o_3, p_4\}, \{n_1, o_3, p_4\}, \{o_3, p_4\}, \{m_2, p_4\}, \{m_2, o_3\}, \{p_4\}, \{o_3\}, \{m_2\}\}$. The $g\delta$-CSs are $\{\emptyset, J, \{m_2\}, \{o_3\}, \{m_2, o_3\}\}$ and the $\delta g\alpha$-CSs are $\{J, \emptyset, \{n_1\}, \{m_2\}, \{o_3\}, \{n_1, m_2, o_3\}\}$. Let the function $f : (I, \tau_1) \to (J, \tau_2)$ be defined by $f(n_1) = m_2, \quad f(m_2) = p_4, \quad f(o_3) = o_3, \quad f(p_4) = n_1$. Then the inverse images are $f^{-1}(m_2) = n_1, \quad f^{-1}(p_4) = m_2, \quad f^{-1}(o_3) = o_3, \quad f^{-1}(n_1) = p_4$. Thus, f is $\delta g\alpha$-continuous but not $g\delta$-continuous, since the inverse image $f^{-1}(\{o_3\}) = \{o_3\}$ is not a $g\delta$-CS in (I, τ_1).

Theorem 3.18: Let (I, τ_1) and (J, τ_2) be any two topological spaces (2-TSs). A function $f : (I, \tau_1) \to (J, \tau_2)$ is $\delta g\alpha$-continuous if and only if it is αg-continuous.

Proof: Let $f : (I, \tau_1) \to (J, \tau_2)$ be a $\delta g\alpha$-continuous map. Let G be a closed set (CS) in (J, τ_2). Since f is $\delta g\alpha$-continuous, $f^{-1}(G)$ is a $\delta g\alpha$-CS in (I, τ_1). Since every $\delta g\alpha$-CS is an αg-CS, it follows that $f^{-1}(G)$ is an αg-CS in (I, τ_1).

$\therefore$ f is αg-continuous.

The following example illustrates that the above theorem's converse is untrue.

Illustration 3.19: Let $I = \{j_1, j_2, j_3, j_4\}$ with $\tau = \{I, \emptyset, \{j_1\}, \{j_3\}, \{j_4\}, \{j_1, j_3\}, \{j_1, j_4\}, \{j_3, j_4\}, \{j_1, j_2, j_3\}\}$. The CSs are $\{I, \emptyset, \{j_2, j_3, j_4\}, \{j_1, j_2, j_4\}, \{j_1, j_2, j_3\}, \{j_2, j_4\}, \{j_2, j_3\}, \{j_1, j_2\}, \{j_2\}\}$. The $\delta g\alpha$-CSs are $\{I, \emptyset, \{j_1\}, \{j_3\}, \{j_4\}, \{j_1, j_3\}, \{j_1, j_4\}, \{j_3, j_4\}\}$. The αg-CSs are $\{I, \emptyset, \{j_2, j_3, j_4\}, \{j_1, j_2, j_4\}, \{j_1, j_2, j_3\}, \{j_2, j_4\}, \{j_2, j_3\}, \{j_1, j_2\}, \{j_2\}\}$. Let $J = \{j_1, j_2, j_3, j_4\}$ with $\tau_2 = \{J, \emptyset, \{j_1\}, \{j_1, j_2\}\}$. The CSs are $\{J, \emptyset, \{j_2, j_3, j_4\}, \{j_3, j_4\}\}$. Define a function $f : (I, \tau_1) \to (J, \tau_2)$ by:

$$f(j_1) = j_3, \quad f(j_2) = j_4, \quad f(j_3) = j_2, \quad f(j_4) = j_1.$$

Then the inverse images are $f^{-1}(j_3) = j_1, \quad f^{-1}(j_4) = j_2, \quad f^{-1}(j_2) = j_3, \quad f^{-1}(j_1) = j_4$. We have $f^{-1}(\{j_3, j_4\}) = \{j_1, j_2\}$, which is an αg-CS but not a $\delta g\alpha$-CS in (I, τ_1).

$\therefore$ f is αg-continuous but not $\delta g\alpha$-continuous.

Theorem 3.20: Let (I, τ_1) and (J, τ_2) be any two topological spaces. A function $f : (I, \tau_1) \to (J, \tau_2)$ is $g\alpha$-continuous if and only if f is $\delta g\alpha$-continuous.

Proof: Let B be a closed set (CS) in (J, τ_2). Then, since f is $g\alpha$-continuous, the preimage $f^{-1}(B)$ is a $g\alpha$-CS in (I, τ_1). But every $g\alpha$-CS is a $\delta g\alpha$-CS, which implies that $f^{-1}(B)$ is a $\delta g\alpha$-CS in (I, τ_1).

$\therefore$ f is $\delta g\alpha$-continuous.

The following example illustrates that the above theorem's converse is untrue.

Illustration 3.21: Let $I = \{l_1, m_2, n_3\}$ with $\tau_1 = \{I, \emptyset, \{l_1\}, \{m_2\}, \{n_3\}, \{l_1, m_2\}, \{m_2, n_3\}\}$ and the CSs are $\{\emptyset, I, \{l_1, n_3\}, \{m_2, n_3\}, \{l_1, m_2\}, \{n_3\}, \{l_1\}\}$. The $g\alpha$-CSs are $\{\emptyset, I, \{l_1\}, \{m_2\}\}$. Then, the $\delta g\alpha$-CSs are $\{\emptyset, I, \{l_1\}, \{m_2\}, \{m_2, n_3\}\}$. Let $J = \{l_1, m_2, n_3\}$ with $\tau_2 = \{J, \emptyset, \{l_1\}\}$ and the CSs are $\{\emptyset, J, \{m_2, n_3\}\}$. Then, the $\delta g\alpha$-CSs are $\{J, \emptyset, \{l_1\}, \{m_2\}, \{n_3\}, \{l_1, m_2\}, \{m_2, n_3\}, \{l_1, n_3\}\}$. Define the function $f : (I, \tau_1) \to (J, \tau_2)$ by $f(l_1) = l_1$, $f(m_2) = m_2$, $f(n_3) = n_3$. Then, $f^{-1}(l_1) = \{l_1\}$, $f^{-1}(m_2) = \{m_2\}$, $f^{-1}(n_3) = \{n_3\}$. Here, f is $\delta g\alpha$-continuous but not $g\alpha$-continuous since the inverse image $f^{-1}(\{m_2, n_3\}) = \{m_2, n_3\}$ is not a $g\alpha$-CS in (I, τ_1).

Theorem 3.22: Let (I, τ_1) and (J, τ_2) be any 2-TSs. A function $f : (I, \tau_1) \to (J, \tau_2)$ is g^*-continuous, then it is $\delta g\alpha$-continuous.

Proof: Let L be a CS in (J,τ_2). Then $f^{-1}(L)$ is g^*-closed in (I,τ_1) since f is g^*-continuous. But every g^*-CS is $\delta g\alpha$-closed.

$\therefore f^{-1}(L)$ is $\delta g\alpha$-closed. Hence, f is $\delta g\alpha$-continuous.

The following example illustrates that the above theorem's converse is untrue.

Illustration 3.23: Let $I=\{h_1,i_2,j_3,k_4\}$ with $\tau=\{I,\phi,\{h_1\},\{i_2\},\{h_1,i_2\},\{h_1,j_3\},\{h_1,k_4\},\{h_1,i_2,j_3\},\{h_1,i_2,k_4\},\{h_1,j_3,k_4\}\}$. CSs are $\{I,\phi,\{i_2,j_3,k_4\},\{h_1,j_3,k_4\},\{j_3,k_4\},\{i_2,j_3\},\{k_4\},\{j_3\},\{i_2\}\}$. $\delta g\alpha$-CSs are $\{I,\phi,\{h_1\},\{i_2\},\{j_3\},\{h_1,i_2,j_3\}\}$. g^*-CSs are $\{I,\phi,\{i_2\},\{j_3\},\{i_2,j_3\}\}$. Let $J=\{h_1,i_2,j_3,k_4\}$ with $\tau_2=\{J,\phi,\{h_1\},\{i_2\},\{h_1,i_2\},\{h_1,i_2,j_3\}\}$. CSs are $\{J,\phi,\{i_2,j_3,k_4\},\{h_1,j_3,k_4\},\{j_3,k_4\},\{k_4\}\}$. Define a function $f:(I,\tau_1)\to(J,\tau_2)$ by $f(h_1)=k_4$, $f(i_2)=j_3$, $f(j_3)=h_1$, $f(k_4)=i_2$. Then $f^{-1}(k_4)=\{h_1\}$, $f^{-1}(j_3)=\{i_2\}$, $f^{-1}(h_1)=\{j_3\}$, $f^{-1}(k_4)=\{i_2\}$. Then $f^{-1}(k_4)=\{h_1\}$ is $\delta g\alpha$-CS but not g^*-CS in (I,τ_1).

Theorem 3.24: Let (I,τ_1) and (J,τ_2) be any two topological spaces (2-TSs). A function $f:(I,\tau_1)\to(J,\tau_2)$ is pre-continuous if it is $\delta g\alpha$-continuous.

Proof: Let L be a closed set (CS) in (J,τ_2). Then $f^{-1}(L)$ is pre-closed in (I,τ_1) since f is pre-continuous. But every pre-closed set is a $\delta g\alpha$ closed set (CS).

$\therefore$ $f^{-1}(L)$ is a $\delta g\alpha$ closed set. Hence, f is $\delta g\alpha$-continuous.

The converse of the above theorem need not be true as illustrated by the following example.

Illustration 3.25: Let $I=\{m_1,m_2,m_3\}$ and $\tau_1=\{I,\emptyset,\{m_1\},\{m_1,m_2\}\}$. The closed sets (CSs) are $\{I,\emptyset,\{m_2,m_3\},\{m_3\}\}$. The $\delta g\alpha$-closed sets (CSs) are $\{I,\emptyset,\{m_1\},\{m_2\},\{m_3\},\{m_1,m_2\},\{m_1,m_3\},\{m_2,m_3\}\}$. The pre-closed sets (pre-CSs) are $\{I,\emptyset,\{m_1\},\{m_3\},\{m_1,m_3\},\{m_2,m_3\}\}$. Let $J=\{m_1,m_2,m_3\}$ and $\tau_2=\{J,\emptyset,\{m_1\},\{m_2\},\{m_3\},\{m_1,m_2\},\{m_1,m_3\},\{m_2,m_3\}\}$. The closed sets (CSs) of (J,τ_2) are $\{J,\emptyset,\{m_1\},\{m_2\},\{m_3\},\{m_1,m_2\},\{m_1,m_3\},\{m_2,m_3\}\}$. Define a function $f:(I,\tau_1)\to(J,\tau_2)$ by $f(m_1)=m_1$, $f(m_2)=m_2$, $f(m_3)=m_3$. Then, $f^{-1}(\{m_1\})=\{m_1\}$, $f^{-1}(\{m_2\})=\{m_2\}$, $f^{-1}(\{m_3\})=\{m_3\}$. Also, $f^{-1}(\{m_1,m_2\})=\{m_1,m_2\}$. Here, $\{m_1,m_2\}$ is a $\delta g\alpha$-closed set (CS) but not a pre-closed set (pre-CS) in (I,τ_1).

Theorem 3.26: Let (I,τ_1) and (J,τ_2) be any two topological spaces (2-TSs). A function $f:(I,\tau_1)\to(J,\tau_2)$ is $\delta g\alpha$-continuous if it is gp-continuous.

Proof: Let $f:(I,\tau_1)\to(J,\tau_2)$ be a $\delta g\alpha$-continuous map. Let L be a closed set (CS) in (J,τ_2). Since f is $\delta g\alpha$-continuous, $f^{-1}(L)$ is a $\delta g\alpha$-closed set (CS) in (I,τ_1). Since every $\delta g\alpha$-closed set is a gp-closed set (CS), it follows that $f^{-1}(L)$ is a gp-closed set in (I,τ_1).

$\therefore$ f is gp-continuous.

The following example illustrates that the converse of the above theorem need not be true.

Illustration 3.27: Let $I=\{f_1,f_2,f_3,f_4\}$ and $\tau_1=\{I,\emptyset,\{f_1\},\{f_2\},\{f_1,f_2\},\{f_1,f_3\},\{f_1,f_4\},\{f_1,f_2,f_3\},\{f_1,f_2,f_4\},\{f_1,f_3,f_4\}\}$. The closed sets (CSs) are $\{I,\emptyset,\{f_2,f_3,f_4\},\{f_1,f_3,f_4\},\{f_3,f_4\},\{f_2,f_4\},\{f_2,f_3\},\{f_4\},\{f_3\},\{f_2\}\}$. The $\delta g\alpha$-closed sets (CSs) are $\{I,\emptyset,\{f_1\},\{f_2\},\{f_3\},\{f_1,f_2,f_3\}\}$. The gp-closed sets (gp-CSs) are $\{I,\emptyset,\{f_1\},\{f_2\},\{f_3\},\{f_1,f_2\},\{f_1,f_3\},\{f_2,f_3\},\{f_1,f_2,f_3\}\}$. Let $J=\{f_1,f_2,f_3,f_4\}$ and $\tau_2=\{J,\emptyset,\{f_1\},\{f_2\},\{f_1,f_2\},\{f_1,f_2,f_3\}\}$. The closed sets (CSs) of (J,τ_2) are $\{\{f_2,f_3,f_4\},\{f_1,f_3,f_4\},\{f_3,f_4\},\{f_4\}\}$. Define a function $f:(I,\tau_1)\to(J,\tau_2)$ by $f(f_1)=f_2$, $f(f_2)=f_3$, $f(f_3)=f_4$, $f(f_4)=f_1$. Then, $f^{-1}(\{f_2\})=\{f_1\}$, $f^{-1}(\{f_3\})=\{f_2\}$, $f^{-1}(\{f_4\})=\{f_3\}$, $f^{-1}(\{f_1\})=\{f_4\}$. Also, $f^{-1}(\{f_3,f_4\})=\{f_2,f_3\}$. Here, $\{f_2,f_3\}$ is a gp-closed set (gp-CS) but not a $\delta g\alpha$-closed set (CS) in (I,τ_1).

Theorem 3.28: Let (I,τ_1) and (J,τ_2) be any two topological spaces (2-TSs). A function $f:(I,\tau_1)\to(J,\tau_2)$

is *gpr*-continuous if it is $\delta g\alpha$-continuous.

Proof: Let L be a closed set (CS) in (J,τ_2). Then $f^{-1}(L)$ is a *gpr*-closed set (gpr-CS) in (I,τ_1) since f is *gpr*-continuous. But every *gpr*-closed set is a $\delta g\alpha$-closed set (CS).

$\therefore$ $f^{-1}(L)$ is a $\delta g\alpha$-closed set (CS). Hence, f is $\delta g\alpha$-continuous.

The following example illustrates that the converse of the above theorem need not be true.

Illustration 3.29: Let $I=\{g_1,h_2,i_3,j_4\}$ and $\tau_1=\{I,\emptyset,\{g_1\},\{h_2\},\{g_1,h_2\},\{g_1,h_2,i_3\}\}$. The closed sets (CSs) are $\{I,\emptyset,\{h_2,i_3,j_4\},\{g_1,i_3,j_4\},\{i_3,j_4\},\{j_4\}\}$. The $\delta g\alpha$-closed sets (CSs) are $\{I,\emptyset,\{g_1\},\{i_3\}\}$. The *gpr*-closed sets (gpr-CSs) are $\{I,\emptyset,\{i_3\}\}$.

Let $J=\{g_1,h_2,i_3,j_4\}$ and $\tau_2=\{J,\emptyset,\{g_1\},\{h_2\},\{g_1,h_2\},\{g_1,h_2,i_3\}\}$. The closed sets (CSs) of (J,τ_2) are $\{\{h_2,i_3,j_4\},\{g_1,i_3,j_4\},\{i_3,j_4\},\{j_4\}\}$. Define a function $f:(I,\tau_1)\to(J,\tau_2)$ by $f(g_1)=j_4$, $f(h_2)=g_1$, $f(i_3)=h_2$, $f(j_4)=i_3$. Then, $f^{-1}(\{j_4\})=\{g_1\}$, $f^{-1}(\{g_1\})=\{h_2\}$, $f^{-1}(\{h_2\})=\{i_3\}$, $f^{-1}(\{i_3\})=\{j_4\}$. Also, $f^{-1}(\{j_4\})=\{g_1\}$. Here, $\{g_1\}$ is a $\delta g\alpha$-closed set (CS) but not a *gpr*-closed set (gpr-CS) in (I,τ_1).

Theorem 3.28: Let (I,τ_1) and (J,τ_2) be any two topological spaces (2-TSs). A function $f:(I,\tau_1)\to(J,\tau_2)$ is β-continuous if it is $\delta g\alpha$-continuous.

Proof: Let L be a closed set (CS) in (J,τ_2). Then $f^{-1}(L)$ is β-closed in (I,τ_1) since f is β-continuous. But every β-closed set (CS) is a $\delta g\alpha$-closed set (CS).

$\therefore$ $f^{-1}(L)$ is a $\delta g\alpha$-closed set (CS). Hence, f is $\delta g\alpha$-continuous.

The following example illustrates that the converse of the above theorem need not be true.

Illustration 3.29: Let $I=\{a_1,b_2,c_3\}$ and $\tau_1=\{I,\emptyset,\{a_1\},\{b_2\},\{a_1,b_2\}\}$. The closed sets (CSs) are $\{I,\emptyset,\{b_2,c_3\},\{a_1,c_3\},\{c_3\}\}$. The $\delta g\alpha$-closed sets (CSs) are $\{I,\emptyset,\{a_1\},\{b_2\},\{c_3\},\{a_1,b_2\},\{b_2,c_3\},\{a_1,c_3\}\}$. The β-closed sets (CSs) are $\{\{a_1\},\{b_2\},\{c_3\},\{b_2,c_3\},\{a_1,c_3\}\}$. Let $J=\{a_1,b_2,c_3\}$ and $\tau_2=\{J,\emptyset,\{a_1\},\{b_2\},\{a_1,b_2\},\{a_1,c_3\}\}$. The closed sets (CSs) of (J,τ_2) are $\{J,\emptyset,\{b_2,c_3\},\{a_1,c_3\},\{c_3\},\{b_2\}\}$. Define a function $f:(I,\tau_1)\to(J,\tau_2)$ by $f(a_1)=a_1$, $f(b_2)=c_3$, $f(c_3)=b_2$. Then, $f^{-1}(\{a_1\})=\{a_1\}$, $f^{-1}(\{c_3\})=\{b_2\}$, $f^{-1}(\{b_2\})=\{c_3\}$. Also, $f^{-1}(\{a_1,c_3\})=\{a_1,b_2\}$. Here, $\{a_1,b_2\}$ is a $\delta g\alpha$-closed set (CS) but not a β-closed set (CS) in (I,τ_1).

Result 3.30: The composition of two $\delta g\alpha$-continuous needs not always is a $\delta g\alpha$-continuous as seen from the following example.

Illustration 3.31: Let $I=\{u_1,v_2,w_3\}$ with $\tau_1=\{I,\emptyset,\{u_1\},\{v_2\},\{w_3\},\{u_1,v_2\},\{v_2,w_3\}\}$. The closed sets (CSs) are $\{\emptyset,I,\{u_1,w_3\},\{v_2,w_3\},\{u_1,v_2\},\{w_3\},\{u_1\}\}$. The $\delta g\alpha$-closed sets (CSs) are $\{\emptyset,I,\{u_1\},\{v_2\},\{v_2,w_3\}\}$. Let $J=\{u_1,v_2,w_3\}$ with $\tau_2=\{J,\emptyset,\{u_1\}\}$. The closed sets (CSs) are $\{\emptyset,J,\{v_2,w_3\}\}$. The $\delta g\alpha$-closed sets (CSs) are $\{J,\emptyset,\{u_1\},\{v_2\},\{w_3\},\{u_1,v_2\},\{v_2,w_3\},\{u_1,w_3\}\}$. Let $W=\{u_1,v_2,w_3\}$ with $\tau_3=\{W,\emptyset,\{u_1\},\{v_2\},\{u_1,v_2\},\{u_1,w_3\}\}$. The closed sets (CSs) are $\{\emptyset,W,\{v_2,w_3\},\{u_1,w_3\},\{w_3\},\{v_2\}\}$. The $\delta g\alpha$-closed sets (CSs) are $\{\emptyset,W,\{u_1\},\{w_3\},\{u_1,w_3\}\}$. Define two functions $f:(I,\tau_1)\to(J,\tau_2)$ and $g:(J,\tau_2)\to(W,\tau_3)$. By the above examples, it is clear that these two functions are $\delta g\alpha$-continuous.

However, their composition $(g\circ f):(I,\tau_1)\to(W,\tau_3)$ is not $\delta g\alpha$-continuous because for the closed set $\{u_1,w_3\}$ in (W,τ_3), $(g\circ f)^{-1}(\{u_1,w_3\})=f^{-1}(g^{-1}(\{u_1,w_3\}))=f^{-1}(\{u_1,w_3\})=\{u_1,w_3\}$ is not a $\delta g\alpha$-closed set (CS) in (I,τ_1). Hence, $(g\circ f):(I,\tau_1)\to(W,\tau_3)$ is not $\delta g\alpha$-continuous. Thus, the composition of two $\delta g\alpha$-continuous functions need not always be $\delta g\alpha$-continuous.

Theorem 3.32: Let $f:(I,\tau_1)\to(J,\tau_2)$ and $g:(J,\tau_2)\to(W,\tau_3)$ be two $\delta g\alpha$-continuous functions. Then their composition $g\circ f:(I,\tau_1)\to(W,\tau_3)$ is $\delta g\alpha$-continuous if (J,τ_2) belongs to the class of $T-\delta g\alpha$-TSs.

Proof: Let L be any closed set (CS) in (J,τ_2). Since $f:(I,\tau_1)\rightarrow(J,\tau_2)$ is $\delta g\alpha$-continuous, $f^{-1}(L)$ is a $\delta g\alpha$-CS in (I,τ_1). Let L be a CS in (W,τ_3). Since $g:(J,\tau_2)\rightarrow(W,\tau_3)$ is $\delta g\alpha$-continuous, $g^{-1}(L)$ is a $\delta g\alpha$-CS in (J,τ_2).

Now consider the composition $g\circ f:(I,\tau_1)\rightarrow(W,\tau_3)$. Let L be a CS in (W,τ_3). Then, $(g\circ f)^{-1}(L)=f^{-1}(g^{-1}(L))$, where $g^{-1}(L)$ is a $\delta g\alpha$-CS in (J,τ_2). Since (J,τ_2) is a $T-\delta g\alpha$-space, we have $g^{-1}(L)$ is a CS in (J,τ_2). Thus, $f^{-1}(g^{-1}(L))$ is a $\delta g\alpha$-CS in (I,τ_1), since f is $\delta g\alpha$-continuous. Hence, $g\circ f:(I,\tau_1)\rightarrow(W,\tau_3)$ is $\delta g\alpha$-continuous.

4. $\delta g\alpha$ irresolute functions

In this section, we introduce the new concepts of $\delta g\alpha$ irresolute functions in TSs and characterized some of their properties.

Definition 4.1: A function $f:(I,\tau_1)\rightarrow(J,\tau_2)$ is called $\delta g\alpha$ irresolute function if the inverse image of every $\delta g\alpha$ CS in (J,τ_2) is $\delta g\alpha$ CS in (I,τ_1).

Illustration 4.2: Let $I=\{n_1,m_2,l_3\}$, $\tau_1=\{I,\phi,\{n_1\},\{m_2\},\{l_3\},\{n_1,m_2\},\{m_2,l_3\}\}$ and CSs are $\{I,\phi,\{n_1,l_3\},\{m_2,l_3\},\{n_1,m_2\},\{l_3\},\{n_1\}\}$. $\delta g\alpha$ CSs are $\{I,\phi,\{n_1\},\{m_2\},\{m_2,l_3\}\}$. Let $J=\{n_1,m_2,l_3\}$, $\tau_2=\{J,\phi,\{n_1\}\}$. CSs are $\{J,\phi,\{m_2,l_3\}\}$. $\delta g\alpha$ CSs are $\{I,\phi,\{n_1\},\{m_2\},\{l_3\},\{n_1,m_2\},\{m_2,l_3\},\{n_1,l_3\}\}$. Define a function $f:(I,\tau_1)\rightarrow(J,\tau_2)$ by $f(n_1)=l_3$, $f(m_2)=n_1$, $f(l_3)=m_2$. Then $f^{-1}(\{n_1,m_2\})=\{m_2,l_3\}$ is $\delta g\alpha$ CS in (I,τ_1).

$\therefore f$ is a $\delta g\alpha$-irresolute function.

Theorem 4.3: If $f:(I,\tau_1)\rightarrow(J,\tau_2)$ is a $\delta g\alpha$-irresolute function, then f is $\delta g\alpha$ continuous, but not conversely.

Proof: Let $f:(I,\tau_1)\rightarrow(J,\tau_2)$ be a $\delta g\alpha$-irresolute function. Let L be a CS in (J,τ_2). Since every CS is a $\delta g\alpha$-CS and f is an irresolute map, it follows that $f^{-1}(L)$ is a $\delta g\alpha$-CS in (I,τ_1). This $\Longrightarrow$ that f is $\delta g\alpha$-continuous. Hence, every $\delta g\alpha$-irresolute map is a $\delta g\alpha$-continuous map.

Illustration 4.4: Let $I=\{r_1,s_2,t_3\}$, $\tau_1=\{I,\phi,\{r_1\},\{s_2\},\{r_1,s_2\},\{r_1,t_3\}\}$. CSs are $\{I,\phi,\{s_2,t_3\},\{r_1,t_3\},\{t_3\},\{s_2\}\}$. $\delta g\alpha$-CSs are $\{I,\phi,\{r_1\},\{t_3\},\{r_1,t_3\}\}$. Let $J=\{r_1,s_2,t_3\}$, $\tau_2=\{J,\phi,\{r_1\},\{s_2\},\{r_1,s_2\}\}$. CSs of J are $\{J,\phi,\{s_2,t_3\},\{r_1,t_3\},\{t_3\}\}$. $\delta g\alpha$-CSs are $\{J,\phi,\{r_1\},\{s_2\},\{t_3\},\{r_1,t_3\},\{s_2,t_3\},\{r_1,s_2\}\}$. Define a function $f:(I,\tau_1)\rightarrow(J,\tau_2)$ by $f(r_1)=t_3$, $f(s_2)=s_2$, $f(t_3)=r_1$. The function f is $\delta g\alpha$-continuous but not $\delta g\alpha$-irresolute. Then, $f^{-1}(\{r_1,s_2\})=\{s_2,t_3\}$ is not a $\delta g\alpha$-CS in (I,τ_1), indicating that f is not a $\delta g\alpha$-irresolute function.

Theorem 4.5: A map $f:(I,\tau_1)\rightarrow(J,\tau_2)$ is a $\delta g\alpha$-irresolute function if and only if $f^{-1}(L)$ is a $\delta g\alpha$-OS in (I,τ_1) for every $\delta g\alpha$-OS in (J,τ_2).

Proof: Let $f:(I,\tau_1)\rightarrow(J,\tau_2)$ be a $\delta g\alpha$-irresolute map. Let H be a $\delta g\alpha$-OS in (J,τ_2). Then $f^{-1}(H^c)$ is a $\delta g\alpha$-CS in (I,τ_1). But $f^{-1}(H^c)=(f^{-1}(L))^c$ and so $f^{-1}(L)$ is a $\delta g\alpha$-OS in (I,τ_1). The converse follows similarly.

Theorem 4.6: Let $f:(I,\tau_1)\rightarrow(J,\tau_2)$ and $g:(J,\tau_1)\rightarrow(W,\tau_2)$ be any two maps. Then:

1. $(g\circ f)$ is a $\delta g\alpha$-irresolute function if both f and g are $\delta g\alpha$-irresolute functions.

2. $(g\circ f)$ is a $\delta g\alpha$-continuous function if g is $\delta g\alpha$-continuous and f is $\delta g\alpha$-irresolute.

Proof:

1. Let F be a $\delta g\alpha$-CS in (W,τ_2). Since g is a $\delta g\alpha$-irresolute map, $g^{-1}(L)$ is a $\delta g\alpha$-CS in (J,τ_2). Since f is $\delta g\alpha$-irresolute,
$$f^{-1}(g^{-1}(L)) = (g\circ f)^{-1}(L)$$
is a $\delta g\alpha$-CS in (I,τ_1). Thus, $(g\circ f)$ is a $\delta g\alpha$-irresolute function.

2. Let F be a $\delta g\alpha$-CS in (W,τ_2). Since g is $\delta g\alpha$-continuous,
$$g^{-1}(L) \text{ is a } \delta g\alpha\text{-CS in } (J,\tau_2).$$
Since f is $\delta g\alpha$-irresolute,
$$f^{-1}(g^{-1}(L)) = (g\circ f)^{-1}(L)$$
is a $\delta g\alpha$-CS in (I,τ_1). Thus, $(g\circ f)$ is a $\delta g\alpha$-continuous function.

5. Conclusion

In this study, we present the notion of $\delta g\alpha$-continuous maps in TSs and investigate their connections to well-known continuous map types. By defining this new class of continuous maps, we aim to enhance the understanding of continuity in topology and highlight connections to existing frameworks. Our investigation provides insights into the properties and implications of $\delta g\alpha$-continuity, contributing to the broader discourse in the field.

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