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Two contributions to the absolute-value-Hardy-Hilbert-type integral inequalities

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Abstract: This article presents two new Hardy-Hilbert-type integral inequalities involving the absolute value function. The first result can be viewed as an analog of a well-known inequality of this type, while the second is distinguished by its originality, incorporating the sine function into the integrand. Complete and detailed proofs are provided for both results.

Key words: Hardy-Hilbert integral inequality, absolute value function, trigonometric functions, change of variables technique.

1. Introduction

The Hardy-Hilbert integral inequality is one of the most celebrated results in mathematical analysis. Its precise formulation is given below. Let $p > 1$, $q = p/(p-1)$ such that $1/p + 1/q = 1$, and $f, g : (0, +\infty) \to (0, +\infty)$ be two (measurable) functions such that

$$\int_0^{+\infty} f^p(x)dx < +\infty, \quad \int_0^{+\infty} g^q(x)dx < +\infty.$$

Then we have

$$\int_0^{+\infty}\int_0^{+\infty} \frac{1}{x+y} f(x)g(y)dxdy \leq \frac{\pi}{\sin(\pi/p)} \left[\int_0^{+\infty} f^p(x)dx\right]^{1/p} \left[\int_0^{+\infty} g^q(x)dx\right]^{1/q}. \tag{1}$$

This inequality is known to be sharp. More details and historical developments can be found in [11, 12, 31]. The study of Hardy-Hilbert-type integral inequalities continues to stimulate research. Results concerning the two-dimensional case can be found in [1, 3, 4, 6–10, 16, 20, 24–26, 28–30], while extensions to higher dimensions can be found in [2, 5, 13–15, 17, 18, 21–23, 27, 32, 33].

For the purposes of this article, our focus is on a specific type of Hardy-Hilbert integral inequality involving absolute values, as presented in [19, Corollary 3]. It is stated formally below. Let $\lambda \in (0, 1)$, $p > 1$, $q = p/(p-1)$ and $f, g : (0, +\infty) \to (0, +\infty)$ be two functions such that

$$\int_0^{+\infty} f^p(x)x^{(1-\lambda/2)p-1}dx < +\infty, \quad \int_0^{+\infty} g^q(x)x^{(1-\lambda/2)q-1}dx < +\infty.$$

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Then we have

$$\int_0^{+\infty}\int_0^{+\infty}\frac{1}{|x-y|^{\lambda}}f(x)g(y)dxdy$$
$$\leq 2B\left(\frac{\lambda}{2},1-\lambda\right)\left[\int_0^{+\infty}f^p(x)x^{(1-\lambda/2)p-1}dx\right]^{1/p}\left[\int_0^{+\infty}g^q(x)x^{(1-\lambda/2)q-1}dx\right]^{1/q}, \tag{2}$$

where $B(a,b)=\int_0^1 t^{a-1}(1-t)^{b-1}dt,\ a,b>0$, is the beta function at a and b. Compared with the classical Hardy-Hilbert integral inequality, this result exhibits a more intricate structure: the absolute value function lies at the core of the integrand, the constant factor depends on the beta function, and the norms of f and g are weighted by powers of x. On this basis, several functional generalizations and parameter-dependent extensions have been developed in [19].

In this article, we extend and complement the main results in [19]. In particular, we examine two new absolute-value-Hardy-Hilbert-type integral inequalities that are, in various ways, closely related to the inequality in Equation (2). The first inequality concerns a double integral of the form

$$\int_0^{+\infty}\int_0^{+\infty}\frac{1}{|1-xy|^{\lambda}}f(x)g(y)dxdy,$$

where $f,g:(0,+\infty)\to(0,+\infty)$ are functions that satisfy certain conditions of integrability. This inequality may be regarded as an analog of Equation (2), as it involves a similar integrand depending on the product xy rather than the difference $|x-y|$. The second inequality is associated with the following double integral:

$$\int_0^{\pi/2}\int_0^{\pi/2}\frac{1}{|\sin(x-y)|^{\lambda}}f(x)g(y)dxdy,$$

where $f,g:(0,\pi/2)\to(0,+\infty)$ are functions that satisfy certain conditions of integrability. The corresponding proof essentially relies on the inequality in Equation (2), combined with an appropriate change of variables and the use of trigonometric identities. These two results thus connect several well-known Hardy-Hilbert-type integral inequalities via common analytical techniques. This framework enables further generalizations and could form the basis for future research on weighted integral inequalities.

The rest of the article is as follows: The two main theorems are stated and proved in Sections 2 and 3, respectively. The conclusion is given in Section 4.

2. First theorem

Our first main integral inequality is presented in the theorem below.

Theorem 2.1. *Let $\lambda\in(0,1)$, $p>1$, $q=p/(p-1)$ and $f,g:(0,+\infty)\to(0,+\infty)$ be two functions such that*

$$\int_0^{+\infty}f^p(x)x^{(1-\lambda/2)p-1}dx<+\infty,\quad \int_0^{+\infty}g^q(x)x^{(1-\lambda/2)q-1}dx<+\infty.$$

Then we have

$$\int_0^{+\infty}\int_0^{+\infty}\frac{1}{|1-xy|^{\lambda}}f(x)g(y)dxdy$$
$$\leq 2B\left(\frac{\lambda}{2},1-\lambda\right)\left[\int_0^{+\infty}f^p(x)x^{(1-\lambda/2)p-1}dx\right]^{1/p}\left[\int_0^{+\infty}g^q(x)x^{(1-\lambda/2)q-1}dx\right]^{1/q}.$$

Proof. A suitable decomposition of the integrand using the identity $1/p + 1/q = 1$ and the Hölder integral inequality give

$$\int_0^{+\infty}\int_0^{+\infty} \frac{1}{|1-xy|^{\lambda}} f(x)g(y)dxdy$$
$$= \int_0^{+\infty}\int_0^{+\infty} \frac{y^{(\lambda/2-1)/p}x^{-(\lambda/2-1)/q}}{|1-xy|^{\lambda/p}} f(x) \times \frac{y^{-(\lambda/2-1)/p}x^{(\lambda/2-1)/q}}{|1-xy|^{\lambda/q}} g(y)dxdy$$
$$\leq \left[\int_0^{+\infty}\int_0^{+\infty} \frac{y^{\lambda/2-1}x^{-(\lambda/2-1)p/q}}{|1-xy|^{\lambda}} f^p(x)dxdy\right]^{1/p}$$
$$\times \left[\int_0^{+\infty}\int_0^{+\infty} \frac{y^{-(\lambda/2-1)q/p}x^{\lambda/2-1}}{|1-xy|^{\lambda}} g^q(y)dxdy\right]^{1/q}. \tag{3}$$

Let us determine each of the integrals of this upper bound. By the Fubini-Tonelli integral theorem, we can write

$$\int_0^{+\infty}\int_0^{+\infty} \frac{y^{\lambda/2-1}x^{-(\lambda/2-1)p/q}}{|1-xy|^{\lambda}} f^p(x)dxdy$$
$$= \int_0^{+\infty} f^p(x)x^{-(\lambda/2-1)p/q}\left[\int_0^{+\infty} \frac{y^{\lambda/2-1}}{|1-xy|^{\lambda}}dy\right]dx.$$

Let us now focus on the central integral. Making the change of variables $t = xy$, using the Chasles integral relation, making the change of variables $t = 1/z$ and recognizing the beta function, we get

$$\int_0^{+\infty} \frac{y^{\lambda/2-1}}{|1-xy|^{\lambda}}dy = x^{-\lambda/2}\int_0^{+\infty} \frac{(xy)^{\lambda/2-1}}{|1-xy|^{\lambda}}xdy = x^{-\lambda/2}\int_0^{+\infty} \frac{t^{\lambda/2-1}}{|1-t|^{\lambda}}dt$$
$$= x^{-\lambda/2}\left[\int_0^1 \frac{t^{\lambda/2-1}}{(1-t)^{\lambda}}dt + \int_1^{+\infty} \frac{t^{\lambda/2-1}}{(t-1)^{\lambda}}dt\right]$$
$$= x^{-\lambda/2}\left[\int_0^1 t^{\lambda/2-1}(1-t)^{(1-\lambda)-1}dt + \int_1^0 \frac{(1/z)^{\lambda/2-1}}{(1/z-1)^{\lambda}}\left(-\frac{1}{z^2}dz\right)\right]$$
$$= x^{-\lambda/2}\left[\int_0^1 t^{\lambda/2-1}(1-t)^{(1-\lambda)-1}dt + \int_0^1 z^{\lambda/2-1}(1-z)^{(1-\lambda)-1}dz\right]$$
$$= 2B\left(\frac{\lambda}{2}, 1-\lambda\right)x^{-\lambda/2}.$$

Therefore, we have

$$\int_0^{+\infty}\int_0^{+\infty} \frac{y^{\lambda/2-1}x^{-(\lambda/2-1)p/q}}{|1-xy|^{\lambda}} f^p(x)dxdy$$
$$= \int_0^{+\infty} f^p(x)x^{-(\lambda/2-1)p/q}2B\left(\frac{\lambda}{2}, 1-\lambda\right)x^{-\lambda/2}dx$$
$$= 2B\left(\frac{\lambda}{2}, 1-\lambda\right)\int_0^{+\infty} f^p(x)x^{-(\lambda/2-1)(p-1)}x^{-\lambda/2}dx$$
$$= 2B\left(\frac{\lambda}{2}, 1-\lambda\right)\int_0^{+\infty} f^p(x)x^{(1-\lambda/2)p-1}dx. \tag{4}$$

Similarly, we obtain

$$\begin{aligned}
&\int_0^{+\infty}\int_0^{+\infty}\frac{y^{-(\lambda/2-1)q/p}x^{\lambda/2-1}}{|1-xy|^{\lambda}}g^q(y)dxdy\\
&=\int_0^{+\infty}g^q(y)y^{-(\lambda/2-1)q/p}\left[\int_0^{+\infty}\frac{x^{\lambda/2-1}}{|1-xy|^{\lambda}}dx\right]dy\\
&=2B\left(\frac{\lambda}{2},1-\lambda\right)\int_0^{+\infty}g^q(y)y^{-(\lambda/2-1)(q-1)}y^{-\lambda/2}dy\\
&=2B\left(\frac{\lambda}{2},1-\lambda\right)\int_0^{+\infty}g^q(y)y^{(1-\lambda/2)q-1}dy\\
&=2B\left(\frac{\lambda}{2},1-\lambda\right)\int_0^{+\infty}g^q(x)x^{(1-\lambda/2)q-1}dx. \qquad (5)
\end{aligned}$$

It follows from Equations (3), (4) and (5), and the identity $1/p+1/q=1$, that

$$\begin{aligned}
&\int_0^{+\infty}\int_0^{+\infty}\frac{1}{|1-xy|^{\lambda}}f(x)g(y)dxdy\\
&\le\left[2B\left(\frac{\lambda}{2},1-\lambda\right)\int_0^{+\infty}f^p(x)x^{(1-\lambda/2)p-1}dx\right]^{1/p}\\
&\times\left[2B\left(\frac{\lambda}{2},1-\lambda\right)\int_0^{+\infty}g^q(x)x^{(1-\lambda/2)q-1}dx\right]^{1/q}\\
&=2B\left(\frac{\lambda}{2},1-\lambda\right)\left[\int_0^{+\infty}f^p(x)x^{(1-\lambda/2)p-1}dx\right]^{1/p}\left[\int_0^{+\infty}g^q(x)x^{(1-\lambda/2)q-1}dx\right]^{1/q}.
\end{aligned}$$

This completes the proof. □

The upper bound obtained here coincides with that presented in Equation (2). Consequently, Theorem 2.1 can be considered a meaningful analog of this classical result, extending its applicability to a new framework of integrands. Moreover, the proof is notable for its clarity, relying on precise analytical arguments that emphasize the underlying structure of the inequality.

3. Second theorem

Our second main integral inequality is of a more original nature, involving the sine function within the integrand. It is stated in the theorem below.

Theorem 3.1. *Let $\lambda\in(0,1)$, $p>1$, $q=p/(p-1)$ and $f,g:(0,\pi/2)\to(0,+\infty)$ be two functions such that*

$$\int_0^{\pi/2}f^p(x)[\sin(2x)]^{(1-\lambda/2)p-1}dx<+\infty,\quad \int_0^{\pi/2}g^q(x)[\sin(2x)]^{(1-\lambda/2)q-1}dx<+\infty.$$

Then we have

$$\int_0^{\pi/2}\int_0^{\pi/2}\frac{1}{|\sin(x-y)|^{\lambda}}f(x)g(y)dxdy$$
$$\leq 2^{\lambda}B\left(\frac{\lambda}{2},1-\lambda\right)\left[\int_0^{\pi/2}f^p(x)[\sin(2x)]^{(1-\lambda/2)p-1}dx\right]^{1/p}\left[\int_0^{\pi/2}g^q(x)[\sin(2x)]^{(1-\lambda/2)q-1}dx\right]^{1/q}.$$

Proof. We begin by reformulating the double integral via the change of variables

$$u=\tan(x),\quad v=\tan(y).$$

Then we have $x=\arctan(u)$, $y=\arctan(v)$,

$$dx=\frac{1}{1+u^2}du,\quad dy=\frac{1}{1+v^2}dv,$$

and the limits $x=0$ when $u=0$, $y=0$ when $v=0$, $x=\pi/2$ when $u\to+\infty$ and $y=\pi/2$ when $v\to+\infty$. Furthermore, standard trigonometric formulas give

$$\sin(x-y)=\sin(x)\cos(y)-\cos(x)\sin(y)$$
$$=\frac{u}{\sqrt{1+u^2}}\times\frac{1}{\sqrt{1+v^2}}-\frac{1}{\sqrt{1+u^2}}\times\frac{v}{\sqrt{1+v^2}}$$
$$=\frac{u-v}{\sqrt{1+u^2}\sqrt{1+v^2}}.$$

Therefore, we get

$$\int_0^{\pi/2}\int_0^{\pi/2}\frac{1}{|\sin(x-y)|^{\lambda}}f(x)g(y)dxdy$$
$$=\int_0^{+\infty}\int_0^{+\infty}\frac{(1+u^2)^{\lambda/2}(1+v^2)^{\lambda/2}}{|u-v|^{\lambda}}f(\arctan(u))g(\arctan(v))\left(\frac{1}{1+u^2}\times\frac{1}{1+v^2}dudv\right)$$
$$=\int_0^{+\infty}\int_0^{+\infty}\frac{1}{|u-v|^{\lambda}}f(\arctan(u))(1+u^2)^{\lambda/2-1}g(\arctan(v))(1+v^2)^{\lambda/2-1}dudv$$
$$=\int_0^{+\infty}\int_0^{+\infty}\frac{1}{|u-v|^{\lambda}}f_{\dagger}(u)g_{\dagger}(v)dudv, \tag{6}$$

where

$$f_{\dagger}(u)=f(\arctan(u))(1+u^2)^{\lambda/2-1},\quad g_{\dagger}(v)=g(\arctan(v))(1+v^2)^{\lambda/2-1}.$$

Applying the inequality in Equation (2) to the functions $f_{\dagger}$ and $g_{\dagger}$, we obtain

$$\int_0^{+\infty}\int_0^{+\infty}\frac{1}{|u-v|^{\lambda}}f_{\dagger}(u)g_{\dagger}(v)dudv$$
$$\leq 2B\left(\frac{\lambda}{2},1-\lambda\right)\left[\int_0^{+\infty}f_{\dagger}^p(u)u^{(1-\lambda/2)p-1}du\right]^{1/p}\left[\int_0^{+\infty}g_{\dagger}^q(v)v^{(1-\lambda/2)q-1}dv\right]^{1/q}. \tag{7}$$

Let us examine the first integral of the upper bound. We have

$$\int_0^{+\infty} f_\dagger^p(u)u^{(1-\lambda/2)p-1}du = \int_0^{+\infty} f^p(\arctan(u))u^{(1-\lambda/2)p-1}(1+u^2)^{(\lambda/2-1)p}du$$

$$= \int_0^{+\infty} f^p(\arctan(u))u^{(1-\lambda/2)p-1}(1+u^2)^{(\lambda/2-1)p+1}\left(\frac{1}{1+u^2}du\right)$$

$$= \int_0^{+\infty} f^p(\arctan(u))\left(\frac{u}{1+u^2}\right)^{(1-\lambda/2)p-1}\left(\frac{1}{1+u^2}du\right).$$

We consider the change of variables $u = \tan(x)$. Then we have $x = \arctan(u)$, $dx = 1/(1+u^2)du$, $x = 0$ when $u = 0$, $x = \pi/2$ when $u \to +\infty$ and

$$\frac{u}{1+u^2} = \sin(x)\cos(x) = \frac{1}{2}\sin(2x). \tag{8}$$

Therefore, we get

$$\int_0^{+\infty} f_\dagger^p(u)u^{(1-\lambda/2)p-1}du = \frac{1}{2^{(1-\lambda/2)p-1}}\int_0^{\pi/2} f^p(x)[\sin(2x)]^{(1-\lambda/2)p-1}dx.$$

Similarly, but with the change of variables $v = \tan(x)$, we get

$$\int_0^{+\infty} g_\dagger^q(v)v^{(1-\lambda/2)q-1}dv = \int_0^{+\infty} g^q(\arctan(v))v^{(1-\lambda/2)q-1}(1+v^2)^{(\lambda/2-1)q}dv$$

$$= \int_0^{+\infty} g^q(\arctan(v))\left(\frac{v}{1+v^2}\right)^{(1-\lambda/2)q-1}\left(\frac{1}{1+v^2}dv\right)$$

$$= \frac{1}{2^{(1-\lambda/2)q-1}}\int_0^{\pi/2} g^q(x)[\sin(2x)]^{(1-\lambda/2)q-1}dx. \tag{9}$$

It follows from Equations (6), (7), (8) and (9), and the identity $1/p + 1/q = 1$, that

$$\int_0^{\pi/2}\int_0^{\pi/2} \frac{1}{|\sin(x-y)|^\lambda} f(x)g(y)dxdy$$

$$\leq 2B\left(\frac{\lambda}{2}, 1-\lambda\right)\left[\frac{1}{2^{(1-\lambda/2)p-1}}\int_0^{\pi/2} f^p(x)[\sin(2x)]^{(1-\lambda/2)p-1}dx\right]^{1/p}$$

$$\times \left[\frac{1}{2^{(1-\lambda/2)q-1}}\int_0^{\pi/2} g^q(x)[\sin(2x)]^{(1-\lambda/2)q-1}dx\right]^{1/q}$$

$$= \frac{2}{2^{(1-\lambda/2)-1/p}2^{(1-\lambda/2)-1/q}}B\left(\frac{\lambda}{2}, 1-\lambda\right)\left[\int_0^{\pi/2} f^p(x)[\sin(2x)]^{(1-\lambda/2)p-1}dx\right]^{1/p}$$

$$\times \left[\int_0^{\pi/2} g^q(x)[\sin(2x)]^{(1-\lambda/2)q-1}dx\right]^{1/q}$$

$$= 2^\lambda B\left(\frac{\lambda}{2}, 1-\lambda\right)\left[\int_0^{\pi/2} f^p(x)[\sin(2x)]^{(1-\lambda/2)p-1}dx\right]^{1/p}\left[\int_0^{\pi/2} g^q(x)[\sin(2x)]^{(1-\lambda/2)q-1}dx\right]^{1/q}.$$

This completes the proof. □

The inequality in Equation (2) thus forms the basis of the proof. The argument is concise, relying on a carefully chosen change of variables and precise analytical developments. To the best of our knowledge, Theorem 3.1 represents one of the few Hardy-Hilbert-type integral inequalities in which the sine function is the main component of the integrand. This highlights a distinctive contribution to the theory.

4. Conclusion

In conclusion, we have derived two absolute-value-Hardy-Hilbert-type integral inequalities that extend the findings of [19]. They also reveal new connections between product- and trigonometric-based integrals. Furthermore, the obtained constants are sharp by construction. The methods developed offer a versatile framework for deriving additional weighted and multidimensional integral inequalities. These results suggest promising avenues for future research in functional analysis and inequality theory, as well as their applications in various fields.

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