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Study on fixed point results for G-Expansion onto mappings on CCM-spaces

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Abstract: In this paper, we obtain a fixed point result for G(Generalized)- Expansion onto self- mappings on CCM(Complete Cone Metric)-Spaces. This result is an extension and improved result of the some of the existing results in this literature.

Key words: Cone metric space, continuous mapping fixed point
AMS subject classifications: 47H10, 54H25.

1. Introduction and Preliminaries

Cone metric space was introduced by the mathematicians Huang and Zhang [7]. They obtained some fixed point results in cone metric space. Later on many authors were inspired by these results, they have been extending these results in different dimensions (see for e.g. [1-15]). In recent developments in this area for non explosive map in cone metric spaces (see for e.g. [1-6], [8-11]). Recently, Aaage and Salunke [6] proved some fixed point theorems for expansion onto mappings on cone metric spaces. In this paper, we obtained affixed point theorem for expansion mappings on cone metric spaces. The following ar useful in our main results which are due to [7].

1.1. Sections and subsections

Definition 1.1. Let B be a real Banach space and Q a subset of B. Q is called a cone if and only if:

(a). Q is closed, non empty, and $Q \neq 0$.
(b). $\alpha, \beta \in R$, $\alpha, \beta \geq 0$, $x, y \in Q$ *implies* $\alpha x + \beta y \in Q$.
(c). $x \in Q$ and $-x \in Q$ implies $x = 0$.

Definition 1.2. Given a cone $Q \subset B$, we define a partial ordering $\leq$ with respect to Q by : $x \leq y$ iff $y - x \in Q$. We shall write $x < y$ to indicate that $x \leq y$ but $x \neq y$, while $x << y$ will stand for $y - x \in$ interior of Q.

Definition 1.3. The cone Q is called normal if there is a number $M > 0$ such that for all $x, y \in B$, $0 \leq x \leq y$ implies $\|x\| \leq M\|y\|$, where $\|.\|$ is the norm in B. In this case the number M is called the normal constant of Q.

Definition 1.4. Let X be a non- empty set. Suppose the $\rho : X \longrightarrow X$ satisfies the following:

(a). $\rho(x, y) > 0$ for all $x, y \in X$ and $\rho(x, y) = 0$ if and only if $x = y$.
(b). $\rho(x, y) = \rho(y, x)$, for all $x, y \in X$.
(c). $\rho(x, y) \leq \rho(x, z) + \rho(z, y)$, for all $x, y, z \in X$.

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Then ρ is called a cone metric on X, and (X, ρ) is called a cone metric space.

Definition 1.5. Let (X, ρ) be a cone metric space. Let (x_n) be a sequence in X and $x \in X$. If for every $d \in M$, $d >> 0$ there is N such that for all $n > N$, $\rho(x_n, x) << d$, then (x_n) is said to be convergent to x and x is the limit of (x_n). We denote this $x_n \longrightarrow x$ as $n \longrightarrow \infty$.

Definition 1.6. Let (X, ρ) be a cone metric space. Let (x_n) be a sequence in X and $x \in X$. If for every $d \in M$, $d >> 0$ there is N such that for all $n, m > N$, $\rho(x_n, x_m) << d$, then (x_n) is said to be Cauchy sequence in X.

Definition 1.7. Let (X, ρ) be a cone metric space. If every Cauchy sequence is convergent, then X is called a CCM(Complete Cone Metric)-space.

2. Main Result

Theorem 2.1. *Let (X, ρ) be a CCM-Space and the mapping $A : X \longrightarrow X$ is continuous onto and satisfies the generalized contractive condition:*

$$\rho(Ax, Ay) \leq \alpha\rho(x, y) + \beta[\rho(x, Ax) + \rho(y, Ay)] + \gamma[\rho(x, Ay) + \rho(y, Ax)]. \tag{1}$$

For all $x, y \in X$, where $\alpha, \beta, \gamma \geq 0$ and $\frac{1}{2} < \frac{\alpha}{2} + \beta < 1$ is constant. Then A has a fixed point in X.

Proof. For each $x_0 \in X$. Sine A is onto there exists $x_1 \in X$ such that $x_0 = Ax_1$ similarly for each $n \geq 1$ there exists $x_{n+1} \in X$ such that $x_n = Ax_{n+1}$. If $x_{n-1} = x_n$ then x_n is a fixed point of A. Thus we suppose that $x_{n-1} \neq x_n$ for all $n \geq 1$. Then by (1) we have

$$\begin{aligned}
\rho(x_n, x_{n-1}) &= \rho(Ax_{n+1}, Ax_n), \\
&\geq \alpha\rho(x_{n+1}, x_n) + \beta[\rho(x_{n+1}, Ax_{n+1}) + \rho(x_n, Ax_n)] + \gamma[\rho(x_{n+1}, Axn) + \rho(x_n, Ax_{n+1})], \\
&= \alpha\rho(x_{n+1}, x_n) + \beta[\rho(x_{n+1}, x_n) + \rho(x_n, x_{n-1})] + \gamma[\rho(x_{n+1}, x_{n-1}) + \rho(x_n, x_n)], \\
&\geq (\alpha + \beta + \gamma)\rho(x_n, x_{n+1}) + (\beta + \gamma)\rho(x_{n-1}, x_n), \\
\rho(x_n, x_{n-1}) &\leq \frac{1 - (\beta + \gamma)}{\alpha + \beta + \gamma}\rho(x_{n-1}, x_n), \\
&\leq k\rho(x_{n-1}, x_n),
\end{aligned}$$

where, $k = \frac{1-(\beta+\gamma)}{\alpha+\beta+\gamma} < 1$ *and*$0 \ \leq k < 1$.

From this, we get that

$$\rho(x_n, x_{n-1}) \leq k^n\rho(x_0, x_1),$$

where $k = \frac{1-(\beta+\gamma)}{\alpha+\beta+\gamma} < 1$ *and* $0 \leq k < 1$.

Now for $n > m$, we have

$$\rho(x_n, x_m) \leq \rho(x_n, x_{n+1}) + \rho(x_{n+1}, x_{n+2}) + \ldots + \rho(x_{m-1}, x_m),$$
$$\leq (k^n + k^{n+1} + \ldots + k^{m-1})\rho(x_0, x_1),$$
$$\leq \frac{k^n}{1-k}\rho(x_0, x_1).$$

Let $0 \leq e$ be given, Choose a natural number N_1 such that $\frac{k^n}{1-k}\rho(x_0, x_1) \leq e$, for all $n \geq N_1$.

Thus, $\rho(x_n, x_m) \leq e$ for $n > m$, Therefore, x_n, $n \geq 1$ is a Cauchy sequence in (X, ρ), Since (X, ρ) is a CCM-Space there exists $x^* \in X$ such that $x_n \longrightarrow x^*$ *as* $n \longrightarrow \infty$, Since, A is continuous then $\rho(Ax^*, x^*) \leq \rho(Ax_n, Ax^*) + \rho(Ax^*, x^*) \longrightarrow 0$, as $n \longrightarrow \infty$, Since $x_n \longrightarrow x^*$ and $Ax_n \longrightarrow Ax^*$, As $n \longrightarrow \infty$, Therefore, $\rho(Ax^*, x^*) = 0$ and so $Ax^* = x^*$, Then A has a fixed point in X, This completes the proof of the theorem. □

Remark 2.1. *If we take $\alpha = 0$ in the above theorem we get the Theorem 2.3, of [6].*

3. Conclusion

Our results are more general than the results of [6]

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